Achieving very high accuracy for the time integration of multiscale processes using extrapolation methods

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ABSTRACT (discrete methods and multiscale phenomena)

Multiphysics multiscale systems are characterized by multiple simultaneous physical processes evolving at very different time scales. As an example consider advection-diffusion-reaction systems where the advection is slow while the diffusion and chemistry are fast evolving. The dynamics of a process determines the best numerical solution strategy. Explicit time discretizations are effective for slow processes, whereas implicit methods are more efficient for fast processes. Implicit-explicit (IMEX) approaches have been developed for processes that have both fast and slow components.

This study constructs extrapolated implicit-explicit time stepping methods that allow to efficiently solve problems with multiple scale components. The proposed methods can provide very high order discretizations of ODEs, index-1 DAEs, and PDEs in the method of lines framework. Implicit-explicit schemes based on extrapolation are simple to construct, easy to implement, and straightforward to parallelize. This work establishes the existence of perturbed asymptotic expansions of global errors, explains the convergence orders of these methods, and studies their linear stability properties. Numerical results with stiff ODE, DAE, and PDE test problems confirm the theoretical findings and illustrate the potential of these methods to solve multiphysics multiscale problems.

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