

Better Relaxations of Classical Discrete Optimization Problems

Robert Carr* and Goran Konjevod[†] and Giuseppe Lancia[‡] and Ojas Parekh[§]

A *mathematical program* is an optimization problem expressed as an objective function of multiple variables subject to set of constraints. When the optimization problem has specific structure, the problem class usually has a special name. A linear program is the optimization of a linear objective function subject to linear constraints. An integer program is a linear program where some of the variables must take only integer values. A semidefinite program is a linear program where the variables are arranged in a matrix and for all feasible solutions, this matrix must be positive semidefinite. There are general-purpose solvers for each of these classes of mathematical program. There are usually many ways to express a problem as a correct, say, linear program. However, equivalent formulations can have significantly different practical tractability. In this poster, we present new formulations for two classic discrete optimization problems, maximum cut (max cut) and the graphical traveling salesman problem (GTSP), that are significantly stronger, and hence more computationally tractable, than any previous formulations of their class. Both partially answer longstanding open theoretical questions in polyhedral combinatorics.

The GTSP is a variant of the classical TSP where a salesman must visit every city while traveling a minimum total distance. In GTSP, the legal city-to-city routes are restricted. Therefore, the salesman can travel through a city multiple times if this minimizes the total travel time. The TSP problem has applications in stocking warehouses, VLSI, and computational biology, and serves as a fundamental subroutine in many other optimization problems. We have developed the first tractable linear programming relaxation for the GTSP. The current solution technology for finding lower bounds for the GTSP, used for intelligent search, involves translating to a dense TSP. That is, the salesman can move between any pair of cities. The new formulation allows direct solution of the GTSP. When the problem is sparse, for example, when the number of legal city-to-city trips is proportional to the number of cities, this direct computation can be orders of magnitude faster than previous methods. In preliminary computational experiments with small examples, we have observed speed up proportional to the number of cities.

In maximum cut problem, we are given a graph $G = (V, E)$. We wish to partition the vertices to maximize the number of edges that go between the partitions. One can calculate an exact ground state of a spin glass in the Ising model by determining a maximum cut in the associated graph of interactions (see http://www.informatik.uni-koeln.de/lis_juenger/research/spinglass/) and max cut is a useful tool for other discrete optimization problems. The formulation with the best lower bound is a semidefinite program due to Goemans and Williamson that provably computes a bound within 87.8 percent of optimal. We have developed a linear program that provides a bound that is provably at least as strong.

*Discrete Mathematics and Complex Systems Department, Sandia National Laboratories, Albuquerque, NM, 87185-1318. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94-AL85000.

[†]Arizona State University

[‡]University of Udine

[§]Emory University