Multiscale Mathematics for Magnetic Fusion Plasmas*

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Using the mixed representations of Lagrangian and Eulerian coordinates, the magnetic fusion community, over the years, have developed a powerful numerical tool for solving the governing Vlasov-Maxwell equations, called the Particle-In-Cell (PIC) method, see for example, [Dawson, 1983]. To mitigate the intrinsic multiscale nature of these equations, for both time and space, the gyrokinetic model was developed to simplify the physics of these equations by replacing the spiral gyromotion of a particle in the magnetic field by a rotating rigid ring [Lee, 1983; Lee, 1987]. In doing so, orders of magnitude improvement were obtained in terms of time steps and grid spacings when carrying out PIC simulations. Furthermore, to reduce the intrinsic numerical noise due to the use of finite number of particles in the simulation, the perturbative PIC method was developed, [Parker and Lee, 1993]. These techniques are now standards in present fusion research codes, for example, the GTS code [W. X. Wang, 2006]. The multiscale mathematics involved will be discussed.

Recently, a new high frequency gyrokinetic mathematical model using a rubber-band-like Kruskal ring to replace the rigid ring has enabled us to venture into the physics of ion cyclotron waves [Kolesnikov, 2007; Kolesnikov, 2008], which we will describe. Moreover, in the process of implementing Alfven physics in GTS, we have also encountered the numerical difficulties associated with the Poisson's equation of the type,

$$\nabla^2\psi - \psi/\delta_e^2 = -J_e, \tag{1}$$

in the regime of interest of $k^2\delta_e^2 \ll 1$, where ψ is related to the vector potential, δ_e the electron skin depth and J_e the electron current. Another interesting numerical problem comes from the solutions of Poisson's equation in the presence of short wavelength small amplitude perturbations as well as the large amplitude long wavelength zonal flows. Thus, for the GTS code, we are solving the equation of the form of

$$\nabla_\perp^2\phi - [\phi - \langle\phi\rangle] = \delta n_i, \tag{2}$$

where $\langle\phi\rangle$ is the flux-surface average and $\perp$ is referred to the direction perpendicular to the external magnetic field. How we accomplish the numerical convergence for these multiscale equations, especially in the presence of high order velocity space moments in J_e, will be reported.

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