Multiphysics Solvers for Implicitly Coupled Electromechanical and Electromagnetic Transients Simulation

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***Abstract*—By exploiting the structure of the Jacobian matrix for combined electromechanical-electromagnetic transients simulation, numerically efficient multiphysics solvers can be composed. Three multiphysics solvers: Block-Jacobi, Block-Gauss-Seidel, and Schur-complement based are presented for the implicitly coupled electromechanical and electromagnetic transients simulation. Results for a 9-bus and 118 bus system show the computational efficiency of the proposed multiphysics solvers as compared with direct solution vwith LU factorization.**

***Index Terms*—Hybrid simulator, implicitly-coupled solution approach, block composable solvers, transient stability, electromagnetic transients.**

I. INTRODUCTION

The dynamic behavior of electrical power systems is simulated by using transient stability (TS) simulators and electromagnetic transient (EMT) simulators. A TS simulator, running at large time steps, is used for studying relatively slower dynamics (e.g., electromechanical interactions among generators) and can be used for simulating large-scale power systems. In contrast, an EMT simulator models the same components in finer detail and uses a smaller time step for studying fast dynamics (e.g., electromagnetic interactions among power devices). Because of small step size, simulating large-scale power systems with an EMT simulator is computationally inefficient. A hybrid simulator attempts to interface the TS and EMT simulators, which are running at different time steps. By modeling the bulk of the large-scale power system in a TS simulator and a small portion of the system in an EMT simulator, the fast dynamics of the smaller area can be studied in detail, while providing a global picture of the slower dynamics for the rest of the power system. In the existing hybrid simulation interaction protocols, the two simulators run independently, exchanging solutions at regular intervals.

In [1], [2], we proposed a novel implicitly coupled approach for combined TS and EMT simulation, unlike the existing approach of interfacing TS and EMT at the application level. This implicitly coupled formulation entails the solution of an augmented nonlinear system that consists of equations for one TS time step and multiple EMT time steps. These equations are solved simultaneously by Newton's method. The linear system required to be solved at each Newton iteration possesses a structure that can be exploited to compose efficient multiphysics solvers. Multiphysics solvers can be thought of as "block composable" solvers that can exploit the structure of each physics.

In this paper we present several efficient multiphysics solvers for the solution of combined electromechanical and electromagnetic transients simulation (referred to in this paper as TSEMT) as proposed in [2]. The results presented on a test 9-bus and 118-bus system show the numerical efficiency of the proposed multiphysics solvers.

II. HYBRID SIMULATORS

The idea of combined TS-EMT simulation was first proposed by Heffernan et al. [9] to simulate combined HVAC-HVDC systems. They modeled an HVDC link in detail within a stability-based AC system framework, thus exploiting the advantages of both EMT and TS. Specifically, they executed TS and EMT alternately with periodic coordination of the results. Reeve and Adapa [12] proposed that the boundary of the interface should be extended further into the AC network in order to take into consideration the effect of harmonics generated by power electronics on the AC network. Anderson et al. [4] presented another approach to take the harmonics into account. In their approach, the network equivalent for the TS network is represented by a frequency-dependent equivalent, instead of a simple fundamental frequency equivalent circuit. Sultan et. al. [16] basically adopted the approaches described above, extending the interface location into the AC network to some extent, and at the same time having a frequency-dependent TS network equivalent. Kasztenny [10] discussed a general method for linking different modeling techniques such as waveform-type, phasor-type, and algebraic-type simulation techniques into one complete model. Over the years, many researchers have further explored the combined TS-EMT simulation in terms of both modeling and algorithm. The term *hybrid simulator* is commonly used to refer to a combined TS and EMT simulator.

In the hybrid simulator, the power system network is partitioned into two subnetworks: a large network (TS domain of operation) and a smaller network run with EMT. The large network has been called the external system [12], [4], [14], electromechanical transient network [13], or TS-program subsystem [8], while the smaller system has been called the detailed system [12], [4], [14], EMT network [13], or

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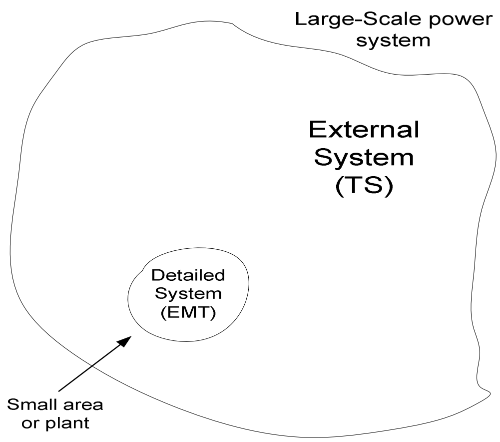

Fig. 1. Detailed and external system

instantaneous network [17]. In this paper, the larger network will be called the external system and the smaller system the detailed system (see Figure 1).

