Parallel Preconditioners for the Incompressible Navier-Stokes Equations

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A family of pressure convection–diffusion preconditioners is considered for the solution of linear systems arising from the incompressible Navier-Stokes equations. The matrices have the form

$$\begin{pmatrix} F & B^T \\ B & C \end{pmatrix} \tag{1}$$

where B^T is a discrete gradient, B is a divergence, F contains terms coming from the linearized momentum equation, and C may be zero or may be a stabilization operator. Solution of these systems is often a significant bottleneck within implicit fluid simulations as they must be resolved within every time step and/or every nonlinear iteration. New methods are proposed to address challenges (user requirements, stabilization, and boundary conditions) associated with the original methods of Kay, Loghin, Elman, Silvester, and Wathen. Results are given illustrating the computational advantages within several fluid applications.

Pressure convection-diffusion preconditioners are based on block factorization ideas where the key computational obstacle is to accurately and inexpensively approximate the inverse Schur complement. A relatively inexpensive approximation is constructed based on commuting properties which essentially allow a differential operator associated with F to be approximately commuted with a gradient (or divergence) operator. In the original pressure convection–diffusion preconditioner a user is required to construct a convection-diffusion operator associated with F on the pressure space. It has been demonstrated that appropriate choices for this operator lead to a an effective preconditioner within a GMRES iteration.

We show how an algebraic method can also be developed based on ideas of differential commuting. A key element of this algebraic method is to make the distinction between discrete operators and differential operators. The algebraic method often converges noticeably faster than the original method and does not require users to define a somewhat awkward convection-diffusion operator on the pressure space. Special attention needs to be given to discretizations requiring stabilization ($C \neq 0$). In this case, techniques can be used to stabilize the preconditioner which are similar to those used to stabilize discretizations. One significant issue which has been a source of some confusion is that of boundary conditions. When deriving the preconditioners, the differential operators that one considers do not have any imposed boundary conditions. We have recently shown that it is possible to extend a notion of commuting to operators defined with boundary conditions. This leads to a new set of preconditioning boundary conditions for the original pressure convection–diffusion method as well as a slight modification to the algebraic method. We show that significant convergence improvements are achieved by paying careful attention to these boundary conditions.

Parallel results are given for several benchmark problems including one microfluidic mixing application.

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