Accommodating Irregular Subdomains in Domain Decomposition

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In the theory for domain decomposition methods, we have previously often assumed that each subdomain is the union of a small set of coarse shape-regular triangles or tetrahedra. In this talk, we discuss recent progress which makes it possible to analyze cases with irregular subdomains such as those provided by mesh partitioners.

Our goal is to extend our analytic tools to problems on subdomains that might not even be Lipschitz and to characterize the rates of convergence of our methods in terms of a few, easy to understand, geometric parameters of the subregions. For two dimensions, we have already obtained some best possible results for scalar elliptic and linear elasticity problems: the subdomains should be John or Jones domains and the rate of convergence is determined using the parameters that define such domains and that of an isoperametric inequality. Progress on three dimensions will also be reported.

New results have also recently been obtained concerning variants of classical two-level additive Schwarz preconditioners. Our family of overlapping Schwarz methods, borrows and extends coarse spaces from older iterative substructuring methods, i.e., methods based on non-overlapping subdomains. The local components of these preconditioners, on the other hand, are based on Dirichlet problems defined on a set of overlapping subdomains which cover the original domain.

Our methods are robust even in the presence of large changes, between subdomains, of the materials being modeled in the finite element models. An extra attraction is that our methods can be applied directly to problems where the stiffness matrix is available only in its fully assembled form. We note that these new methods have been used successfully as part of a production-level iterative solver in the parallel structural dynamics code Salinas, developed at Sandia National Laboratories in Albuquerque, NM.

We will also discuss several applications of the new tools. They include new results on almost incompressible elasticity and mixed finite elements using spaces of discontinuous pressures. We will also touch on recent work on Maxwell's equations in two dimensions.

Our work is carried out in close collaboration with Clark R. Dohrmann of the Sandia National Laboratories, Albuquerque, NM and Axel Klawonn and Oliver Rheinbach of the University of Duisburg-Essen, Germany.

Boldface Olof Widlund will present the work.

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