Oil spill response planning with consideration of physicochemical evolution of the oil slick: A multiobjective optimization approach

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Abstract

This paper addresses the optimal planning of oil spill response operations under the constraints of economic and responsive criteria, with consideration of oil transport and weathering process. The economic criterion is measured in terms of total cost, while the responsiveness is measured by the time span of the entire response operations. A bi-criterion, multiperiod mixed-integer linear programming (MILP) model is developed that minimizes the total cost and the response time span and simultaneously predicts the optimal time trajectories of the oil slick's volume and area, transportation profile, usage levels of response resources, oil spill cleanup schedule, and coastal protection plan. The MILP model also integrates with the prediction of an oil transport and weathering model that takes into account oil physiochemical properties, spilled amount, hydrodynamics, and weather and sea conditions. The multi-objective optimization model is solved with the ε-constraint method and produces a Pareto-optimal curve that reveals how the optimal total cost, oil spill cleanup operations, and coastal protection plans change under different specifications of the response time span. We present two case studies for oil spill incidents in the Gulf of Mexico area and the New England region to illustrate the application of the proposed approach.

Key words: planning, oil spill response, MILP, multi-objective optimization, ODE

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1. Introduction

The recent *Deepwater Horizon* oil spill in the Gulf of Mexico has become the focus of public attention due to its significant ecological, economic and social impacts (see http://www.deepwaterhorizonresponse.com/ and http://www.restorethegulf.gov/). This incident, coupled with previous catastrophic oil spills (e.g., 1989 *Exxon Valdez* oil spill), has demonstrated the importance of developing responsive and effective oil spill response planning strategies for the government and for the oil exploration and production industries (NOAA, 1992). Under the Oil Pollution Act of 1990 (OPA 90) the party responsible for the spill is obligated to conduct response operations that satisfy all requirements set by the Coast Guard. Oil spill response usually occurs within a complex environment that requires timely decisions to balance the response cost and responsiveness and to address such issues as weathering and movement of the oil slick, selection of response/cleanup methods, coordination of coastal protection activities, availability of cleanup facilities, legal constraints, and performance degradation with bad weather. This is usually a nontrivial task for decision makers (incident commanders), who must coordinate considerable resources and must plan many operations. For instance, in the response to the *Deepwater Horizon* oil spill, over 39,000 personnel, 5,000 vessels, and 110 aircraft were involved, over 700 kilometer boom has been deployed, 275 controlled burns have been carried out, approximately 27 million gallons of oily liquid has been recovered by skimmers, and more than 1.5 million gallons of chemical dispersant (Corexit) have been used as of July 1, 2010. Therefore, development of optimal planning strategies for oil spill response operations is critical, in order to balance the total cost and responsiveness.

Planning problems have been studied extensively by the process systems engineering community in the recent years. General reviews on this topic are given by Reklaitis (2000), by Kallrath (2002), and by Verderame et al (2010). Although most of the existing planning models have only one objective function to maximize the economic performance, some use a multi-objective optimization scheme to account for such additional objectives as environmental damage (Bojarski et al., 2009; Guillen-Gosalbez & Grossmann, 2009; Duque et al., 2010), service level (You et al., 2010), financial risk (Rodera et al., 2002; You et al., 2009), responsiveness (You & Grossmann, 2008, 2009 &

2010), and flexibility (Ierapetritou & Pistikopoulos, 1994). The methodology of process planning has been used in a wide spectrum of applications in process industries, including chemical production (Verderame & Floudas, 2009), refinery operations (Neiro & Pinto, 2005) new production development (Maravelias & Grossmann, 2001), pharmaceutical manufacturing (Gatica et al., 2003), and water network (Laird et al., 2006). None, however, has been extended to deal with the decision-making problems in oil spill response.

On the other hand, most modeling literature on oil spill is focused on the simulation of oil transport and weathering process (Brebbia, 2001; Reed et al., 1999), and few have addressed the response planning problem. A review of the planning models for oil spill response is given by Iakovou et al (1994). For example, Psaraftis and Ziogas (1985) developed an integer programming model for optimal dispatching of oil spill cleanup equipments with the objective to minimize the total response costs. Wilhelm and Srinivasa (1997) developed an integer programming model for prescribing the tactical response of oil spill cleanup operations with the objective of minimizing the total response time of equipments. Limited literature exists that addresses the integration of oil properties, the weathering model, and the planning model (Ornitz & Champ, 2003). In a recent work by Gkonis et al,(2007) the authors presented a mixed-integer linear programming model that considers the oil weathering process, an important factor for decision making in response operations. None these planning models, however, has taken into account coastal protection planning, which is usually required for massive oil spills. Moreover, only a single objective is used in the existing literature; and the time span of the entire response operations, which is the measure of the responsiveness, has not been considered by the existing optimization models.

The objective of this paper is to develop an optimization approach to tactical oil spill response planning under the constraints of economic and responsiveness criteria. The economic criterion is measured by the total response cost, and responsiveness is measured by the time span of the entire response operations. We consider both oil spill cleanup and coastal protection operations in this work. A bi-criterion, multiperiod mixed-integer linear programming (MILP) model is developed and coupled with the predictions from an oil transport and weathering model that takes into account the time-dependent oil

properties, spilled amount, hydrodynamics, weather, and sea conditions. The planning model simultaneously predicts the optimal time trajectories of the oil slick's volume and area, transportation and usage levels of response resources, oil spill cleanup schedule, and coastal protection plan. The multi-objective optimization model is solved with the ε-constraint method and produces a Pareto-optimal curve that reveals how the optimal total cost, oil spill cleanup operations, and coastal protection plans change under different specifications of the response time span. The application of the proposed optimization approach is illustrated through two examples based on the incidents of *Deepwater Horizon* oil spill and the *Argo Merchant* oil spill.

This paper includes several novel features. First, we use a multi-objective optimization approach to consider both economic and responsiveness objectives in the oil spill response planning. To the best of our knowledge, the responsiveness objective measured by the time span of the entire response operations has not been considered in the existing literature. Besides, the multi-objective optimization scheme for the tradeoff between these two important attributes of oil spill response operations, economics and responsiveness, was not addressed by any previous work. Second, the proposed model considers not only the mechanical cleanup method but also in situ burning and chemical dispersant methods, which were not taken into account in previous work. Third, coastal protection planning is integrated with oil weathering process and cleanup planning decisions in the proposed model; this issue has not been reported before as far as we know.

The rest of this paper is organized as follows. Section 2 presents background on oil transport, the weathering process, and response operations. We provide a general problem statement in Section 3. The detailed model formulation of the oil spill response planning model is given in Section 4, which is followed by the solution approach introduced in Section 5. In Section 6, we present computational results for two case studies. Section 7 summarizes our conclusions and briefly discusses future work. The formulation of an oil transport and weathering model is given in the appendix.

2. Background

Oil spill response planning requires an effective integration of the physical and

chemical properties, transport, and weathering of spilled oil; weather and sea conditions; planning of coastal protection operations; selection of cleanup methods; and scheduling of cleanup facilities. The spilled oils are normally mixtures of hydrocarbon compounds whose chemical and physical properties vary among oil types. When an oil spill occurs, the nature of the oil undergoes a series of changes in chemical and physical properties over time that, in combination, are termed "weathering." In this section, we briefly review the oil weathering processes including short-term processes such as spreading, drift, evaporation, emulsification, dispersion, and dissolution and long-term processes such as photo-oxidation, biodegradation, and sedimentation (see Figure 1). Also reviewed in this section are the widely used protective and cleanup methods, including boom, skimming, in situ burning, and chemical dispersant.

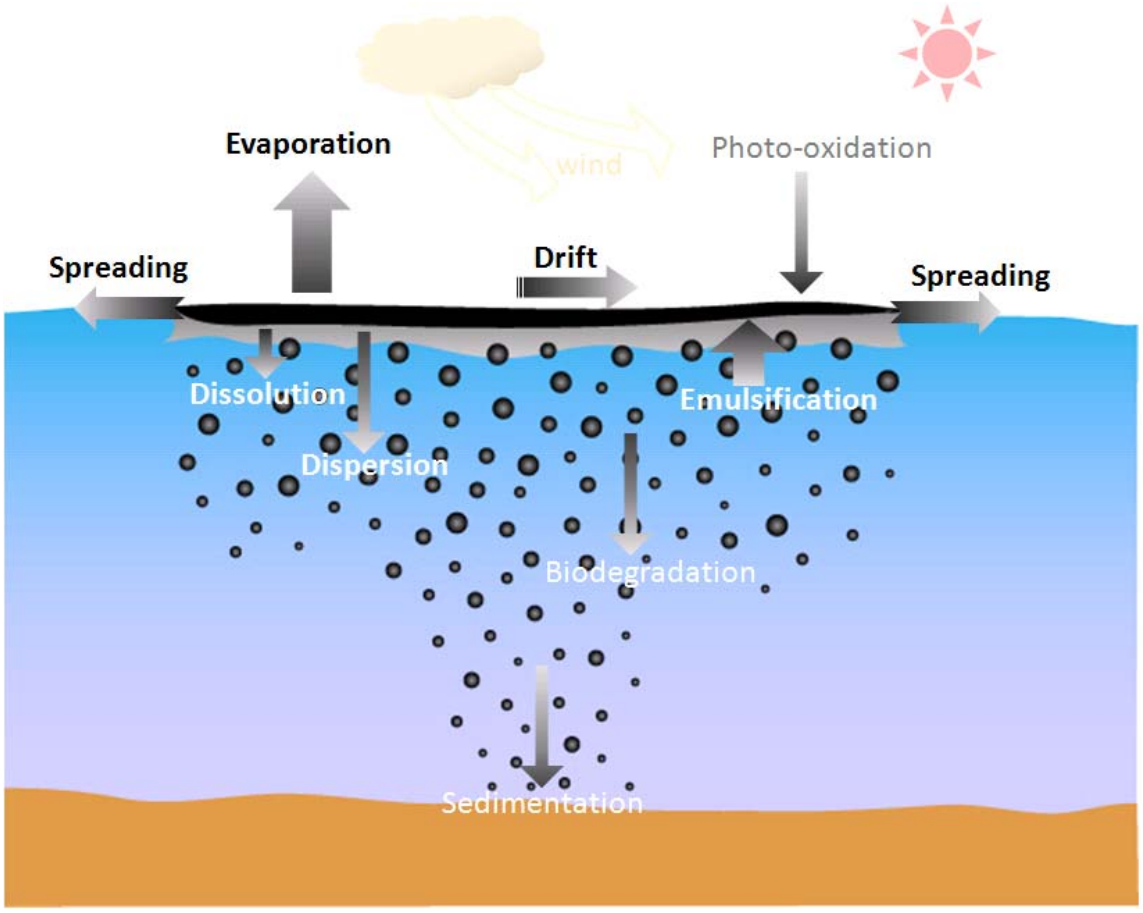

Figure 1. Oil spill weathering process

2.1 Oil transport process and weathering models

Spreading and Drift

As soon as oil is released on water, the oil begins to spread by gravity, wind, and current, with the process resisted by inertia, viscosity, and surface tension, until the slick reaches a thickness of ~0.1 mm or less. The surface transport process, or spreading, can affect other weathering processes such as evaporation, dispersion, and emulsification (see Figure 1). The environmental impact of oil spills largely depends on the spreading area. The coastal protection and cleanup operations also require information on the spill size.

Because of the influence of the winds and wind-induced surface currents, the oil slick may move downward with respect to the wind direction. This movement, called drift, results in a displacement of the center of the oil slick and contributes to the nonsymmetrical spreading. The drift speed is around 2.5~4.5% of the wind speed as measured in various laboratory studies and field studies (Brebbia, 2001; Reed et al., 1999). The drift velocity and the trajectory of the oil slick can significantly affect the coastal protection plan.

Evaporation

Evaporation is the process that the lower-molecular-weight volatile component of the spilled oil mixture comes from the surface slick into air. Evaporation is usually the most important weathering process, which can account for the loss of 20~50% of many crude oils and 75% or more of refined petroleum products (Brebbia, 2001; Michel et al., 2005). The evaporation rate of oil depends primarily on its physicochemical properties and is increased by spreading, high temperature, wind and waves. The composition and physicochemical properties of oil can change significantly with the extent of evaporation. For example, if about 40% of the oil evaporates, its viscosity could increase by as much as a thousandfold.

Natural Dispersion

Natural dispersion is the process of forming small droplets of oil that are incorporated into the water column by wave action or turbulence. Natural dispersion is the net result of three processes: (1) globulation, which is the formation of oil droplets from slick under influence of breaking waves; (2) dispersion, which is the transport of the oil droplets into

the water column as a net result of the kinetic energy of oil droplets supplied by the breaking waves and the rising forces; and (3) coalescence of the oil droplets with the slick (CONCAWE, 1983).

Natural dispersion reduces the volume of slick on the sea surface and the evaporative loss, but it does not lead to changes in the physicochemical properties of the spilled oil in the way those other processes (e.g., evaporation) do. If droplets are small enough, natural turbulence will prevent the oil from resurfacing. The rate of natural dispersion is an important factor for the life of an oil slick on the sea surface. In practice, natural dispersion can be significant, accounting for a major part of the removal of oil from the sea surface. The effect of natural dispersion depends on both the oil properties and the amount of sea energy.

Emulsification

Emulsification is the process whereby water droplets are entrained into the oil layer and remain in the oil slick in unstable, semi-stable, and stable forms. Emulsification can change the physicochemical properties of oils dramatically, especially for viscosity. The emulsified oil can contain up to ~70% water. More significant, the oil viscosity can increase as much as a thousandfold, making the emulsion very difficult to clean up. Once stable emulsion forms, other weathering processes are also affected. The evaporation and biodegradation slow, and the spreading and dissolution almost cease. Whether the emulsification occurs depends on the oil properties. Light, refined oils generally will not emulsify since they do not contain the right hydrocarbon components to stabilize the water droplets. Crude oil will emulsify when the wax and asphaltene content reach 5%. Some oils will emulsify only after they have been weathering to an extent. Emulsions are characterized as stable, meso-stable, and unstable when the maximum amount of contained water is 60–80%, 40–60%, and 30–40%, respectively (Sebastiao & Sores, 1995). Most emulsification models are derived from the formulation proposed by Mackay & McAuliffe (1988) to predict the water content, viscosity, and density of the emulsion.

