Accelerated Monte Carlo Methods for Coulomb Collisions

Russel E. Caflisch[*], Richard Wang[*], Yanghong Huang[*], Bruce I. Cohen[†] and Andris M. Dimits[†]

We present an accelerated computational method for Coulomb collisions in a plasma, based on significant improvements in our earlier hybrid method that combines a Monte Carlo particle simulation and a fluid dynamic solver in a single uniform method throughout phase space. The hybrid method uses a hybrid representation of the velocity distribution function $f(v)$, as a combination of a Maxwellian equilibrium $M(v)$ and a collection of discrete particles $g(v)$. The Maxwellian M evolves in space and time through fluid-like equations, and the particles in g convect and collide through Nanbu's Monte Carlo particle method (PRE 1997). Interactions between M and g are represented by a thermalization process that removes particles from g and includes them in M and a dethermalization process that samples particles from M and inserts them into g. We present several improvements to this method: We derive an improved formulation of the detailed balance constraint on the thermalization and dethermalization probabilities. We define a parameterized set of thermalization and dethermalization probabilities and optimize the choice of parameters to achieve the fastest computation time for a specified accuracy level. We mathematically analyze the validity of the thermalization and dethermalization step in the context of a simple drift-diffusion model that includes long range interactions as in Coulomb collisions. Finally, we formulate a higher order stochastic method for solving the drift diffusion model using a Milstein correction.

[*]University of California, Los Angeles. Work performed for US DOE by UCLA under grant DE-FG02-05ER25710.
[†]Lawrence Livermore National Laboratory. Work performed for US DOE by LLNL under contract DE-AC52-07NA27344.