Since the TS and EMT run at different time steps, synchronization of these simulators is required for data exchange. This synchronization is done through predefined sequential actions that coordinate the data exchange between TS and EMT simulators [11]. Both serial [9], [4] and parallel [14], [15] interaction protocols have been proposed so far. In serial protocols, only one simulator, either TS or EMT, runs while the other is idle. In parallel protocols, both simulators run at the same time. A comprehensive overview of the state of the art in hybrid simulators is given in [11]. Figure 2 describes the data exchange between the TS and EMT simulators, for one TS time step, in a serial interaction protocol.

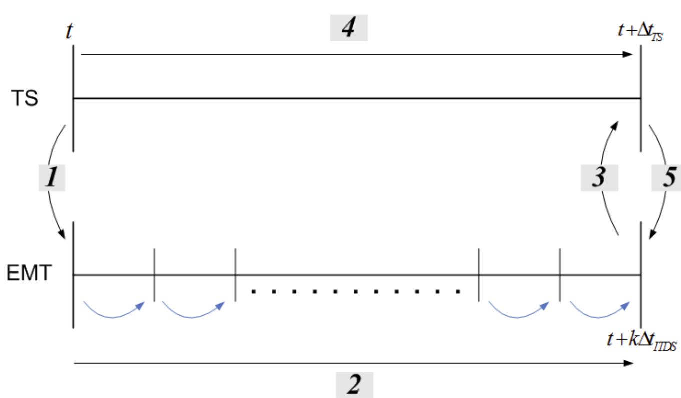

Fig. 2. Serial interaction protocol for one TS time step

We note here that the external system equivalent is not updated when EMT is running; it is held constant for all the EMT time steps within a TS time step. This equivalent can be also derived from some extrapolated history data, but either way, it may not accurately predict the conditions at the next TS time step. While such an approach would be sufficient if the TS system was evolving slowly (i.e., there is a small difference between the voltages and currents at two consecutive time steps), for large changes this approach may not be suitable.

Another point to note here is that no iterations are done between TS and EMT to check whether the solutions at each TS and EMT boundary are consistent. Having no iterations is probably sufficient when the external system equivalent does not change much, and it may be adequate for the gradually changing external system voltage profile. However, for large changes in voltages between consecutive TS time steps, iterations would be needed to update the external system equivalent repeatedly. Because of the explicit coupling, more iterations would be required, and the solution still might diverge [2].

III. Implicitly coupled Electromechanical and Electromagnetic Simulation (TSEMT)

In [2], we proposed a novel implicitly coupled solution approach for the combined transient stability and electromagnetic transient simulation. To combine the two sets of equations with their different time steps, and ensure that the TS and EMT solutions are consistent, the equations for TS and coupled-in-time EMT equations are solved simultaneously. While computing a single time step of the TS equations, a simultaneous calculation of several time steps of the EMT equations is proposed.

In compact form, the TS system DAE model equations are

$$\begin{aligned} \frac{dx_{TS}}{dt} &= F(x_{TS}, y_{TS}) \\ 0 &= G(x_{TS}, y_{TS}) \end{aligned} \tag{1}$$

In (1), x_{TS} represents the dynamic variables for the synchronous generators and the associated control circuitry (i.e., exciters, voltage regulators, turbine governors), while y_{TS} represent the network phasor bus voltages. The differential equations for EMT are described by

$$\frac{dx_{EMT}}{dt} = f(x_{EMT}) \tag{2}$$

Adding the coupling between TS and EMT, the equations for TSEMT in compact form are

$$\begin{aligned} \frac{dX_{TS}}{dt} &= F(x_{TS}, y_{TS}) \\ 0 &= G(x_{TS}, y_{TS}, I_{BDRY}) \\ \frac{dx_{EMT}}{dt} &= f_1(x_{EMT}, i_{bdry}) \\ \frac{di_{bdry}}{dt} &= f_2(x_{EMT}, i_{bdry}, v_{thev}) \end{aligned} \tag{3}$$

The coupling variables I_{BDRY}, i_{bdry}, and v_{thev} result from choice of network equivalents (thevenin equivalent for the detailed system and dependent current source for the external system. The reader is referred to [1] and [2] for details on the formulation of network equivalents and coupling variables used in this work.