Other Weathering Processes

Other weathering processes include dissolution, photo-oxidation, sedimentation, and biodegradation. Dissolution occurs immediately after the oil spill, and the amount is

usually much less than that from evaporation. Photo-oxidation can change the composition of spilled oil, but it is not considered to be an important process because it affects less than 1% of the oil in the slick. Sedimentation is the adhesion of oil to solid particles in water; it has little effect in removing oil in open-sea conditions. Biodegradation is a slow, long-term process, and there is no general mathematical model to describe the biodegradation rate of crude oil in a marine environment. Because of these limitations, these processes are generally not considered in the mass balance or physicochemical property changes of the oil weathering model.

2.2 Cleanup and coastal protection methods

When a massive oil spill occurs, quick and effective response is critical in order to minimize the economic impact and the damage to both the ecology and the quality of human life. Four methods are commonly used to contain and clean up a spill: booms, skimmers, chemical dispersants, and in situ burning (see Figure 2).

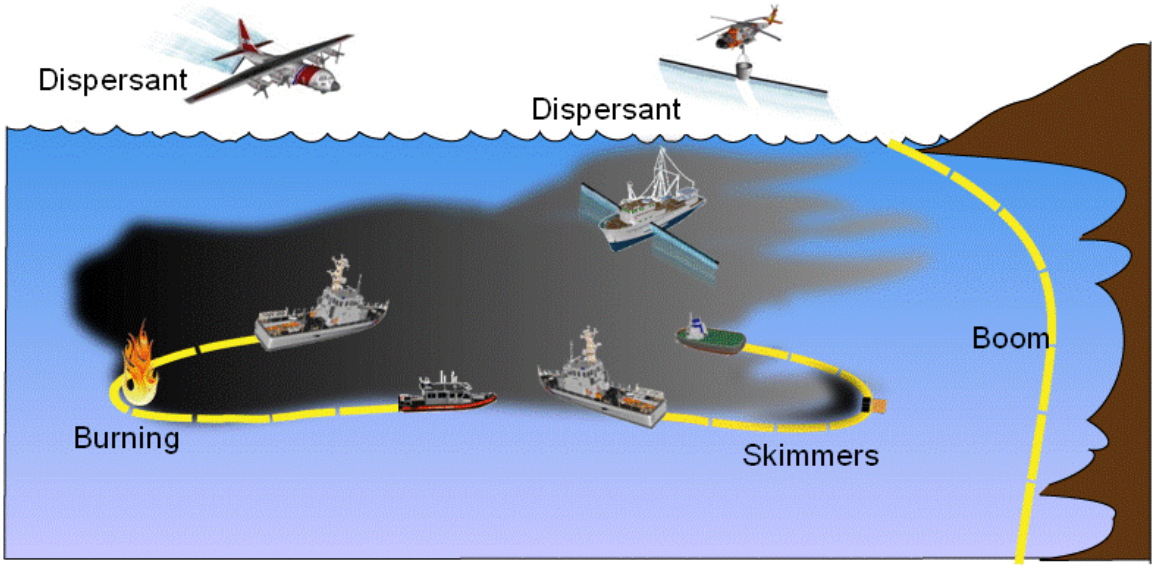

Figure 2. Oil spill cleanup and coastal protection operations

Booms

Booms are floating mechanical barriers capable of controlling the movement of oil slick on the sea surface. Booms are generally the first equipment deployed after an oil spill and are often used throughout the response process. Booms can be used to prevent

oil from spreading, to protect shorelines, to divert oil to areas where it can be treated or recovered, or to concentrate oil so that skimmers can be used or in situ burning can be applied. All booms need to be placed and maintained in a coordinated strategy with other response approaches to ensure their effectiveness. Booms must be deployed before the arrival of oil for effective coastal protection. The boom's performance and ability to contain oil are affected by currents and winds. When the current speed exceeds a critical velocity or booms are damaged, boom failure or loss of oil can result. Thus, boom maintenance, including periodic checks, repairs, and resets, is necessary for effective coastal protection. There are several types of booms, including conventional hard booms, fire booms, and sorbent booms. Conventional booms are subject to damages over a certain time period. Fire booms can withstand high temperatures and are usually used to contain or concentrate oils for in situ burning. Sorbent booms are made of porous sorbent material; they are used both to contain and to recover oil, specifically by removing traces of oil or sheen when oil slick is relatively thin. They are also used as a backup for other booms and are widely used to improve the performance of conventional hard booms by absorbing oil Sorbent booms require continuous maintenance, including reposition and turning to expose a clean surface, and must be replaced when they are saturated by either oil or water.

Skimmers

Skimmers are mechanical devices designed to recover oil or oil-water mixtures from the water surface. The effectiveness of a skimmer is rated according to the amount of oil it recovers. Most skimmers function best when the oil slick is relatively thick; hence, they are usually placed in front of the boom or where the oil is most concentrated, in order to recover as much oil as possible. Skimmers can be classified according to their basic operating principles as oleophilic surface skimmers, weir skimmers, suction/vacuum skimmers, elevating skimmers, submersion skimmers, and vortex/centrifugal skimmers. A skimmer's performance is affected by a number of factors including the thickness of the oil, the extent of weathering and emulsification of the oil, the presence of debris, and weather conditions.

Chemical Dispersants

Chemical dispersants are mixtures of solvent, surfactant, and other additives that can be applied to oil on the water surface to reduce the oil-water interfacial tension. They are able to enhance the entrainment of small oil droplets into the water column at lower energy inputs. Whether an oil slick is dispersible largely depends on oil properties (density, viscosity, wax/asphaltene content, boiling point fractions). Response actions using dispersants should be initiated as soon as possible. Every effort should be made to spray the chemical dispersant before significant oil weathering occurs. The longer the oil slick undergoes weathering, the less effective the chemical dispersants are. Dosage control is another key operational factor for dispersant application. The typical dispersant to oil ratio is 1:20, though ratios of 1:40 or even 1:60 are achievable for some dispersants and oils, and oil ratios as high as 1:10 could be required for some of the more emulsified and viscous heavy oils (Michel et al., 2005). Dispersion, both natural and chemically enhanced, increases with wave energy; and more dispersant is required for application when sea energy is low. A decision maker should also evaluate the potential environmental consequences for dispersant use. For application, dispersants are usually sprayed on oil surface by aircrafts, helicopters, and vessels (Michel et al., 2005).

In Situ Burning

In situ burning involves controlled burning of the oil at or near the spill site. This technique is more effective in massive oil spills, where it can remove large amounts of oil in less time than can other techniques. The slick thickness is the most important factor for in situ burning. Almost all types of oil can be burned on the sea surface if the slick is thick enough. The burning efficiency depends on the slick thickness: The oil slick must be at least 2 to 3 mm thick to be ignited, and it can be burned down to about 1 to 2 mm thick (Fingas, 2001). If a 10 mm thick pool of oil is ignited and burned down to 1 mm, the efficiency can be as high as 90% (Fingas, 2001). Therefore, fire-resistant booms often are used to contain the oil in thicker slicks to facilitate the in situ burning process. The main concern about in situ burning of oil spill is the toxic emissions from the large, black smoke plume.

3. Problem Statement

An integrated approach is needed in which the oil weathering process, coastal protection planning, and oil spill cleanup operations are considered simultaneously, in order to resolve the trade-offs between economics and responsiveness in oil spill response in an optimal manner. The problem addressed in this work can be formally stated as follows.

A given type of oil spills at a specific location. The initial spill volume, the constant release rate, and the release duration are all known. We are also given the physical and chemical parameters of the oil (API degree, initial boiling point, density, initial viscosity, etc.) and those of the water (seawater density, kinematic viscosity of seawater, water temperature, oil-water interfacial tension, etc.) as well as the weather data (wind speed, temperature, etc.). We assume in this work that the oil weathering process follows the prediction of an oil transport and weathering model (the formulation of an oil weathering model is presented in the appendix).

We are given a set of time periods $t \in T$ with fixed lengths of time periods and a set of staging areas $i \in I$ along the shoreline near the spill sites. The staging areas can stage cleanup equipment and response resources, including coastal protection booms and chemical dispersants. Each staging area *i* is also responsible for deploying booms with a minimum length of $\underline{L}_i$ to protect the nearby shoreline if the oil slick may hit the coast. Because of the spreading and drift processes, the oil slick may hit the coast around staging area *i* at time period *t* if the slick area is larger than a given value $\overline{AREA}_{i,t}$, which is a given parameter in this work and can be derived from the wind and current speeds. The minimum and maximum boom deployment rates at staging area *i* are given as BDL_i and BDU_i, respectively. The fixed and variable costs of deploying coastal protection booms near staging area *i* are given. All the booms deployed around staging area *i* will lose effect after a lifetime φ_i. After deployment, the existing booms are subject to maintenance cost depending on the weather condition reflected by a weather factor for boom maintenance $\omega_{i,t}^{Boom}$ for staging area *i* at time period *t*. Booms are shipped to the staging areas from a set of boom storage/supplier locations $j \in J$, and chemical

dispersants are similarly shipped from a set of supplier locations $k \in K$. The corresponding available amount at the supplier location, transportation time, unit purchase and shipping costs, transportation capacity, and the inventory holding cost of booms and chemical dispersants are given.

The major cleanup methods include mechanical cleanup and recovery (skimming), in situ burning, and chemical dispersant application. The cleanup facilities for mechanical, burning, and dispersant application are indexed by *m*, *b,* and *d*, respectively. For instance, the set of chemical dispersant application types may include C-130 Hercules, helicopters, and vessels mounted with dispersant spray systems. The maximum number of each type of cleanup facilities that can be staged to each staging area is given; and the corresponding total response time to notify, mobilize, dispatch, and deploy the system is known. The operating capacities of the cleanup systems and the corresponding fixed and variable costs of operations are given. The weather factor for each cleanup method at time period *t* is given. The minimum thickness of the oil slick that each in situ burning system can handle is known, and the price of the recovered oil through each mechanical skimming system is given. For chemical dispersant systems, the maximum number of sorties that can be dispatched at time period *t*, and the corresponding effectiveness factor and accuracy factor are given. Note that the effectiveness factor may decrease over time due to oil weathering. There is a limit on the total amount of chemical dispersants that can be applied in the entire cleanup operation following federal regulations and ecological impact concerns. The entire response operation finishes when the volume of the oil on the sea surface is less than or equal to a predefined cleanup target $\underline{V}$.

In order to determine the optimal plan of oil spill response operations, one objective is to minimize the total time span of the response operations. Another objective is to minimize the total cost over the entire time span. Since the two conflicting objectives need to be optimized simultaneously, the corresponding problem yields an infinite set of trade-off solutions. These solutions are Pareto-optimal in the sense that it is impossible to improve both objective functions simultaneously. This situation implies that any solutions for which the total response cost and the response time span can be improved simultaneously are "inferior" solutions that do not belong to the Pareto-optimal curve. The aim of this work is to determine the tactical oil spill cleanup and coastal protection

decisions that define the Pareto optimal curve by minimizing the total cost and the total response time span.

4. Oil Spill Response-Planning Model

In this work we propose an optimization approach for tactical decision making in oil spill response. The optimization model is coupled with the prediction of physicochemical evolution of the oil slick from a dynamic oil weathering model. Given the characteristics of the specific oil spill that define the initial conditions and oil-specific model parameters (see the Appendix), the oil weathering model is simulated up to the time when natural weathering process reduces the volume of oil on the sea surface to the cleanup target. Note that in the natural weathering process, no cleanup or coastal protection actions are taken. The oil volume, slick area, and water content predicted by the oil weathering model at each discrete time period are then used as input to the optimization problem. The formulation of the dynamic oil-weathering model, which is not the focus of this work, is given in the appendix.

The tactical planning model is formulated as a bi-criterion, multiperiod mixed-integer linear programming (MILP) problem, which predicts the optimal time trajectories of the oil slick's volume and area, transportation and usage levels of response resources, oil spill cleanup schedule, and coastal protection plan with different specifications of the response time span. Different from the oil weathering model, where a continuous-time representation is used, in the planning model we discretize the planning horizon into $|T|$ time periods with H_t as the length of time period $t \in T$. The multiperiod formulation is widely used in oil spill response-planning problems as it can greatly simplify the modeling and solution process of the planning models (Gkonis et al., 2007; Psaraftis & Ziogas, 1985; Srinivasa & Wilhelm, 1997; Wilhelm & Srinivasa, 1997). This representation is also consistent with the real-world practice that most oil spill cleanup and shoreline protection decisions are made on a hourly or daily basis (i.e. using one hour or one day as a time period). Three types of constraints are included in this multiperiod planning model: oil slick constraints (1)–(5), coastal protection constraints (6)–(20), and cleanup planning constraints (21)–(33). Equation (34) defines the total response time span, and equation (35) defines the total cost, both of which are objective functions to be

optimized. A list of indices, sets, parameters, and variables is given in the Nomenclature section after the appendix. The detailed model formulation is presented in the following subsections.

4.1 Oil slick constraints

Based on the multiperiod formulation, we define δ_t as the thickness of the oil slick at the end of time period t. A major assumption of this work is that all the cleanup operations are local effects that change the slick area but not the average slick thickness; that is, the thickness remains the same as the prediction by the oil natural-weathering model. Thus, we have the slick thickness at each time period given by $\delta_t = V^*(t)/A^*(t)$, where $V^*(t)$ and $A^*(t)$ are the volume and area of the oil slick, respectively, at time t predicted by the oil weathering model. With the time trajectory of the slick thickness fixed, the volume of oil slick should be equal to the product of slick area and slick thickness in all the time periods:

$$v_t = area_t \cdot \delta_t, \qquad \forall t \in T, \tag{1}$$

where v_t is the volume of oil slick and $area_t$ is the slick area. Both depend on the cleanup operations and are variables to be optimized.

We further define θ_t as the percentage of oil removed from the slick at time period t by natural weathering (evaporation and dispersion). The value of this parameter can also be derived from the solution of dynamic oil-weathering model by using the following equation: $\theta_t = \left[V^*(t-1) - V^*(t) + VI_t \cdot H_t\right] / V^*(t-1)$, where the term $VI_t \cdot H_t$ accounts for the volume of oil newly released to the sea surface in time period t. This parameter is introduced in order to take into account the effects of evaporation and natural dispersion in the oil spill response-planning model. To some extent, the use of this parameter is similar to piecewise linear approximation of the time trajectory of the oil volume. Volume balance of the oil slick shows that the volume of the oil slick in the previous time period plus the newly released volume of oil to the sea surface through the current time period should be equal to the summation of the oil volume at the end of the time period and the oil volume removed from the slick by natural weathering and cleanup operations. Thus,

we can model the volume balance with the following equations:

$$V0 + VI_{t=1} \cdot H_{t=1} = v_{t=1} + \theta_{t=1} \cdot V0 + u_{t=1}^{M} + u_{t=1}^{B} + u_{t=1}^{D}, \tag{2}$$

$$v_{t-1} + VI_t \cdot H_t = v_t + \theta_t \cdot v_{t-1} + u_t^{M} + u_t^{B} + u_t^{D} \qquad \forall t \geq 2, \tag{3}$$

where u_t^M, u_t^B, and u_t^D are the volumes of oil removed from the sea surface by using mechanical systems (skimmers), in situ burning, and chemical dispersants, respectively.

