Uncertainty and Sensitivity in Multilevel Hierarchical Modeling

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Characterizing the uncertainty and sensitivity in models and simulations of complex multiscale phenomena is critical to establishing a scientific basis for policy decisions as well as effectively driving scientific research in many DOE applications, including carbon sequestration, aquifer assessment and protection, and nuclear waste disposal. This problem arises because fully resolved simulations are intractable, the mathematical model may be inaccurate or incomplete, and its parameters may be poorly characterized by sparse noisy data at disparate lengths scales. Developing efficient and robust techniques that facilitate the accurate quantification of uncertainty and sensitivity in these situations is the objective of the "Predictability with Stochastic Partial Differential Equations" project at LANL (D. Moulton,PI).

In this presentation we focus on the class of flow problems in which the fine-scale continuum model is well understood, but the medium may be highly heterogeneous and it may be represented stochastically. Here, homogenization (or upscaling) techniques are necessary to develop computationally feasible models on scales coarser than the variation of the coefficients of the continuum model. Although the accuracy of these techniques depends dramatically on the assumptions that underlie the particular upscaling methodology used, a mechanism to estimate or control the error in the model or solution is not provided. Recent work on multilevel hierarchical modeling techniques, such as Multilevel Upscaling (MLUPS) and the Multilevel Multiscale Mimetic (M^3) Method, provides a framework that naturally supports the development of error estimation and control. To highlight the potential of these approaches, we consider a fine-scale stochastic representation of a heterogeneous porous medium and explore the propagation of uncertainty through a hierarchy of flow models at coarser resolutions. Specifically, for a fine-scale Darcy flow we study the scale-dependent tensorial nature of the coarsened parameters and the implicit closure models. In addition, we examine the corresponding uncertainty in common quantities of interest, such as the average flux through this medium given a uniform coarse-scale pressure gradient.

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