Discretizing the TS equations with the TS time step, Δt_{TS}, and EMT equations with EMT time step, Δt_{EMT}, and using an implicit trapezoidal integration scheme, one obtains the complete set of equations (4)-(11) to solve at each TS time step. Equations (4) and (5) represent the equations for the external system for one TS time step while (6)-(11) are the coupled-in-time EMT equations:

$$
\begin{aligned}
&x_{TS}(t_{N+1}) - x_{TS}(t_N) - \\
&\qquad \frac{\Delta t_{TS}}{2}(F(t_{N+1}) + F(t_N)) = 0 && (4)\\
&G(t_{N+1}) = 0 && (5)\\
&x_{EMT}(t_{n+1}) - x_{EMT}(t_n) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_1(t_{n+1}) + f_1(t_n)) = 0 && (6)\\
&i_{bdry}(t_{n+1}) - i_{bdry}(t_n) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_2(t_{n+1}) + f_2(t_n)) = 0 && (7)\\
&x_{EMT}(t_{n+2}) - x_{EMT}(t_{n+1}) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_1(t_{n+2}) + f_1(t_{n+1})) = 0 && (8)\\
&i_{bdry}(t_{n+2}) - i_{bdry}(t_{n+1}) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_2(t_{n+2}) + f_2(t_{n+1})) = 0 && (9)\\
&\quad \vdots \\
&\quad \vdots \\
&x_{EMT}(t_{n+k}) - x_{EMT}(t_{n+k-1}) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_1(t_{n+k}) + f_1(t_{n+k-1})) = 0 && (10)\\
&i_{bdry}(t_{n+k}) - i_{bdry}(t_{n+k-1}) - \\
&\qquad \frac{\Delta t_{EMT}}{2}(f_2(t_{n+k}) + f_2(t_{n+k-1})) = 0 && (11)
\end{aligned}
$$

where

$$
\begin{aligned}
I_{BDRY}(t_{N+1}) &= h_{EMT->TS3ph}(i_{bdry}(t_{n+1}), \\
&\quad i_{bdry}(t_{n+2}), \dots, i_{bdry}(t_{n+k})) \\
(v_{thev}(t_{n+1}), &v_{thev}(t_{n+2}), \dots, v_{thev}(t_{n+k})) \\
&= h_{TS3ph->EMT}(V_{thev,TS}(t_N), V_{thev,TS}(t_{N+1}))
\end{aligned}
$$

represents the coupling between TS3ph and EMT. Here, $h_{EMT->TS3ph}$ denotes a fourier analysis of the EMT instantaneous boundary currents i_{bdry} over a running window of one cycle of fundamental frequency to obtain the fundamental frequency TS phasor boundary currents $I_{BDRY}(t_{N+1})$ as described in equation (12).

$$
\begin{aligned}
I_{BDRY,D}(t+\Delta t_{TS}) &= \frac{2}{T}\int_{\tau=t}^{t+\Delta t_{TS}} i_{bdry}(\tau)\sin(\omega\tau)d\tau \\
I_{BDRY,Q}(t+\Delta t_{TS}) &= \frac{2}{T}\int_{\tau=t}^{t+\Delta t_{TS}} i_{bdry}(\tau)\cos(\omega\tau)d\tau
\end{aligned} \qquad (12)
$$

Here $I_{BDRY,D}$ and $I_{BDRY,Q}$ represent the real and the imaginary components of the phasor TS boundary current I_{BDRY}. Since EMT uses instantaneous voltages, the phasor voltage V_{thev} needs to be converted to instantaneous waveform v_{thev}. This conversion, represented by $h_{TS3ph->EMT}$, is done by a fundamental fixed-frequency sine wave generator.