To model the time span of the oil spill response operations, we introduce a 0-1 variable f_t. If the volume of the oil slick is greater than the cleanup target, then f_t equals 1; otherwise, it equals zero. Thus, we have the following constraint:

$$v_t \leq \underline{V} + U \cdot f_t \qquad \forall t \in T, \tag{4}$$

where $\underline{V}$ is the cleanup target and U is a sufficiently large number for the upper bound of the volume.

If the oil spillage has not yet stopped at time period t, the cleanup target is not achieved in that time period regardless of the volume of oil remaining on the sea surface:

$$\sum_{t'=t}^{T} VI_{t'} \leq U \cdot f_t \qquad \forall t \in T. \tag{5}$$

4.2 Coastal protection constraints

Because of the spreading and drift processes, the oil slick may hit the coast and lead to significant environmental damage. Booms can be deployed along the coast to protect sensitive shorelines, but boom protection can be effective if and only if sufficient lengths of booms are deployed in the corresponding shorelines. In order to protect the coast, either the slick area must be controlled through effective cleanup operations so that it will not hit the shore, or coastal protection booms must be fully deployed around those staging areas that might be hit by the oil slick ($z_{i,t}$=1). However, this constraint can be relaxed after the cleanup target is achieved (f_t=0); in other words, there is no need to protect the coasts after the volume of oil is reduced to the cleanup target. The following inequality models this relationship:

$$area_t \leq \overline{AREA}_{i,t} + U \cdot z_{i,t} + U \cdot (1 - f_t) \qquad \forall i \in I, t \in T, \tag{6}$$

where $area_t$ is the area of the oil slick at the end of time period t, $\overline{AREA}_{i,t}$ is the area of the oil slick that will hit the shore around staging area i at time period t, z_{it} is a binary variable that equals 1 if sufficient booms have been deployed to protect the shoreline around staging area i at time period t, and U is a sufficiently large number for the upper bound of oil slick area. $\overline{AREA}_{i,t}$ depends primarily on the drifting effect; its value may decrease over time if wind and current push the oil slick toward the shore, and vice versa.

If sufficient booms have been deployed to protect the shoreline around staging area i at time period t ($z_{i,t}$), the booms must have been deployed at any time before t. This relationship can be modeled with the following logic proposition:

$$z_{i,t} \Rightarrow \bigvee_{t' \le t-1} zd_{i,t'}$$

where $zd_{i,t}$ is a binary variable that equals 1 if the booms are being deployed around staging area i to protect the nearby shoreline in time period t. The logic propositions can be further transformed into inequalities (Raman & Grossmann, 1993):

$$z_{i,t} \le \sum_{t' \le t-1} zd_{i,t'} \qquad \forall i \in I, t \in T \,. \tag{7}$$

If booms are being deployed at time period t-1 along the shoreline near staging area i ($zd_{i,t-1}$), then the boom deployment will continue during time period t ($zd_{i,t}$) or until sufficient booms have been deployed to protect the shoreline around this staging area at the beginning of time period t ($z_{i,t}$). In other words, once the boom deployment at a staging area begins, it will not stop until sufficient booms have been deployed. The corresponding logic proposition is

$$zd_{i,t-1} \Rightarrow zd_{i,t} \vee z_{i,t} \,,$$

which can be transformed into the inequality

$$z_{i,t} \ge zd_{i,t-1} - zd_{i,t} \qquad \forall i \in I, t \in T \,. \tag{8}$$

Boom maintenance is required for the shoreline around staging area i at time period t ($zm_{i,t}$) if and only if the cleanup target has not been achieved (f_t) and the coastal protection booms are being deployed at time period t ($zd_{i,t}$) or fully deployed to protect the shoreline around staging area i ($z_{i,t}$). This relationship can be modeled with a logic

proposition as follows:

$$zm_{i,t} \Leftrightarrow f_t \wedge \left(zd_{i,t} \vee z_{i,t} \right),$$

where $zm_{i,t}$ is a binary variable that equals 1 if maintenance is required for booms around staging area *i*. This logic proposition can be transformed into the following inequalities.

$$zm_{i,t} \le f_t \qquad \forall i \in I, t \in T \tag{9}$$

$$zm_{i,t} \le zd_{i,t} + z_{i,t} \qquad \forall i \in I, t \in T \tag{10}$$

$$zm_{i,t} \ge f_t + zd_{i,t} - 1 \qquad \forall i \in I, t \in T \tag{11}$$

$$zm_{i,t} \ge f_t + z_{i,t} - 1 \qquad \forall i \in I, t \in T \tag{12}$$

The shoreline around staging area *i* is fully protected by the booms at time period *t* if and only if the length of boom ($bl_{i,t}$) is no less than the required length ($\underline{L}_i$) both at the beginning and at the end of time period *t*. Since the length of boom deployed at the beginning of time period *t* is the same as the one at the end of time period *t*-1, we use the following inequalities to model this constraint:

$$\underline{L}_i \cdot z_{i,t} \le bl_{i,t-1} \le \underline{L}_i + U \cdot z_{i,t} \qquad \forall i \in I, t \in T \tag{13}$$

$$\underline{L}_i \cdot z_{i,t} \le bl_{i,t} \le \underline{L}_i + U \cdot z_{i,t} \qquad \forall i \in I, t \in T, \tag{14}$$

where $\underline{L}_i$ is the length of boom required to protect the shore around staging area *i* and $bl_{i,t}$ is the length of boom deployed along the shore of staging area *i* at the end of time period *t*.

Coastal protection booms deployed at staging area *i* can be effective for only a certain lifetime (φ_i) after deployment. The length of the boom that fails at time period *t* is the same as the length deployed at time period $t - \varphi_i$.

$$bfail_{i,t} = bdep_{i,t-\varphi_i} \qquad \forall i \in I, t \in T \tag{15}$$

The length of the boom around the shore of staging area *i* at the end of time period *t* ($bl_{i,t}$) is equal to the boom length at the end of the previous time period ($bl_{i,t-1}$) plus the length of the boom deployed at the current time period ($bdep_{i,t}$) minus those that fail at this time period ($bfail_{i,t}$). Thus, the balance of boom length is given by the following

equation.

$$bl_{i,t} = bl_{i,t-1} + bdep_{i,t} - bfail_{i,t} \qquad \forall i \in I, t \in T \tag{16}$$

If booms are deployed at time period t along the shoreline near staging area i ($zd_{i,t}$), then the length of the boom deployed at this time should be bounded by the maximum and minimum deployment rates times the length of time period t (H_t). Thus, we have

$$BDL_i \cdot H_t \cdot zd_{i,t} \le bdep_{i,t} \le BDU_i \cdot H_t \cdot zd_{i,t} \qquad \forall i \in I, t \in T, \tag{17}$$

where BDL_i and BDU_i are the minimum and maximum boom deployment rates, respectively.

The balance of the boom length at staging area i at the end of time period t is given by the following equation:

$$binv_{i,t-1} + \sum_j btr_{i,j,t-\tau_{i,j}^{boom}} = bdep_{i,t} + binv_{i,t} \qquad \forall i \in I, t \in T, \tag{18}$$

where $binv_{i,t}$ is the length of available boom at staging area i at the end of time period t, $btr_{i,j,t}$ is the length of coastal protection boom transported from storage location j to staging area i at the beginning of time period t, $bdep_{i,t}$ is the length of boom deployed in staging area i at time period t, and $\tau_{i,j}^{boom}$ is the transportation time of coastal protection boom from storage location j to staging area i. Equation (18) shows that the length of the coastal protection boom available at the end of previous time periods plus those booms that arrive at the current time period (after transportation time $\tau_{i,j}^{boom}$) equals the sum of the lengths of boom that are deployed and remaining at the end of the current time period.

All the booms shipped from storage location j should not exceed the available length,

$$\sum_{i \in I} \sum_{t \in T} btr_{i,j,t} \le BA_j \qquad \forall j \in J, \tag{19}$$

where BA_j is the available length of coastal protection booms in storage location j. The transportation capacity of the booms is also subject to the following capacity constraint:

$$btr_{i,j,t} \le BTU_{i,j,t}^{boom} \qquad \forall i \in I, j \in J, t \in T, \tag{20}$$

where $BTU_{i,j,t}^{boom}$ is the transportation capacity from storage location j to staging area i at time period t.

4.3 Cleanup planning constraints

In order to respond to an oil spill, specialized cleanup facilities must be deployed to clean (and recover) the oil. Three common cleanup methods are mechanical cleanup and recovery, in situ burning, and chemical dispersants. Mechanical systems can skim the oil slick and recover oil from the emulsion; in situ burning and chemical dispersants remove oil only from the surface of the sea.

The total number of mechanical systems *m* that are notified to be staged at staging area *i* for the cleanup operations should not exceed the corresponding available number. This relationship is given by the following inequality:

$$\sum_{t} y_{i,m,t}^{M} \leq N_{i,m}^{M} \qquad \forall i \in I, m \in M, \tag{21}$$

where $y_{i,m,t}^{M}$ is the number of mechanical systems *m* that are notified at time period *t* to be staged at staging area *i* and $N_{i,m}^{M}$ is the available number of mechanic systems *m* that can be staged to staging area *i*.

We define $x_{i,m,t}^{M}$ as the number of mechanical systems *m* from staging area *i* that is operating atn the scene at time period *t*. It should not exceed the total number of mechanical systems *m* that have been notified to join the cleanup operation at or before time period $t - \lambda_{i,m}^{M}$, where $\lambda_{i,m}^{M}$ is the response time of mechanic systems *m* at staging area *i* (including the time to notify, mobilize, dispatch, and deploy the system).

$$x_{i,m,t}^{M} \leq \sum_{t'=1}^{t-\lambda_{i,m}^{M}} y_{i,m,t'}^{M} \qquad \forall i \in I, m \in M, t \in T \tag{22}$$

The volume of oil cleaned and recovered from the sea surface with mechanical systems at time period *t* (u_t^M) is given by the following equation.

$$u_t^M = \sum_{i} \sum_{m} \eta_t \cdot H_t \cdot \omega_t^M \cdot Q_{i,m}^M \cdot x_{i,m,t}^M \qquad \forall t \in T, \tag{23}$$

where $Q_{i,m}^M$ is the operating capability of mechanical system *m* dispatched from staging area *i* , ω_t^M is the weather factor (between 0 and 1) for the performance of mechanic systems at time *t*, and η_t is the percentage of oil that can be recovered in the emulsion collected by skimmers at time *t*. The value of the weather factor ω_t^M can be determined

by weather forecasting. For instance, a sunny day with zero wind speed and zero wave height (i.e., calm sea) has a weather factor close to 1 because skimmers perform best under such conditions. On the other hand, strong winds and high waves coupled with heavy rain may lead to a weather factor close to zero. The percentage of oil in the emulsion (η_t) can be derived from the fractional water content Y_W through the following equation: $\eta_t = 1 - Y_W^*(t)$, where $Y_W^*(t)$ is the fractional water content at time *t* based on the prediction of the oil weathering model.

For in situ burning response systems, we have availability constraints similar to those of the mechanical systems.

$$\sum_t y_{i,b,t}^B \le N_{i,b}^B \qquad \forall i \in I, b \in B \tag{24}$$

$$x_{i,b,t}^B \le \sum_{t'=1}^{t-\lambda_{i,b}^B} y_{i,b,t'}^B \qquad \forall i \in I, b \in B, t \in T, \tag{25}$$

where $y_{i,b,t}^B$ is the number of in situ burning response systems *b* that are notified at time period *t* to be staged at staging area *i*, $x_{i,b,t}^B$ is the number of in situ burning response systems *b* from staging area *i* operating at the scene at time period *t*, $N_{i,b}^B$ is the available number of in situ burning response systems *b* that can be staged to staging area *i*, and $\lambda_{i,b}^B$ is the corresponding response time.

The volume of oil burned by the in situ burning response systems at time period *t* (u_t^B) is given by the following equation:

$$u_t^B = \sum_i \sum_m H_t \cdot \omega_t^B \cdot Q_{i,b}^B \cdot x_{i,b,t}^B \qquad \forall t \in T, \tag{26}$$

where $Q_{i,b}^B$ is the operating capability of in situ burning system *b* dispatched from staging area *i* and ω_t^B is the weather factor for the application of in situ burning at time *t*.

In situ burning response system *b* cannot operate when the thickness of the oil slick is less than the minimum requirement ($THICK_b$):

$$x_{i,b,t}^B = 0 \qquad \forall i \in I, b \in B, t \in T \text{ and } \delta_t \le THICK_b \tag{27}$$

where *U* is a sufficiently large number for the upper bound of the oil slick thickness.

The availability constraint of chemical dispersant application systems is given by,

$$\sum_{t} y_{i,d,t}^{D} \le N_{i,d}^{D} \qquad \forall i \in I, d \in D \ , \tag{28}$$

where $y_{i,d,t}^{D}$ is the number of chemical dispersant application systems d that are notified at time period t to be staged at staging area i and $N_{i,d}^{D}$ is the corresponding availability.

We define $x_{i,d,t}^{D}$ as the number of sorties of chemical dispersant application systems d dispatched from staging area i at time period t to spray dispersants. Note that the number of sorties is different from the number of dispersant application systems since a dispersant application system can have multiple sorties per time period (e.g., a helicopter may operate more than 10 sorties per day for an offshore oil spill within 100 miles from the air station). Thus, we similarly have the following constraint to restrict the maximum number of sorties per time period:

$$x_{i,d,t}^{D} \le \gamma_{i,d,t} \cdot \sum_{t'=1}^{t-\lambda_{i,d}^{D}} y_{i,d,t'}^{D} \qquad \forall i \in I, d \in D, t \in T \ , \tag{29}$$

where $\gamma_{i,d,t}$ is the maximum number of sorties of dispersant application systems d from staging area i to spray dispersant on the oil slick at time period t.

