High-order, Finite Volume Discretization of Gyrokinetic Vlasov-Poisson Systems on Mapped Grids

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We describe our approach and progress in the development of high-order, finite volume discretizations of hyperbolic and elliptic systems on mapped grids. The motivation for this work is the need for efficient numerical methods to solve continuum gyrokinetic Vlasov-Poisson equations, which model the kinetic evolution of plasma and fields in the core and edge regions of tokamak fusion reactors.

The numerical solution of gyrokinetic Vlasov-Poisson systems in tokamak geometries poses several algorithmic challenges. A discretely conservative algorithm is needed that simultaneously addresses issues of long-time fidelity, positivity, large anisotropies and high dimensionality. Our core discretization is a new class of multidimensional higher-order finite volume methods (at least fourth-order in space and time). The use of a finite volume method guarantees discrete conservation of the distribution function, while the use of higher-order methods provides a substantial improvement in the long-time accuracy over more traditional second-order finite-volume methods. In the discretization of the hyperbolic gyrokinetic Vlasov equation, we combine a flux-corrected transport scheme with a new limiter to maintain distribution function positivity and fourth-order accuracy, even near smooth extrema. We employ a fourth-order discretization of the elliptic variable-coefficient gyrokinetic Poisson equation.

We also generalize these semi-structured discretizations to mapped multiblock grids, thereby achieving sufficient flexibility to accommodate tokamak edge geometries and the large anisotropies aligned with the magnetic field. The hyperbolic and elliptic discretizations represented by the gyrokinetic Vlasov and Poisson equations are treated using the same mapped grid, finite volume formalism. The use of a mapped semi-structured computational grid offers additional advantages, including the availability of multigrid-preconditioned elliptic solvers, natural extensions to local mesh refinement, and efficient parallel implementation.

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