IV. MULTIPHYSICS SOLVERS FOR TSEMT

Equations (4)-(11) are solved simultaneously by using Newton's method at each TS time step. The linear system to be solved at each Newton iteration is

$$
\begin{bmatrix} J_{TS,TS} & J_{TS,EMT} \\ J_{EMT,TS} & J_{EMT,EMT} \end{bmatrix} \begin{bmatrix} \Delta X_{TS} \\ \Delta X_{EMT} \end{bmatrix} = \begin{bmatrix} -F_{TS} \\ -F_{EMT} \end{bmatrix} \qquad (13)
$$

where, $X_{TS} \equiv \{x_{TS}(t_{N+1}), y_{TS}(t_{N+1})\}$, and $X_{EMT} \equiv \{x_{EMT}(t_{n+1}) \dots x_{EMT}(t_{n+k-1}),$ $i_{bdry}(t_{n+1} \dots i_{bdry}(t_{n+k-1})$

The solution of the linear system in (13) is the main computational burden for the implicitly coupled TSEMT simulation. A typical way of solving (13) is by LU factorization. However, LU factorization is a memory-intensive operation, especially for larger systems, if there are more fill-in elements.

The Jacobian matrix in (13) possesses a block structure as seen in Fig. 3, where the diagonal blocks ($J_{TS,TS}$ and $J_{EMT,EMT}$) represent the partial derivatives of TS and EMT functions, respectively, while the off-diagonal blocks ($J_{TS,EMT}$ and $J_{EMT,TS}$) are the partial derivatives of the coupling between TS and EMT. This structure is typical in coupled multiphysics applications and offers the possibility of constructing an efficient linear solver for the entire system by composing individual linear solvers for each physics (or block). Moreover, it also allows tuning of the individual physics linear solvers (linear solver method, preconditioner, matrix reordering). Brown et al. [7] discusses the use of multiphysics solvers for applications such as geodynamics and ice sheet dynamics.

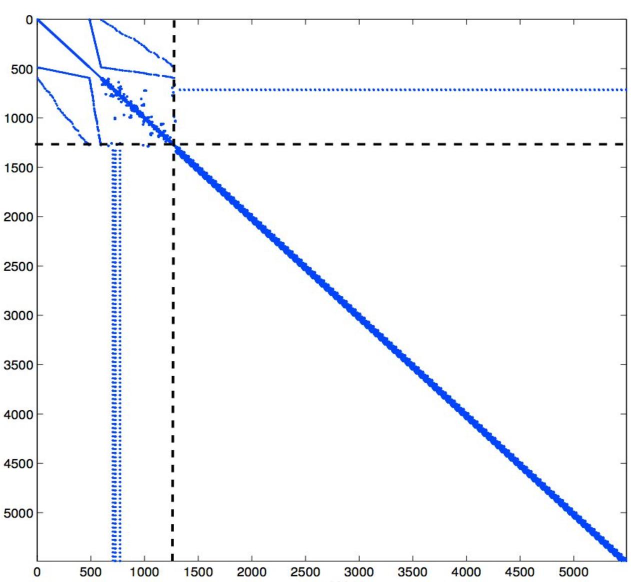

Fig. 3. TSEMT Jacobian structure for 118 bus system

Rewriting (13) with the EMT equations first followed by TS equations, we have

$$
\begin{bmatrix} J_{EMT,EMT} & J_{EMT,TS} \\ J_{TS,EMT} & J_{TS,TS} \end{bmatrix} \begin{bmatrix} \Delta X_{EMT} \\ \Delta X_{TS} \end{bmatrix} = \begin{bmatrix} -F_{EMT} \\ -F_{TS} \end{bmatrix} \qquad (14)
$$

We discuss three multiphysics solvers for the solution of (13): block-Jacobi, block Gauss-Seidel, and Schur-complement based. The details of these solvers are discussed below:

1) *Block-Jacobi or Additive*: The simplest multiphysics solver, this ignores the off-diagonal blocks and leads to solving the block-diagonal linear system for TS and EMT.

$$\begin{bmatrix}\Delta X_{EMT} \\ \Delta X_{TS}\end{bmatrix} = \begin{bmatrix}J_{EMT,EMT} & \\ & J_{TS,TS}\end{bmatrix}^{-1} \begin{bmatrix}-F_{EMT} \\ -F_{TS}\end{bmatrix} \tag{15}$$

2) *Block-Gauss-Seidel or Multiplicative*: The multiplicative solver involves solving the EMT system first, followed by the TS system similar to a Gauss-Seidel scheme.