The volume of oil removed from the sea surface by using chemical dispersants at time period t (u_t^D) is given by the following equation:

$$u_t^D = \sum_{i} \sum_{d} \omega_t^D \cdot \rho_t^{effect} \cdot \rho_d^{accuracy} \cdot Q_{i,d}^D \cdot x_{i,d,t}^D \qquad \forall t \in T \ , \tag{30}$$

where $Q_{i,d}^D$ is the operating capacity of chemical dispersant application systems d dispatched from staging area i , ω_t^D is the weather factor for the application of chemical dispersants at time t, ρ_t^{effect} is the effectiveness factor (ratio between oil dispersed and dispersant sprayed) for chemical dispersant application at time t, and $\rho_d^{accuracy}$ is the accuracy factor (percentage of sprayed dispersant that can reach the oil slick) of chemical dispersant application systems d.

The balance of chemical dispersants at staging area i at the end of time period t is given by the following equation:

$$dinv_{i,t-1} + \sum_{k} dtr_{i,k,t-\tau_{i,k}^{dispersant}} = dinv_{i,t} + \sum_{d} Q_{i,d}^D \cdot x_{i,d,t}^D \qquad \forall i \in I, t \in T \ , \tag{31}$$

where $dinv_{i,t}$ is the amount of available chemical dispersant at staging area i at the end of

time period t, $dtr_{i,k,t}$ is the amount of chemical dispersant shipped from supplier location k to staging area i at the beginning of time period t, and $\tau_{i,k}^{dispersant}$ is the transportation time for moving the chemical dispersant from the supplier location k to staging area i . The term $\sum_d Q_{i,d}^D \cdot x_{i,d,t}^D$ is the total amount of chemical dispersant used by dispersant application systems dispatched from staging area i at time period t. The equation shows that the amount of chemical dispersant available at the end of previous time periods plus the amount of dispersant arriving at the current time period (after transportation time $\tau_{i,k}^{dispersant}$) equals the sum of the amount of chemical dispersant that is used in the current time period and the remaining amount at the end of the current time period. Note that we do not consider the selection of chemical dispersants in this work, because only one type of dispersant is used in most oil spill responses (Michel et al., 2005).

The chemical dispersants shipped from supplier location k to all the staging areas should be less than or equal to the available amount ($CDS_{k,t}$).

$$\sum_i dtr_{i,k,t} \le CDS_{k,t} \qquad \forall k \in K, t \in T \tag{32}$$

The total amount of chemical dispersant used throughout the entire response operation should not exceed the limit set by the regulator (*DLIMIT*).

$$\sum_i \sum_k \sum_t dtr_{i,k,t} \le DLIMIT \tag{33}$$

4.4 Objective functions

Two objective functions are included in this model: responsiveness and economics. Responsiveness is measured by the total time span of the entire response operations, which can be modeled by the following equation.

$$\min: \ TimeSpan = \sum_t f_t \tag{34}$$

Here f_t is a binary variable that equals 1 if the volume of the oil slick is greater than the cleanup target or there is oil newly released to the sea surface at time period t. Summation of all the f_t over the planning horizon equals the number of time periods used for the response operations, because after the spillage stops, the volume of oil slick can only decrease over time as a result of evaporation and natural dispersion. We should note that

constraint (5) imposes that f_t must be 1 if the oil spillage has not yet stopped at time period t.

Economics is measured by the total cost, as follows:

$$\begin{aligned}\min:\ TotalCost = & \sum_i\sum_m\sum_t FC^M_{i,m}\cdot y^M_{i,m,t} + \sum_i\sum_b\sum_t FC^B_{i,b}\cdot y^B_{i,b,t} + \sum_i\sum_b\sum_t FC^D_{i,d}\cdot y^D_{i,d,t} \\ & + \sum_i\sum_m\sum_t C^M_{i,m,t}\cdot x^M_{i,m,t} + \sum_i\sum_b\sum_t C^B_{i,b,t}\cdot x^B_{i,b,t} + \sum_i\sum_b\sum_t C^D_{i,d,t}\cdot x^D_{i,d,t} \\ & + \sum_i\sum_t CI_i^{dispersant}\cdot dinv_{i,t} + \sum_i\sum_t CI_i^{boom}\cdot binv_{i,t} \\ & + \sum_i\sum_j\sum_t CT^{boom}_{i,j}\cdot btr_{i,j,t} + \sum_i\sum_k\sum_t CT^{dispersant}_{i,k}\cdot dtr_{i,k,t} \\ & + \sum_i\sum_t CDEP^{boom}_{i,t}\cdot bdep_{i,t} + \sum_i\sum_t FCDEP^{boom}_{i,t}\cdot zd_{i,t} \\ & + \sum_i\sum_t \omega^{boom}_{i,t}\cdot CBM^{boom}_{i,t}\cdot bl_{i,t} + \sum_i\sum_t FCBM^{boom}_{i,t}\cdot zm_{i,t} \\ & - \sum_t u^M_t\cdot OC\end{aligned} \tag{35}$$

where the first three terms are for the fixed cost (including mobilization, equipment transportation and set-up costs) of staging response systems, the fourth to sixth terms account for operating cost of the cleanup operations, the seventh to tenth terms are the inventory, purchase and transportation cost of dispersants and coastal protection booms, the eleventh to fourteenth terms are the fixed and variable costs for boom deployment and maintenance, and the last term is the credit resulting from the recovery of the spilled oil.

5. Solution Approach

In this section we first discuss two key model properties. We then describe our optimization procedure.

5.1 Model properties

The proposed MILP model has the following two properties that can be used to improve its computational efficiency.

Property 1. *The binary variable $zm_{i,t}$ can be relaxed as a continuous variable without changing the optimal solution.*

Proof: Because f_t, $zd_{i,t}$, and $z_{i,t}$ are all binary variables, constraints (24)–(26) impose that $zm_{i,t}$ equals either zero or 1. □

Property 2. *The binary variables $y^M_{i,m,t}$ and $y^B_{i,b,t}$ can be relaxed as continuous variables without changing the optimal solution.*

Proof: Variable $y^M_{i,m,t}$ appears only in constraints (34) and (35) except the objective function (48). Because $x^M_{i,m,t}$ is an integer variable and parameter $N^M_{i,m}$ has an integer value, the constraints for variable $y^M_{i,m,t}$ define an integer polyhedron after fixing the values of other variables (Nemhauser & Wolsey, 1988). Therefore, $y^M_{i,m,t}$ takes only integer values, and it can be relaxed as a continuous variable. Similarly, we can prove that $y^B_{i,b,t}$ has the same property. □

Based on these two properties, we can relax integer variables $zm_{i,t}$, $y^M_{i,m,t}$, and $y^B_{i,b,t}$ and improve the computational efficiency of solving the MILP model.

5.2 Solution procedure for multi-objective optimization

In order to obtain the Pareto-optimal curve for the bicriterion optimization problem, one of the objectives is specified as an inequality with a fixed value for the bound that is treated as a parameter. Two major approaches can be used to solve the problem in terms of this parameter. One is simply to solve it for a specified number of points to obtain an approximation of the Pareto-optimal curve. The other is to solve it as a parametric programming problem (Dua & Pistikopoulos, 2004), which yields the exact solution for the Pareto-optimal curve. While the latter approach provides a rigorous solution approach, the former approach is easier to implement. Moreover, the objective of response time span is represented by the number of time periods; hence, solving the problem by minimizing the total cost with all the possible values of time span, which is finite, will yield the exact solution of the Pareto-optimal curve for the proposed model. Therefore, we use the ϵ-constraint method to solve the proposed model. The procedure

comprises three steps. First, we to minimize the response time span to obtain the shortest time span $TimeSpan_S$. Second, we minimize the total cost that in turn yields the longest Pareto-optimal time span $TimeSpan_L$. In this case the objective function is set as

$$\min: \ TotalCost + \chi \cdot TimeSpan \ , \tag{36}$$

where χ is a very small value (on the order of 0.01). Third, we fix ε to discrete integer values between $TimeSpan_S$ and $TimeSpan_L$ and add the following constraint to the model with the objective to minimize *TotalCost*.

$$TimeSpan \le \varepsilon \tag{37}$$

In this way we can obtain the exact solution of the Pareto-optimal curve for the proposed model, together with the optimal solutions for different values of time span.

6. Case Studies

To illustrate the application of the proposed model, we consider two case studies based on the incidents of *Deepwater Horizon* oil spill in the Gulf of Mexico and *Argo Merchant* oil spill in New England. The computational studies were performed on an IBM T400 laptop with Intel 2.53 GHz CPU and 2 GB RAM. The ordinary differential equation (ODE) model for the oil transport and weathering processes was coded in MATLAB and solved with Runge–Kutta fourth/fifth-order method. The MILP model for oil spill response planning was coded in GAMS 23.4.3 and solved by using CPLEX 12. The optimality tolerances were all set to 10^{-9}.

6.1 Case study 1: oil spill in the Gulf of Mexico

In the first case study, we consider the response to an oil spill incident in the Gulf of Mexico area. There are three major staging areas for the response operations: S1, S2, and S3,. Their locations, along with the spill site, are given in the map in Figure 3. The minimum distances between the three staging areas and the oil spill site are 60 kilometers, 120 kilometers, and 180 kilometers, respectively. In this case, we assume the oil slick drifts toward the shore as a result of wind and current directions. The lengths of the booms required to protect the sensitive coastline near the three staging areas are 200 kilometers, 180 kilometers, and 300 kilometers, respectively. The spilled oil is considered

as crude oil with an API degree of 25. The oil releases continue for 42 days with a constant release rate of 10,000m^3/day, and the initial spill amount before time zero is 10,000m^3. The cleanup target is that no more than 1,500m^3 of oil remain on the sea surface after the response. Three types of mechanical systems, two types of in situ burning systems, and three types of dispersant application systems (vessel, helicopter, and C-130) are considered in the cleanup operations. Each type has a corresponding operating capacity, available number, fixed and variable cost, response time, and so forth. All the other input data are available upon request.

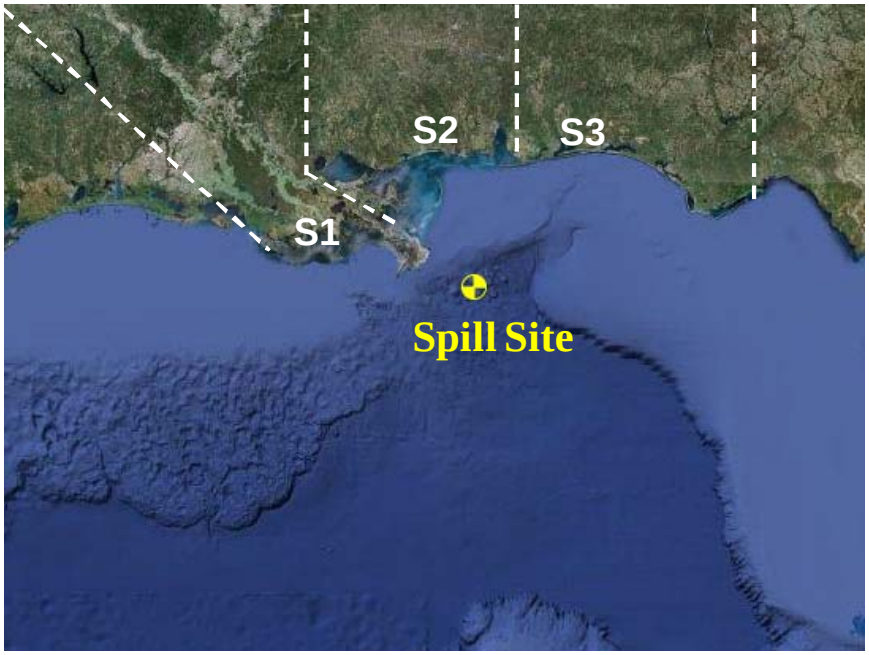

Figure 3. Oil spill site and the locations of the three staging areas S1, S2, and S3 for case study 1

In the oil weathering model given in the appendix, we consider weathering processes such as spreading, evaporation, natural dispersion, and emulsification. Two ODE problems for the weathering process before and after the spillage stops are solved with the ODE45 solver in MATLAB. The solutions are then exported and become inputs of the MILP model for oil spill response planning. The solution of the oil weathering model shows that under natural weathering (i.e., no response actions taken), the volume of oil remaining on the sea surface will reduce to 1,500 m^3, which is the cleanup target, in 179 days. Thus, we consider a planning horizon of 179 days with one day as a time period in

the MILP model. After relaxing integer variables $zm_{i,t}$, $y^{M}_{i,m,t}$, and $y^{B}_{i,b,t}$ to reduce the computational complexity, the MILP-based planning model includes 5,499 discrete variables, 8,543 continuous variables, and 13,884 constraints.

We use the ε–constraint method to obtain the Pareto-optimal curve and determine the trade-off between the total response cost and the responsiveness, which is measured by the response time span. The first step of the ε–constraint method is to determine the optimal lower and upper bounds of the response time span. The lower bound is obtained by minimizing (34) subject to constraints (1)–,(33) and the upper bound can be obtained by minimizing (36) subject to the same constraints. For this problem, we obtain 75 days as the lower bound of time span and 179 days as the optimal upper bound of time span. We then solve the problem with fixed values from 75 days to 179 days (105 instances with increments of one day). The solution process takes a total of 307 CPU-seconds for all 105 instances.

