# Rare Event Simulation for Diffusions

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I will present ongoing work to develop new techniques for approximating probabilities of the form

$$\mathbf{P}\left[X^{\epsilon}(T) \in A\right]$$

where $X^{\epsilon}$ is the solution of the stochastic differential equation

$$dX^{\epsilon}(t) = b(X^{\epsilon}(t))\,dt + \sqrt{\epsilon}\sigma(X^{\epsilon}(t))\,dW(t).$$

Such problems satisfy a large deviations principle, i.e.

$$\lim_{\epsilon \to 0} -\epsilon \log \mathbf{P}\left[X^{\epsilon}(T) \in A\right] = \gamma$$

where $\gamma$ is the minimum value of some action functional. Naive importance sampling (IS) schemes for this problem will behave exponentially poorly as $\epsilon$ becomes small. By solving a deterministic control problem on the fly we attempt to construct schemes with bounded relative error. The equivalent first order Hamilton Jacobi equation is the limit (as $\epsilon \to 0$) of parabolic equations whose solutions can be used to construct zero variance importance sampling schemes for each $\epsilon$. For problems in more than a few dimensions, our method avoids the unreasonable cost of solving these partial differential equations. I will discuss some preliminary numerical results as well as future directions.

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