Uncertainty Quantification: Improved Stochastic Finite Element Approach

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In our work, we introduce a stochastic finite element-based approach to describing the uncertainty of a complex system of differential-algebraic equations with random inputs.

For our test system, we take a 3-dimensional steady-state model of heat distribution in the core of a nuclear reactor. The heat exchange between the fuel and the coolant depends on thermo-dynamical properties of the involved materials; the estimation of temperature dependence of these properties includes experimental error.

The problem of uncertainty quantification may be solved by sampling methods; or through the creation of a *surrogate model* (a valid simplified version of the system). The choices for representing the output of the system include linear approximation, or a projection onto a set of interpolating functions. A generic stochastic finite element method (SFEM) approach uses a complete set of orthogonal polynomials (a *polynomial chaos*), of degrees up to 3-5. Given statistical information on the inputs, SFEM model provides explicit description of the distribution of the output. If input uncertainty structure is not provided, the surrogate model can still be used to estimate the range and variance of the output. One disadvantage of the approach is the large dimension of the model, requiring many full integrations of the system for interpolation.

We construct the surrogate model as a goal-oriented projection onto an *incomplete* space of interpolating polynomials; find the coordinates of the projection by collocation; and use derivative information to significantly reduce the number of the required collocation sample points. The basis may be trimmed to linear functions in some variables, and extended to high order polynomials in the others. Derivatives of the output with respect to random parameters are obtained using an efficient adjoint method with elements of automatic differentiation, the relative magnitudes of the derivatives are also used to decide on the importance of the variables.

The resulting model is more computationally efficient that random sampling, or generic SFEM; and has significantly greater precision than linear models.

Currently, we work on applying the analysis to an extended model of the reactor core, with additional uncertainties coming from the description of neutron interaction, non-uniform flow of the coolant, and structural deformation of the core elements. We will investigate the possibilities to further improve the approach through the optimal choice of the collocation points; and a more sophisticated sensitivity analysis resulting in an optimal polynomial basis. We are open to suggestions of future collaboration on the mathematical and applied aspects of the study.

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