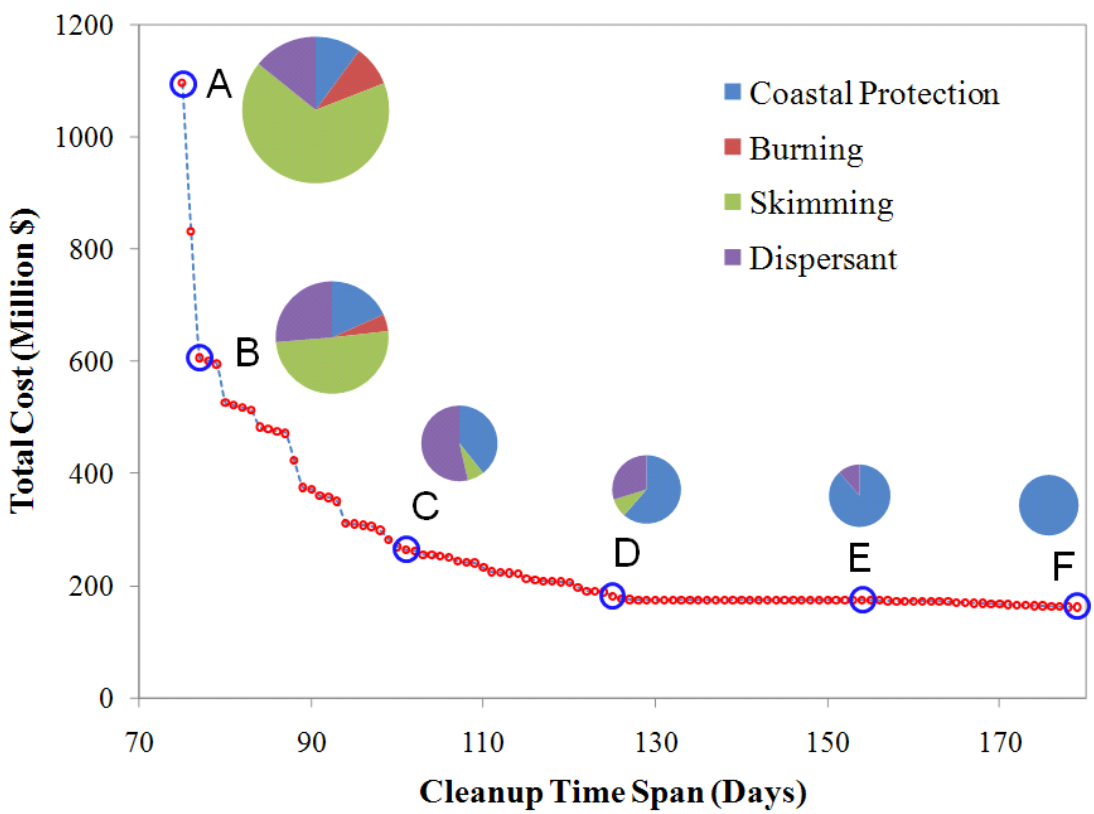

Figure 4. Pareto curve and cost breakdown for case study 1

The results are given in Figures 4–11. The line in Figure 4 is the Pareto-optimal curve of this problem. As can be seen, the total cost ranges from $1,095MM to $162MM, while the response time span ranges from 75 days to 179 days. Thus, the total cost decreases as the time span increases. Since the time span is a measure of responsiveness, we can conclude that the more responsive the response operations is, the more cost it requires. In particular, when the response time span increases from 75 days (Point A) to 77 days (Point B), the total cost reduces almost by half. This suggests that 77 days might be a better choice for the oil spill response based on the trade-off between economics and responsiveness. Moreover, we can see that when the response time span decreases from 178 days (Point F) to 125 days (Point D), the total cost increases only from $162MM to $182MM. In other words, a 10% increase of the cost can reduces the response time span from half a year to four months. Clearly, Point D is a better choice than Point F in the oil spill response operations.

The pie charts in Figure 4 are for the breakdown of the total costs for Points A–F. For short time spans, most of the cost is for oil spill cleanup (skimming, burning, and dispersant). Because of the high operational responsiveness in these cases, most of the shoreline will not be hit by the oil slick, and thus there is relatively low cost for boom deployment for coastal protection. As the time span increases and the total cost decreases, more is spent on coastal protection than on oil spill cleanup. The reason is that the least-cost option for this case study is to deploy booms to protect the sensitive shorelines while leaving the oil slick on the sea surface until natural weathering reduces the oil volume to the cleanup target; that is, no cleanup efforts are taken in the least-cost instance.

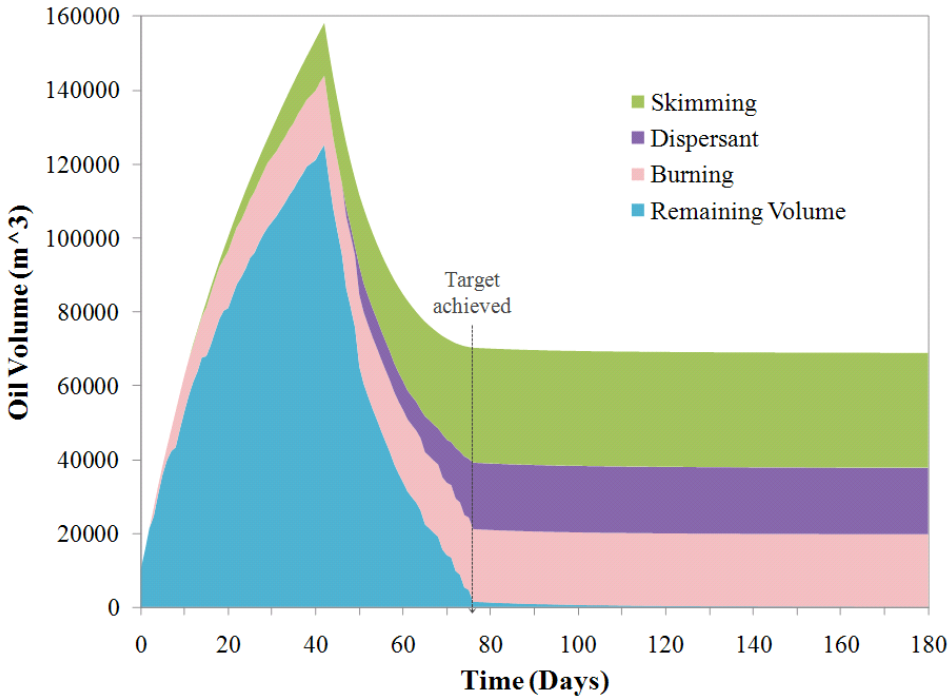

Figure 5. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 75 days (Point A in Figure 4)

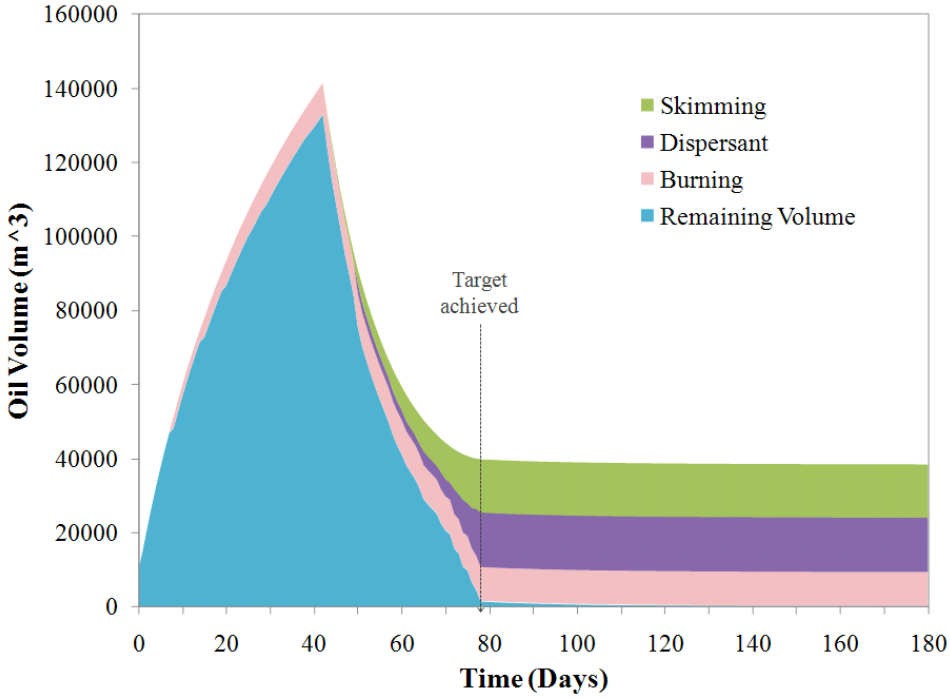

Figure 6. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 77 days (Point B in Figure 4)

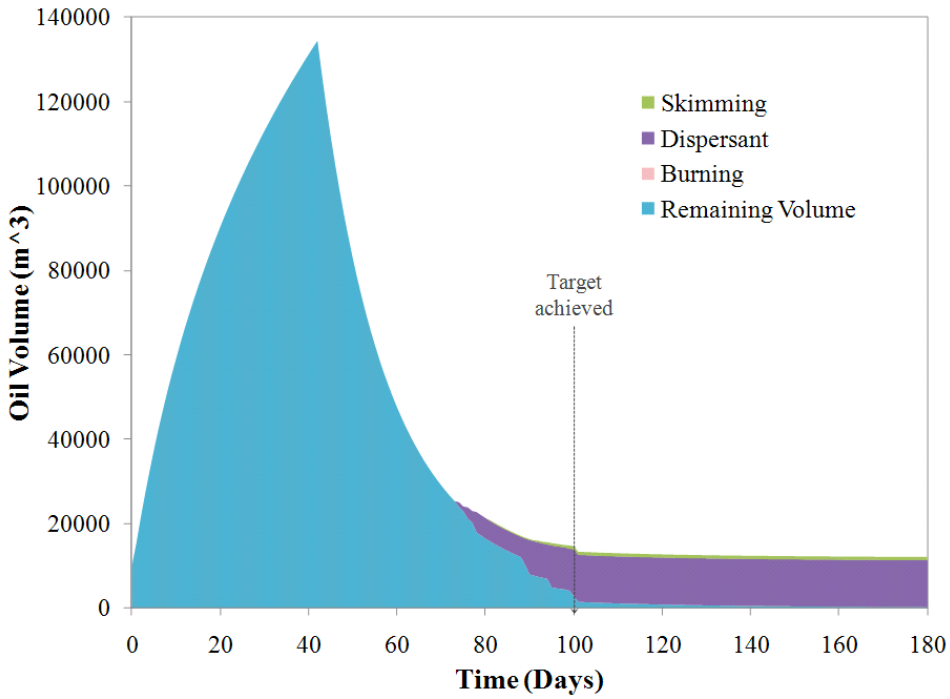

Figure 7. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 100 days (Point C in Figure 4)

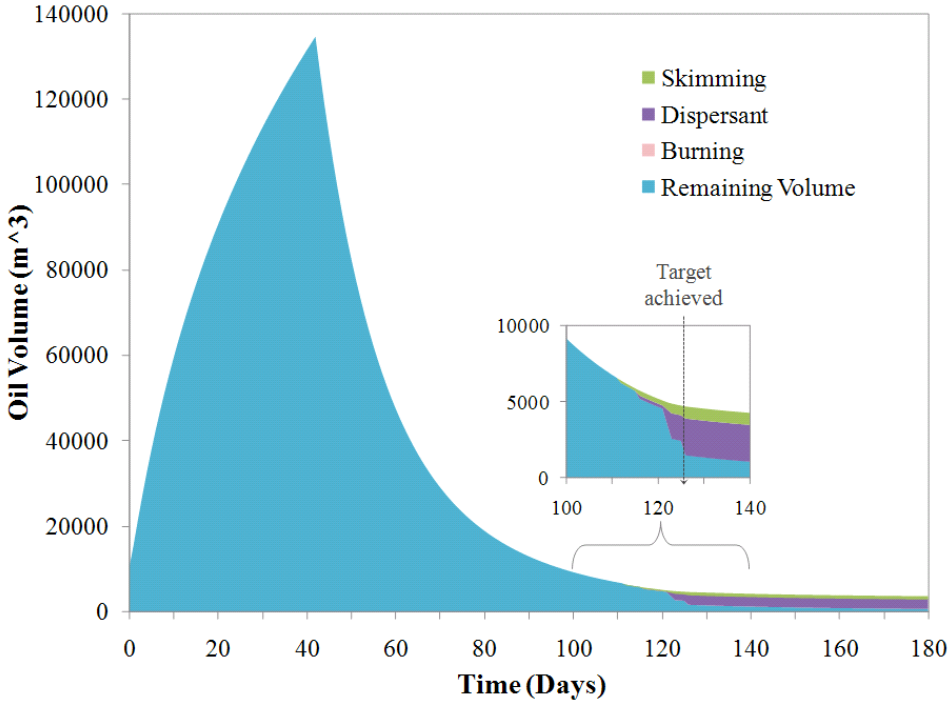

Figure 8. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 125 days (Point D in Figure 4)

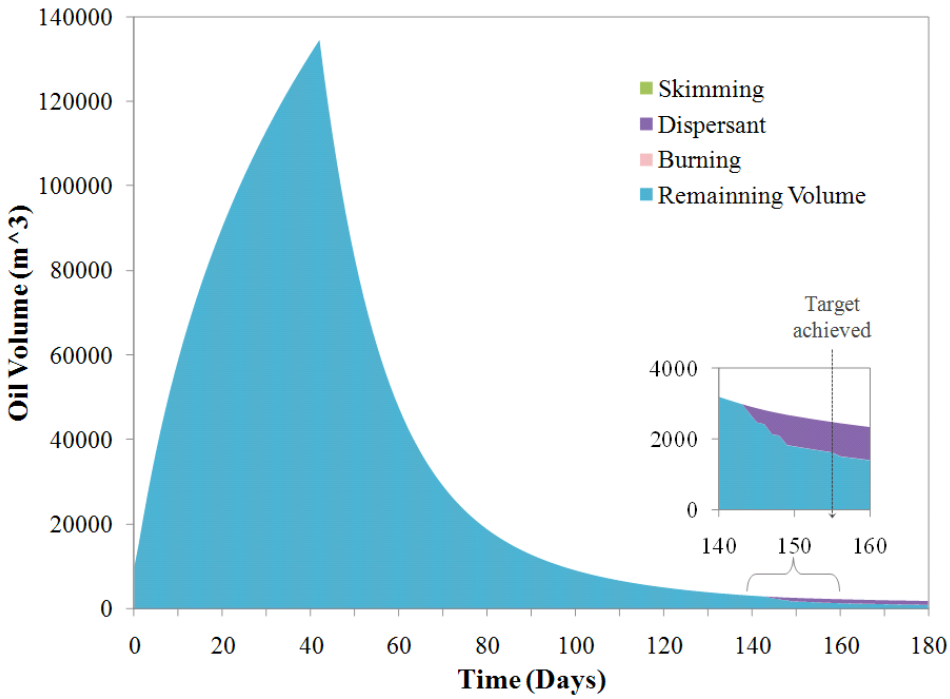

Figure 9. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 155 days (Point E in Figure 4)

