Analytic Integration Technique for Linear Elliptic PDEs

S. Nintcheu Fata* and L. J. Gray†

This study is concerned with the development of accurate and efficient techniques for solving linear elliptic systems such as the Poisson and Lamé equations in three-dimensional domains. The methodology is investigated in the context a Galerkin boundary integral equation approach with both singular and hypersingular formulations.

Taking the 3D Laplace equation as the example to discuss the algorithm, surface integrals are defined as limits to the boundary and linear surface elements are employed to approximate the geometry and boundary functions. In the inner integration procedure, it is shown that all singular and non-singular integrals over a triangular boundary element can be expressed exactly as a sum of analytic formulae over the edges of the designated element. It is also established that weakly-singular, strongly-singular and hypersingular integrals are simply special cases of non-singular integrals. In the outer integration scheme, closed-form expressions are obtained for the coincident case, wherein the divergent hypersingular terms are identified explicitly, and shown to cancel with corresponding terms from the edge-adjacent case. The remaining edge-adjacent, vertex-adjacent and non-coincident integrals contain only weak singularities and can be carried out easily by use of suitable numerical cubature. This semi-exact treatment does not seem to suffer from the usual inaccuracies associated with *near-singular* (or *quasi-singular*) integrals; i.e., integrals over a pair of triangular elements that are "very close". The performance of the proposed algorithm is illustrated with several numerical examples.

*† Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831. This research was supported by the Office of Advanced Scientific Computing Research, U.S. Department of Energy, under contract DE-AC05-00OR22725 with UT-Battelle, LLC. *E-mail: nintcheufats@ornl.gov