Multiscale Study of Biosuspensions: Homogenization of Biofilament Networks Dilute Bacterial Suspensions

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We study macroscopic behavior arising from interactions of large numbers of components in suspensions of biofilaments interacting by means of molecular motors, as well as suspensions of self-propelled elongated bacteria. Both systems have exhibited transitions from disorder to a state with stable large-scale patterns, such as whorls, jets and vortices. While pattern formation of this kind has been studied for a considerable time, we are still far from a understanding of the underlying mechanisms of pattern formation and the their control methods. While this work represents fundamental science research, it is expected to have a wide range of technological implications, including the design of new materials with hitherto unacheavable properties.

The systems we study can be abstractly viewed as collectiions of interacting active *rods*, subject to thermal effects of the environment. The main tool of investigation of the patterns, including their onset and stability, is the derivation of a master equation characterizing the detailed state of the system, including the local density and orientation of rods. The internal activity of the rods competes with the diffusive thermal noise of the environment so the relative strength of the two processes results in the observable patterns.

In the case of bacterial suspensions, hydrodynamic interactions between the rods (bacteria) result in local alignment and an effective reduction in viscosity of the suspension – the result that was for the first time quantified in our recent works. Systems of biofilaments (polymer proteins, such as microtubules or actin filaments) exhibit a new kind of stochasticity, resulting from the probabilistic nature of the molecular motor action, giving rise to a new *active* multiplicative noise possessing many pecular features, including the ability to saturate. The competition of the two kinds of noise gives rise to a rich set of achievable patterns, which we characterize using a range of techniques. The resulting behavior of the filament matrix may lead to the explanation of the elastic properties of the cellular cytoskeleton and the cell wall dynamics.

This research gives rise to a range of applied mathematical tools, expanding the classical homogenization theory and enriching applied stochastic analysis. We will review the new mathematical approaches arising from this research, illustrate their use and the results achieved with the two systems of main interest, and discuss new potential avenues for the applications of these mathematical methods.

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