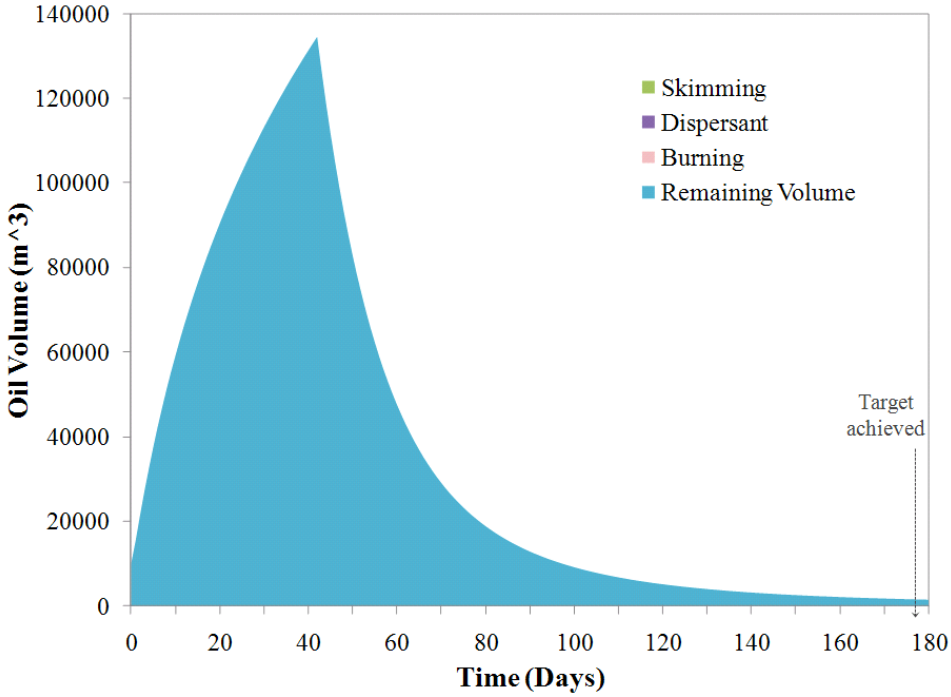

Figure 10. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 179 days (Point F in Figure 4)

Figures 5–10 show the time trajectories of the oil volume throughout the response operations for the six points A–F in Figure 4, where time spans are 75 days, 77 days, 100 days, 125 days, 155 days, and 179 days. We note that the time trajectory of the oil volume shown in Figure 10 is the same as that for natural weathering, where no cleanup effort was taken throughout the operations. For all these figures, we can see a similar trend that the volume of remaining oil first increases from Day 0 to Day 42 and then decreases. The reason is that the oil was being released at a constant rate to the sea surface before Day 42, and this release rate is much higher than the removal capability of natural weathering and all cleanup facilities.

By comparing Figures 5 and 10, however, we can conclude that the maximum volume of oil on the sea surface can be reduced from around 140,000m^3 to around 120,000m^3 if sufficient cleanup efforts are taken in the early stage of the spill. These figures also reveal that the more cleanup operations are taken, the earlier the cleanup target can be achieved. Comparison between the three major cleanup methods in terms of the volumes of oil removed by them shows that dispersant application is usually the most favorable cleanup approach due of its flexibility in various weather conditions. For the most responsive instance, however, where a lot of oil needs to be removed by cleanup operations, skimming becomes as important as dispersant application. Presumably the main reasons are that the maximum amount of chemical dispersant that can be applied is controlled by the regulator due to ecological concerns and that skimming has relatively low requirement of weathering conditions compared to burning. An additional reason is that mechanical cleanup can gain credit from oil recovery, which in turn reduces the total cost.

The time trajectories of the length of coastal protection booms deployed around the three staging areas for the least-cost solution (time span of 179 days) are given in Figure 11. As discussed before, no cleanup operations were taken in this instance, and coastal protection booms were deployed before the oil slick hit the shore. We can see from Figure 11 that the three staging areas start to deploy booms from Day 5, Day 16, and Day 22, respectively. The different starting days are due to the different distances between the staging area and the oil spill site. Although the three staging areas have different boom deployment rates, they generally follow the deployment-maintenance trend. Because the availability and transportation time of booms also affect coastal protection operations, we

can see from Figure 11 that the initial boom deployment rate of staging area 3 at Day 22 is less than the maximum deployment capability.

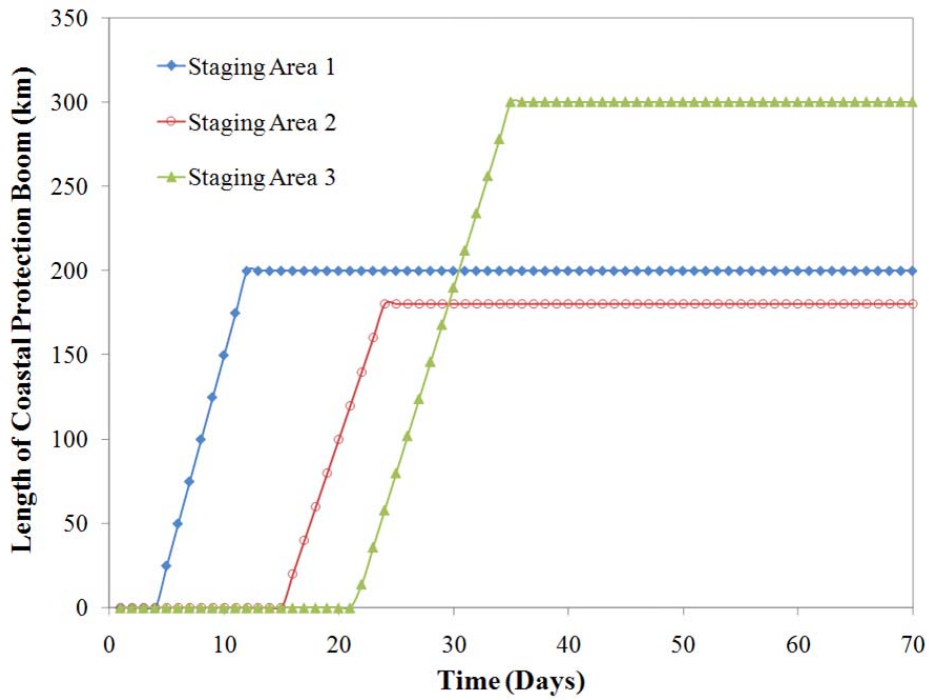

Figure 11. Optimal length of coastal protection boom when time span is 179 days

6.2 Case study 2: oil spill in New England

The second case study involves the response to an oil spill in the New England region. We again consider three major staging areas, whose locations, along with the spill site, are given in Figure 12. The minimum distances between the three staging areas and the oil spill site are 55 kilometers, 100 kilometers, and 80 kilometers, respectively. In this case, we assume the oil slick is moving off the shore as a result of drift. The lengths of boom required to protect the sensitive coastline near the three staging areas are 100 kilometers, 60 kilometers, and 60 kilometers, respectively. The spilled oil is No. 6 fuel oil with an API degree of 14. The oil releases continue for 6 days, with a constant release rate of 5,000m^3/day; no oil is released before time zero. The cleanup target is that no more than 100m^3 oil remains on the sea surface after the response. Similarly, three types of mechanical systems, two types of in situ burning systems, and three types of dispersant application systems are considered in the cleanup operations. All the other input data are available upon request.

Figure 12. Oil spill site and the locations of the three staging areas S1, S2, and S3 for case study 2

Similar to the previous case study, two ODE problems are solved with MATLAB for the process before and after the spillage stops. The results show that under natural weathering the cleanup target can be achieved in 57 days. Thus, we consider a planning horizon of 57 days with one day as a time period in the MILP model. The MILP-based planning model includes 1,568 discrete variables, 2,682 continuous variables, and 4,316 constraints. With the ε–constraint method, we obtain 12 days as the lower bound of the time span and 57 days as the optimal upper bound of the time span. We then solve the problem with fixed values of from 12 days to 57 days (46 instances with increments of one day). The entire solution process takes a total of 20 CPU-seconds for all 46 instances.

The Pareto-optimal curve of this case study is given in Figure 13. As can be seen, the total cost ranges from $70MM to $0, while the response time span ranges from 12 days to 57 days. We can similarly observe that the total cost decreases as the time span increases. In particular, we can see that when the response time span increases from 12 days (Point A') to 14 days (Point B'), the total cost reduces from $69MM to $43MM. When the response time span further increases to 22 days (Point C'), the total cost further reduces to $10MM. The pie charts in Figure 13 indicate the breakdown of the total costs for

Points A'–E'. It is interesting to note that none of the instances includes the cost for coastal protection. The reason is that the oil slick is moving offshore because of drift and it is not going to hit any shoreline. The cost breakdown of Points A'–E' shows that the optimal cleanup method changes under different time spans. For the most responsive case, a combination of three cleanup methods is required; but as the time span increases, either dispersant application or burning is required to achieve the cleanup target. In the longest time span case, we have the minimum total cost of $0, which implies that the natural weathering process will remove the oil from the sea surface and the cleanup target can be achieved in around two months.

Figures 14–18 show the time trajectories of oil volume throughout the response operations for the six points A'–E' in Figure 13, where time spans are 12 days, 14 days, 22 days, 42 days, and 57 days, respectively. The time trajectories are consistent with the cost breakdown shown in Figure 13. In particular, the time trajectory of oil volume shown in Figure 18 for the case that response time span is 57 days is the same as that for natural weathering, where no cleanup effort was taken throughout the operations.

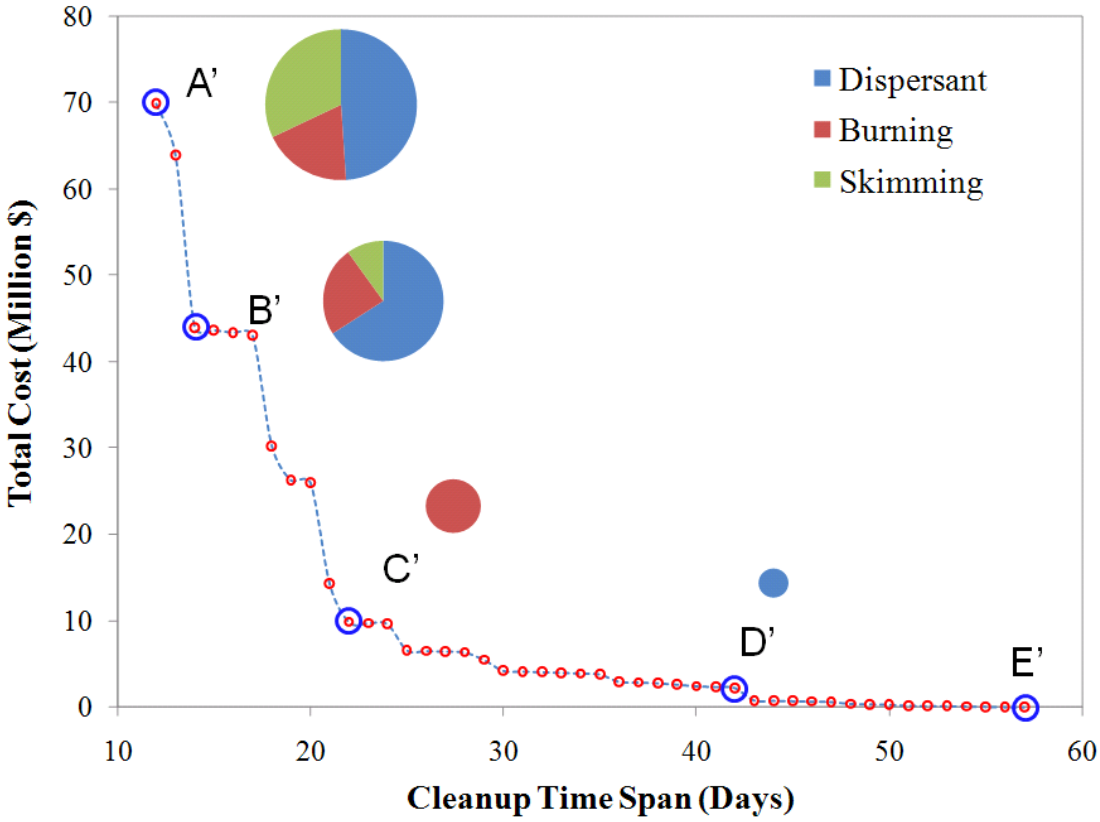

Figure 13. Pareto curve and cost breakdown for case study 2

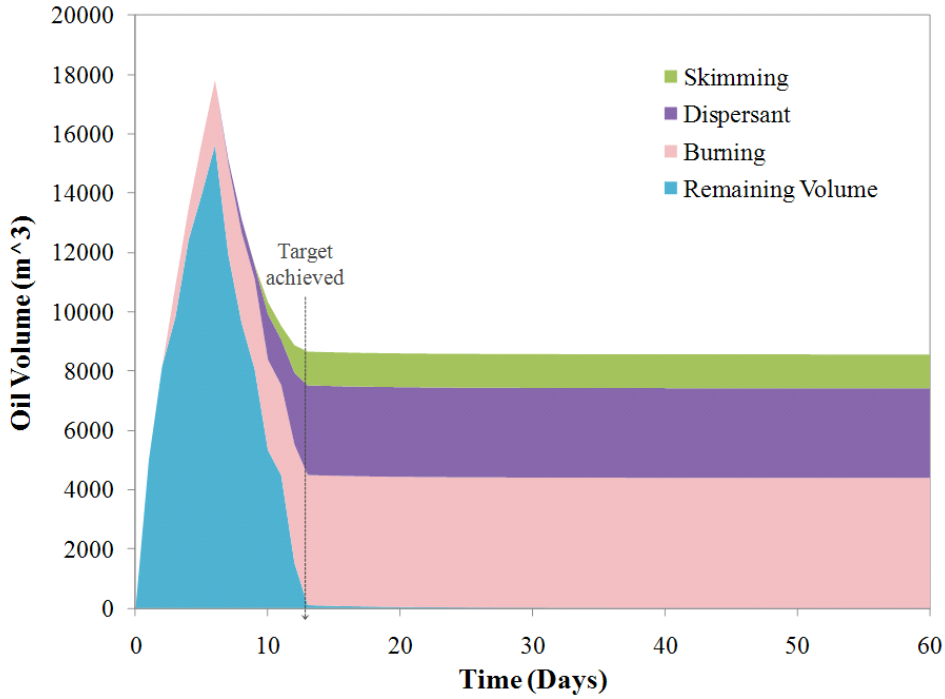

Figure 14. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 12 days (Point A' in Figure 13)

