

**Energetics of eddy scales in the inertial and dissipative ranges:
a numerical study of the parameters α and β in the
Navier–Stokes- $\alpha\beta$ equations for turbulence**

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We perform numerical studies of the Navier–Stokes- $\alpha\beta$ equations for homogeneous, isotropic turbulent flow. These equations generalize the Navier–Stokes- α equations by allowing for separate length scales α and β associated, respectively, with regularizing terms of dispersive and dissipative origin; setting β equal to α equal reduces the Navier–Stokes- $\alpha\beta$ equations to the Navier–Stokes- α equations. In particular, we examine the influences of α and β on the energy spectrum for spatially periodic flow in a cubic domain, including, for comparison, the limiting cases of the Navier–Stokes- α and Navier–Stokes equations. A significant increase in the accuracy of the energy spectrum at large wave numbers arises for $\beta < \alpha$, but an optimal choice of these scales depends on the grid resolution.

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