Infeasibility Detection in Nonlinear Optimization

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Optimization algorithms are confronted with two tasks: the minimization of a function and the satisfaction of constraints. In cases where an optimal feasible solution exists, a variety of rapidly convergent methods can be employed. However, when most of these methods are applied to infeasible problems, the iteration can progress very slowly and a great number of function evaluations is often required before a declaration of infeasibility can be made.

The focus of this talk is to address the need for optimization algorithms that can both efficiently solve feasible problems and rapidly detect when a given problem instance is infeasible. The fast detection of infeasibility has become increasingly important due to the central role it plays in branch and bound methods for mixed integer nonlinear programming and in parametric studies of optimization models, but is also a concern in its own right for general nonlinear programming problems that may include constraint incompatibilities.

One way to address these concerns is to employ a switch in an algorithm to decide whether the current iteration should seek a solution of the nonlinear program or, in constrast, to solely minimize some measure of feasibility. A challenge with such an approach lies in the difficulty of designing effective criteria for determining when such a switch should be made, since an inappropriate technique can lead to inefficiencies. In particular, since the objective function is ignored during feasibility iterations, the iterates may stray from an optimal solution, thus complicating the optimization process.

We propose an alternative approach involving a single optimization strategy, and show that it is effective for finding an optimal feasible solution (when one exists) or finding the minimizer of a feasibility measure (when no feasible point exists). Our algorithm is an active-set method that uses the penalty parameter to emphasize optimality over infeasibility detection, or vice versa. An important feature of our approach is that it is able to generate an accurate estimate of the set of constraints satisfied as equalities at a solution point. This makes it possible to implement the algorithm so that superlinear or quadratic convergence is achieved in most cases – including infeasible problems. We present the results of numerical experiments demonstrating the advantages of our approach.

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