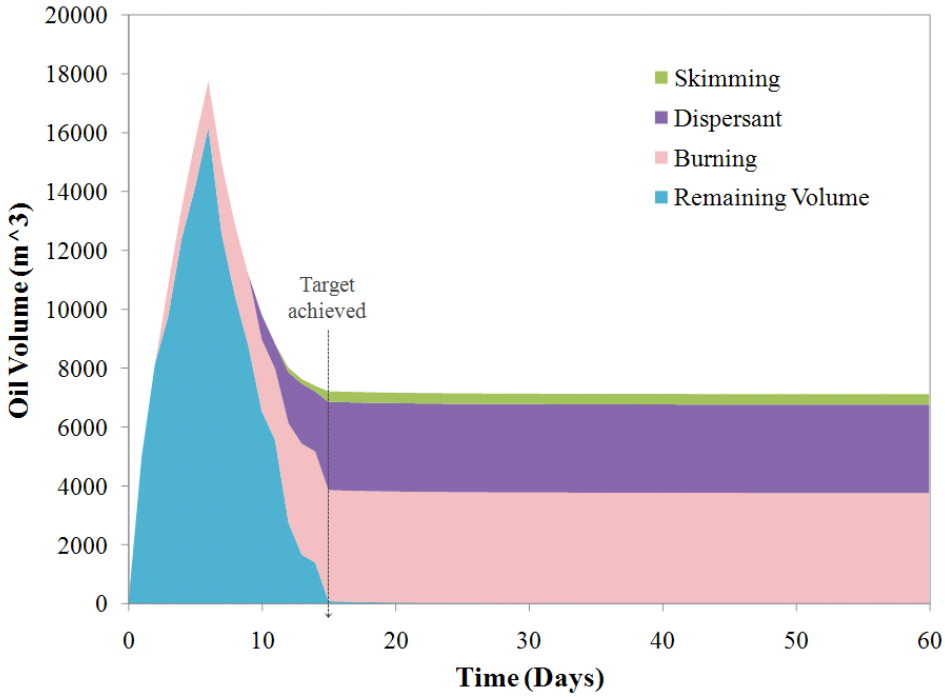

Figure 15. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 14 days (Point B' in Figure 13)

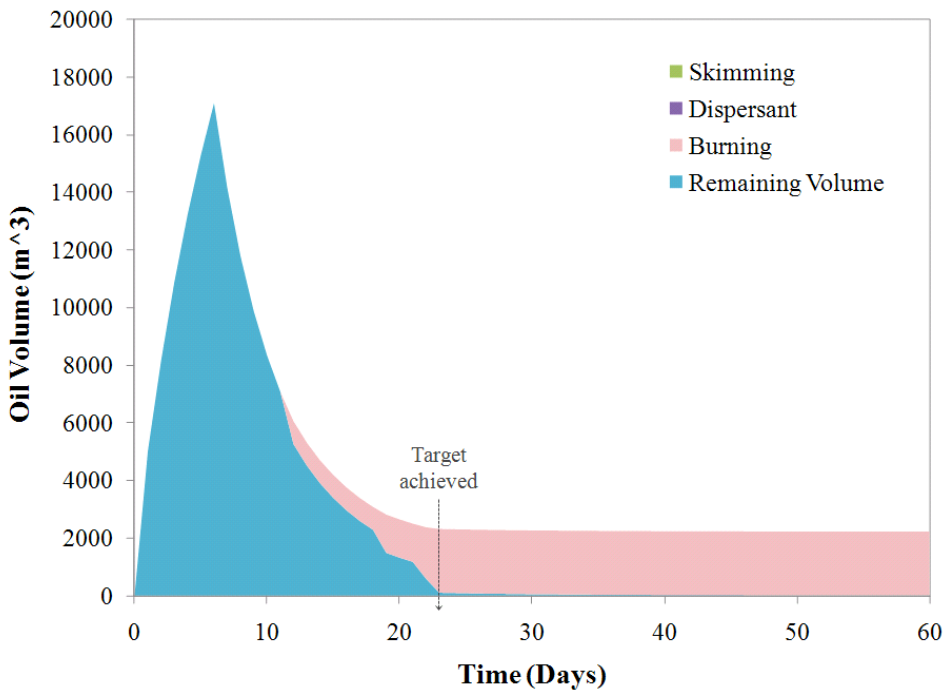

Figure 16. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 22 days (Point C' in Figure 13)

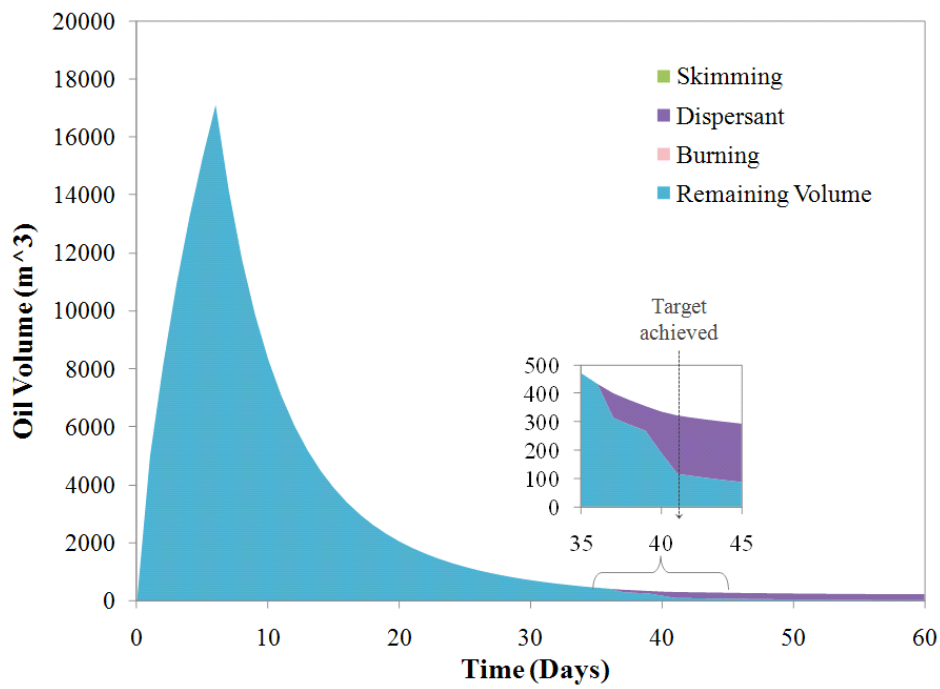

Figure 17. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 42 days (Point D' in Figure 13)

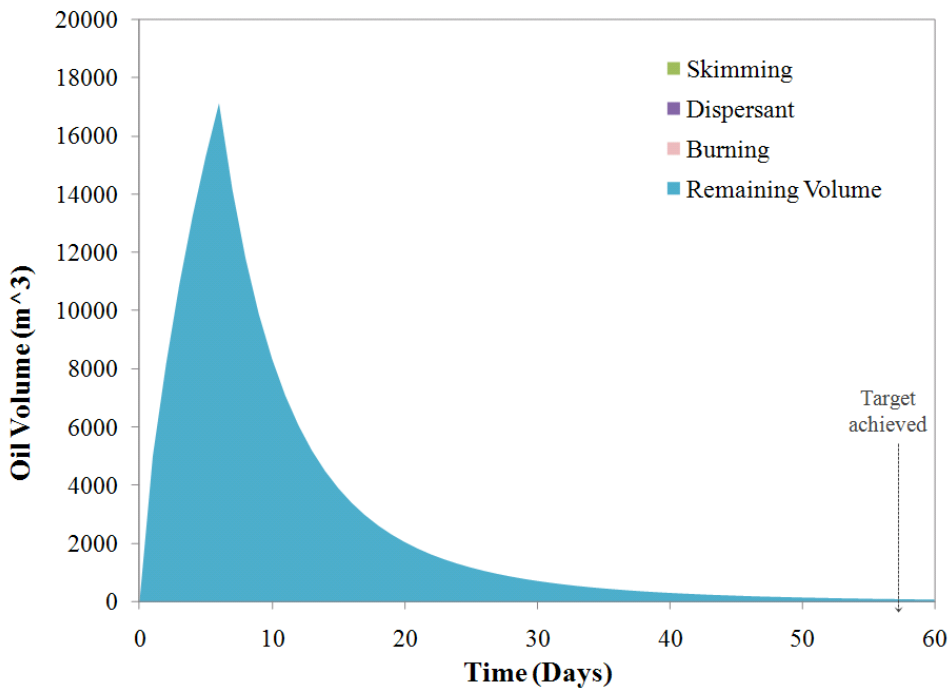

Figure 18. Time trajectories of the oil volumes removed by three methods and remaining on the sea surface when the time span is 57 days (Point E' in Figure 13)

7. Conclusion

In this paper, we have developed an optimization approach for oil spill response planning under the constraints of economic and responsive criteria, with consideration of the physiochemical evolution of oil slicks. A multiperiod MILP model was developed for optimizing these two criteria and was integrated with the predictions from an oil weathering model that takes into account the oil properties, spilled amount, hydrodynamics, and weather and sea conditions. The oil spill response-planning model simultaneously predicts the optimal time trajectories of the oil slick's volume and area, transportation and usage levels of response resources, oil spill cleanup schedule, and coastal protection plan. The multi-objective optimization model was solved with the ε-constraint method and produces a Pareto-optimal curve. Two examples based on realistic oil spill incidents were solved to illustrate the application of this model. The results show that small changes in response time span can lead to significant changes in the total cost and the corresponding response operations. These results in turn suggest the importance of simultaneously considering responsiveness and economics in oil spill response

planning.

A future extension of this research is to develop a mixed-integer dynamic optimization (MIDO) approach that seamlessly integrates the planning model with the oil weathering model. The solution of the resulting MIDO model is a nontrivial task and may require an initialization step based on the approach proposed in this work.

Acknowledgment

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Appendix: Oil Transport and Weathering Model

In the oil transport and weathering process, a variety of complex physical, chemical and biological phenomena take place simultaneously. The weathering process depends on the initial oil properties, the spilled amount, hydrodynamics, and weather conditions. All these factors vary with time. Thus, it is usually nontrivial to determine the weathering process of the spilled oil and the subsequent consequences. It is estimated that over 50 oil weathering models, based mainly on empirical and semi-empirical approaches, have been developed (Brebbia, 2001; Buchanan & Hurford, 1988; D. Mackay & McAuliffe, 1988; Reed et al., 1999; Sebastiao & Sores, 1995; Spaulding, 1988). Although any oil weathering model can, in principle, be used in the approach proposed in this paper, we present in this section an ODE model for the dynamic oil transport and weathering processes that predicts important parameters, such as time trajectories of the oil volume, spill size, and other basic oil properties (e.g., viscosity and water content), for the proposed response planning model. A list is given in the nomenclature section.

A.1 Spreading

The dominant processes that cause significant short-term changes in oil characteristics are spreading, evaporation, dispersion and emulsification. They all occur progressively at different rates depending on the oil properties and the weather condition.

Spreading of oil released on water is probably the most dominant process of a spill,

because it strongly influences other weathering processes such as evaporation and dispersion. Due to the gravity and surface tension, oils spilled on the surface of sea usually spread like a thin continuous layer with a circular pattern. The spreading process includes three phases (Fay, 1969). The first phase is the gravity-inertial spreading which lasts for very short time period (minutes to hours). The second phase, which covers the planning horizon of oil spill response in most cases, is attributed to combined gravity and viscosity phenomena. The third phase, the tension-viscous phase, occurs when the Oil slick is sufficiently thin due to weathering and broken into a few separated slicks. Therefore, most models consider mainly the second phase, known as the gravity-viscous spreading, for the simulation of spreading.

The rate of change of slick area for the second phase can be modeled with equation (38), which is widely used for oil spill models with multiple variables changing simultaneously (Mackay et al., 1980; Reed et al., 1999; Sebastiao & Sores, 1995; Spaulding, 1988).

$$\frac{dA}{dt} = K_1 A^{-1} V^{4/3} \tag{38}$$

where A is the surface area of oil slick (m^2), V is the volume of oil (m^3), and K_1 is the dominant physicochemical parameters of the crude oil in the gravity-viscous spreading process with default value of 150 s^{-1} (Mackay et al., 1980). As reported by CONCAWE (1983), the spreading behavior of two crude oils with almost comparable spreading coefficients but with a considerable difference in other properties is usually very similar.

The initial area of oil slick, A_0 (m^2) can be determined by the gravity-viscous formulation as follows (Fay, 1969):

$$A_0 = \pi \frac{k_2^4}{k_3^2} \left[\frac{(\rho_w - \rho_o) g V_0^5}{\rho_w v_w} \right]^{1/6} \tag{39}$$

where g is the acceleration of gravity ($m \cdot s^{-2}$), ρ_w is the density of seawater, ρ_o is the density of fresh oil, v_w is the kinematic viscosity of seawater ($0.801 \cdot 10^{-6}$ $m^2 s^{-1}$ under 30^oC), V_0 is the initial volume of the oil slick (spilled before time 0), t refers to time (s) and k_2 and k_3 are constants with values of 1.21 and 1.53, respectively (NOAA, 2000).

A.2 Evaporation

Evaporation is the primary initial process involved in the removal of oil from sea. The rate of evaporation is determined by the physicochemical properties of the oil and is increased by spreading, high water temperature, strong wind, and rough sea. By evaporation, low boiling components will rapidly be removed, thus reducing the volume of the remaining slick.

The rate that oil evaporates from the sea surface is modeled by the following equation (D. Mackay & Matsugu, 1973; Stiver & Mackay, 1984),

$$\frac{dF_E}{dt} = \frac{K_{ev}A}{V}\exp\left(A_{ev} - \frac{B_{ev}}{T_K}\left(T_o + T_G F_E\right)\right), \tag{40}$$

where F_E is the volume fraction of oil that has been evaporated until time t, T_K is the oil temperature (K), and A_{ev} and B_{ev} are empirical constants with fixed values of 6.3 and 10.3, respectively (NOAA, 2000). K_{ev} is the mass transfer coefficient for evaporation (ms^{-1}) and can be calculated by (Buchanan & Hurford, 1988),

$$K_{ev} = 2.5 \times 10^{-3} WIND^{0.78},$$

where $WIND$ is the wind speed (ms^{-1}). T_O and T_G are the initial boiling point and the gradient of the oil distillation curve, respectively. Their values can be obtained from the distillation curve of the specific oil spilled or can be calculated through functions of the oil API (American Petroleum Institute) degree as follows (NOAA, 2000):

$$T_O = 457.16 - 3.3447 \cdot \text{API}$$
$$T_G = 1356.7 - 247.36 \cdot \ln(\text{API})$$

At the beginning of the evaporation process, none of the oil has been evaporated. Thus, initial value of evaporative fraction at time 0 is set as zero.

$$F_{E(t=0)} = 0 \tag{41}$$

A.3 Emulsification

In emulsification, water droplets are entrained in the oil. This process results in significant changes in multiple physicochemical properties of oil slicks, such as viscosity. Crude oil will emulsify when the asphaltene content is higher than 5 mass percent of the

spilled oil. The dynamic emulsification process that incorporates water into oil can be computed with the following equation (Mackay et al., 1980):

$$\frac{dY_W}{dt} = K_{em} \cdot \left(WIND + 1\right)^2 \cdot \left(1 - \frac{Y_W}{C_3}\right), \tag{42}$$

where Y_W is the fractional water content in the emulsion, C_3 is a viscosity constant for the final fraction water content (~0.7 for crude oils), and K_{em} is an empirical constant between 1 x 10^{-6} and 2 x 10^{-6}.

Similar to the initial condition of evaporation process, the fractional water content at beginning of emulsification process is zero.

$$Y_{W(t=0)} = 0 \tag{43}$$

As a result of both Mousse formation and evaporation, the viscosity of oil slick may significantly increase during the emulsification process. The rate of changes in viscosity is given by (Mackay & McAuliffe, 1988; Mooney, 1951):

$$\frac{d\mu}{dt} = \frac{2.5\mu}{\left(1 - C_3 Y_W\right)^2}\frac{dY_W}{dt} + C_4 \mu \frac{dF_E}{dt}, \tag{44}$$

where μ is the viscosity of oil slickand C_4 is an oil-dependent constant equal to 10 for crude oils (Mackay et al., 1980; Sebastiao & Sores, 1995). Note that the first term in (44) corresponds to the Mooney equation for the viscosity increment rate due to Mousse formation, and the second term is the contribution by evaporation.

The initial value of the viscosity is the same as that of the parent oil viscosity, which can be calculated by the following equation (Buchanan & Hurford, 1988):

$$\mu_0 = 224 \times \sqrt{AC}, \tag{45}$$

where AC is the asphaltene content (%) of the parent oil.

A.4 Dispersion

Natural dispersion of crude oils after spillage at sea is the process of forming small droplets of oil to be transferred in the water column. Natural dispersion may account for a significant part of removal of oil from the sea surface in addition to evaporation. Besides the total volume of oil on the sea surface and the slick area, an important parameter

influencing natural dispersion is the oil/water interfacial tension, which affects globulation and coalescence, as well as the transport (dispersion) of oil droplets into the water column. The viscosity of the spilled oil also affects natural dispersion - the more viscous the oil is, the lower its ability to form oil droplets.

In this work, we use the approach proposed by Mackay et al. (1980) and Sebastiao & Sores (1995) to calculate the rate of dispersion into the water column of floating oil slick at the sea. The formulation is given as follows:

$$\frac{dV_D}{dt} = \frac{0.11 \cdot (WIND+1)^2 \cdot A \cdot V}{A + 50\zeta_t \cdot V \cdot \mu^{1/2}} \quad , \tag{46}$$

where V_D is volume of oil naturally diepersed and ζ_t is the oil-water interfacial tension.

The initial value of the volume of oil that is naturally dispersed is zero.

$$V_{D(t=0)} = 0 \tag{47}$$

A.5 Volume balance

Oil escapes from the surface slick by two major processes: evaporation and dispersion. As can be seen from Equations (40) and (46), these two processes depend on the slick surface, which changes over time due to spreading. Emulsification not only changes the viscosity of oil slick and affects the dispersion process but also results in the degradation of the cleanup capability of skimmers discussed earlier. The volume balance of the oil slick is based on the volume variation rate given by (Sebastiao & Sores, 1995):

$$\frac{dV}{dt} = -V\frac{dF_E}{dt} - \frac{dV_D}{dt} + VI \quad , \tag{48}$$

where the first term on the right-hand side is for the evaporation rate; the second term is for natural dispersion; and the third term, *VI*, is a time-dependent parameter of the oil spill rate. Most oil spill models consider two release types of oil spillage: the instantaneous release mode, which is for oil spilled in an hour or less, and the continuous release mode for oil spilled into the water over a given time duration with a fixed spill rate. If we define tf_1 as the time when the oil spillage stops and tf_2 as the final time of the planning horizon, then the time-dependent parameter *VI* is given by the following expression:

$$VI = \begin{cases} \text{constat spill rate}, & 0 \le t \le tf1 \\ 0, & tf1 \le t \le tf2 \end{cases} \tag{49}$$

The initial volume of oil slick is given as V_0, .

$$V_{(t=0)} = V_0 \tag{50}$$

Note that we have $tf_1 = 0$ in the instataneous release type; that is, $VI = 0$ over the planning horizon but V_0 is nonzero.

A.6 Solution method

The oil weathering model is a system of differential equations consisting of equations (38), (40), (42), (44), (46), and (48). Numerical solution of this model can be obtained by using the Runge-Kutta method. Because of the time-dependent oil spill rate *VI*, two ODE systems might need to be solved. The first problem is for the period that oil is spilling with a constant release rate, namely, from time 0 to tf_1. The initial conditions of this problem are given in (39), (41), (43), (45), (47), and (50). The second problem is for the period from the oil spillage stops to the end of the planning horizon, namely, from tf_1 to tf_2. The solution of the first ODE problem (i.e., the values of oil slick area A, volume V, evaporative volume fraction F_E, water content Y_W, oil slick viscosity μ, and naturally diepersed volume of oil V_D) at the final time tf_1 is used as the initial condition of the second ODE. The solution of the oil weathering model provides time trajectories of important physical and chemical parameters of the oil slicks.

Nomenclature

Nomenclature for the planning model

Sets/Indices

B: Set of in situ burning response system types indexed by *b*

D: Set of chemical dispersant application system types indexed by *d*

I: Set of staging areas (including airlift wing stations) indexed by *i*

J: Set of containment boom storage/supplier locations indexed by *j*

K: Set of dispersant supplier locations indexed by *k*

M: Set of skimming (mechanical cleanup & recovery) system types indexed by m

T: Set of time periods indexed by t, t'

Parameters

$\overline{AREA}_{i,t}$: The expected slick area that the oil slick would hit the shore around staging area i at time period t if no boom in this area was deployed for protection

BDU_i : Maximum deployment rate of boom in staging area i at time period t

BDL_i : Minimum deployment rate of boom in staging area i at time period t

BA_j : Available amount of coastal protection booms from storage location j

$BTU_{i,j,t}^{boom}$: Maximum transportation amount of coastal protection booms from storage location j to staging area i at time period t

$C_{i,m,t}^{M}$: Operating cost of mechanical cleanup and recovery system m dispatched from staging area i at time t

$C_{i,b,t}^{B}$: Operating cost of in situ burning system type b from staging area i at time t

$C_{i,d,t}^{D}$: Cost of dispatching a chemical dispersant application system d from staging area i at time t to spray a full-load dispersant

$CBM_{i,t}^{boom}$: Maintenance cost unit length coastal protection boom in staging area i at time t

$CDEP_{i,t}^{boom}$: Cost of deploying unit length of coastal protection boom in staging area i

$CDS_{k,t}$: Amount of chemical dispersant available in supplier location k at time period t

$CI_i^{dispersant}$: Unit inventory holding cost of chemical dispersant in staging area i at time t

CI_i^{boom} : Unit inventory holding cost of boom in staging area i at time t

$CT_{i,j}^{boom}$: Staging and transportation costs of unit length coastal protection booms from storage location j to staging area i

$CT_{i,k}^{dispersant}$: Purchase and transportation costs of unit volume chemical dispersant from supplier location k to staging area i

$DLIMIT$: Maximum amount of dispersant that can be applied in the cleanup

$FC_{i,m}^{M}$: Fixed cost (including mobilization, equipment transportation, and set-up costs) of staging mechanical cleanup and recovery system m dispatched to staging

area i

$FC_{i,b}^{B}$: Fixed cost (including mobilization, equipment transportation, and set-up costs) of staging in situ burning response system type b to staging area i

$FC_{i,d}^{D}$: Fixed cost (including mobilization, spray system transportation and system set-up costs) of staging chemical dispersant application system d to staging area i

$FCBM_{i,t}^{boom}$: Fixed cost of maintaining booms in staging area i at time period t

$FCDEP_{i,t}^{boom}$: Fixed cost of deploying boom in staging area i at time period t

H_t: Length of time period t

$\underline{L}_i$: Length of boom required to protect the shore around staging area i

$N_{i,b}^{B}$: Available number of in situ burning response system type b that can be dispatched from staging area i

$N_{i,d}^{D}$: Available number of chemical dispersant application system types d that can be dispatched from staging area i

$N_{i,m}^{M}$: Available number of mechanic cleanup and recovery system type m that can be dispatched from staging area i

OC: Unit price of recovered oil

$THICK_b$: Minimum slick thickness that in situ burning response system b can operate

$Q_{i,b}^{B}$: Operating capability of in-situ burning system type b from staging area i

$Q_{i,d}^{D}$: Full0load capacity of dispersant application system d from staging area i

$Q_{i,m}^{M}$: Operating capability of mechanical cleanup system m from staging area i

U: A sufficiently large number

V_0: Initial volume of oil spilled at time 0

VI_t: Volume of oil that was newly released to the sea surface at time t

$\underline{V}$: Cleanup target, maximum volume of oil left on the sea surface after cleanup

ρ_t^{effect}: Effectiveness factor (ratio between oil dispersed and dispersant sprayed) for chemical dispersant application operation at time t

$\rho_d^{accuracy}$: Accuracy factor (percentage of sprayed dispersant that can reach oil slick) of

chemical dispersant application system d

$\tau_{i,j}^{boom}$: Transportation time of boom from storage location j to staging area i

$\tau_{i,k}^{dispersant}$: Transportation time of dispersant from supplier location k to staging area i

$\lambda_{i,b}^{B}$: Total response time of in situ burning response system type b dispatched from staging area i (including the times to notify, mobilize, dispatch, and deploy the system)

$\lambda_{i,d}^{D}$: Total response time of chemical dispersant application system types d dispatched from staging area i (including the times to notify, mobilize, dispatch, and deploy the system)

$\lambda_{i,m}^{M}$: Total response time of mechanic cleanup and recovery system type m dispatched from staging area i (including the times to notify, mobilize, dispatch and deploy the system)

ω_{t}^{M}: Weather factor for mechanic cleanup and recovery operation at time t

ω_{t}^{B}: Weather factor for controlled burning operation at time t

ω_{t}^{D}: Weather factor for chemical dispersant application operation at time t

$\omega_{i,t}^{Boom}$: Weather factor for maintaining booms in staging area i at time t

$\gamma_{i,d,t}$: Maximum number of sorties of dispersant application system types d from staging area i to oil spill site in time period t

φ_{i}: Lifetime before failure for containment booms deployed at staging area i

η_{t}: Percentage of oil that can be recovered in the emulsified oil collected at time t

δ_{t}: Thickness of oil slick at the end of time period t

θ_{t}: Percentage of oil removed from the sea surface due to evaporation and natural dispersion at time t

Integer Variables

f_{t}: 0-1 variable. Equal to 1 if cleanup target is not achieved at the end of time period t

$x^M_{i,m,t}$: Number of mechanical cleanup and recovery system *m* from staging area *i* operating on scene at time period *t*

$x^B_{i,b,t}$: Number of in situ burning response system *b* from staging area *i* operating on scene at time period *t*

$x^D_{i,d,t}$: Number of sorties of chemical dispersant application systems *d* dispatched from staging area *i* at time period *t* to spray dispersants

$y^M_{i,m,t}$: Number of mechanical cleanup and recovery system *m* notified at time period *t* to be staged at staging area *i*

$y^B_{i,b,t}$: Number of in situ burning response system *b* notified at time period *t* to be staged at staging area *i*

$y^D_{i,d,t}$: Number of chemical dispersant application systems *d* notified at time period *t* to be staged at staging area *i*

$zd_{i,t}$: 0-1 variable. Equal to 1 if boom is being deployed around staging area *i* to protect the nearby shoreline at time period *t*

$z_{i,t}$: 0-1 variable. Equal to 1 if the shoreline around staging area *i* is protected by boom

$zm_{i,t}$: 0-1 variable, but can be relaxed as a continuous variable. Equal to 1 if maintenance is required for boom around staging area *i*

Continuous Variables (0 to $+\infty$)

$area_t$: Area of oil slick on the surface at the end of time *t*

$bdep_{i,t}$: Amount of coastal protection booms deployed in staging area *i* at time period *t*

$bfail_{i,t}$: Amount of coastal protection booms failed in staging area *i* at time period *t*

$binv_{i,t}$: Length of available boom in staging area *i* at the end of time period *t*

$bl_{i,t}$: Length of coastal protection boom deployed along the shore of staging area *i* at the end of time period *t*

$btr_{i,j,t}$: Amount of coastal protection booms shipped from storage location *j* to staging area *i* at the beginning of time period *t*

$dinv_{i,t}$: Amount of chemical dispersant in staging area i at the end of time period t

$dtr_{i,k,t}$: Amount of chemical dispersant shipped from supplier location k to staging area i at the beginning of time period t

u_t^M : Volume of oil collected and recovered through mechanical systems at time t

u_t^B : Volume of oil removed by in situ burning at time period t

u_t^D : Volume of oil dispersed due to chemical dispersant application at time period t

v_t : Volume of oil on the surface at the end of time t

Nomenclature for the ODE model

A: Area of oil slick (m^2)

A_{ev}: Constant for oil weathering process

A_0: Initial area of slick (m^2)

AC: Asphaltene content (%) of the parent oil

B_{ev}: Constant for oil weathering process

C_3: Constant for oil weathering process

C_4: Constant for oil weathering process

F_E: Fraction of oil evaporated

g: Acceleration of gravity (ms^{-2})

K_1: Constant for oil weathering process

k_2: Constant for oil weathering process

k_3: Constant for oil weathering process

K_{ev}: Constant for oil weathering process

tf_1: Time when the oil spillage stops (release duration)

tf_2: Time at the end of the planning horizon (final time of the simulation)

T_G: Gradient of the oil distillation curve

T_K: Temperature (K)

T_O: Initial boiling point of oil

V: Volume of oil slick (m^3)

V_0: Initial volume of oil spilled before time 0

V_D: Volume of oil naturally diepersed

VI: Time-dependent oil spilled rate

WIND: Wind speed

Y_W : Fraction of water content in the emulsion

v_w : Kinematic viscosity of seawater

ρ_w : Seawater density

ρ_o : Oil density

μ : Viscosity of oil slick

μ_0 : Viscosity of parent oil

ζ_t : Oil-water interfacial tension (dyne/m)

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