$$\begin{bmatrix}\Delta X_{EMT} \\ \Delta X_{TS}\end{bmatrix} = \begin{bmatrix}J_{EMT,EMT} & \\ J_{TS,EMT} & J_{TS,TS}\end{bmatrix}^{-1} \begin{bmatrix}-F_{EMT} \\ -F_{TS}\end{bmatrix} \tag{16}$$

3) *Schur complement based*: This solver uses the Schur-complement method to solve (13) directly. This involves solving

$$S\Delta X_{TS} = -F_{TS} + J_{TS,EMT}J^{-1}_{EMT,EMT}F_{EMT} \tag{17}$$

for ΔX_{TS} ,where
$S = J_{TS,TS} - J_{TS,EMT}J^{-1}_{EMT,EMT}J_{EMT,TS}$, and

$$J_{EMT,EMT}\Delta X_{EMT} = -F_{EMT} + J_{EMT,TS}X_{TS} \tag{18}$$

These multiphysics solvers allow individual customization or composing the factorization of its self blocks. For the $J_{EMT,EMT}$ block, we use *reverse Cuthill-Mckee* ordering to get the least number of fill-ins for $J^{-1}_{EMT,EMT}$ since this block is already in a block-subdiagonal form. For the TS part, LU factorization with *quotient minimum degree* ordering is used for $J^{-1}_{TS,TS}$. This customization results in efficient numerical solvers, as seen from the results in the following section.

The multiphysics solvers can be used by themselves to solve the linear system directly, inexactly by block-Jacobi and block Gauss-Seidel or exactly using schur-complement, or they can be used as a preconditioner with an iterative linear solver.

V. Numerical results

We present the results on two test systems, a 9-bus system and an 118-bus system. For the 9-bus system the EMT part consists of three buses with two transmission lines and 1 load, while for the 118-bus system the EMT part consists of 4 buses with three transmission lines and load at each bus. The results shown below are for a temporary three-phase balanced fault inside the EMT region applied for 0.1 seconds. The TS time step is 0.01667 seconds while the EMT time step is 0.0001667 seconds (i.e. $\Delta t_{TS}/\Delta t_{EMT}$ = 100). The simulation time length was set to 1 second. For GMRES, we use a stringent absolute tolerance of 10^{-8} and moreover the outer Newton loop also has a tolerance of 10^{-8}. These stringent tolerances ensure that the system is solved to a reasonable level of accuracy. The TSEMT code is written in C language using the high-performance library PETSc [6] and compiled with GNU compiler with -O3 optimization. We experimented with using the multiphysics solvers as the linear solvers and also a preconditioner with the iterative Krylov subspace solver generalized minimal residual algorithm (GMRES) as the linear solver. The numerical results for the two test systems are shown in Tables I and II.

TABLE I
EXECUTION TIMES FOR THE 9-BUS SYSTEM

Linear Solver	Preconditioner	Wall-clock Time (sec)
LU	–	3.71
Block-Jacobi	–	Did not converge
Block Gauss-Seidel	–	Did not converge
Schur	–	1.58
GMRES	LU	3.78
GMRES	Block-Jacobi	2.13
GMRES	Block Gauss-Seidel	1.78
GMRES	Schur	2.08

TABLE II
EXECUTION TIMES FOR THE 118-BUS SYSTEM

Linear Solver	Preconditioner	Wall-clock Time (sec)
LU	–	31.04
Block-Jacobi	–	6.81
Block Gauss-Seidel	–	1.65
Schur	–	1.68
GMRES	LU	33.28
GMRES	Block-Jacobi	2.06
GMRES	Block Gauss-Seidel	1.89
GMRES	Schur	2.18

Note that using multiphysics solvers as the linear solver directly may not be robust and could lead to divergence, as seen for the results for the 9-bus system. Instead, using these as a preconditioner was found to be robust and efficient.

The speedup for the multiphysics solvers is a result of reduced nonzeros for the factored matrices. Using block solvers allows factorizing individual TS and EMT matrices with optimal reordering strategies that results in fewer nonzeros as compared to a monolithic LU solve on the entire matrix. Table III provides a comparison of the nonzeros in the factored matrices for LU and multiphysics solvers. The nonzeros for multiphysics solvers is the sum of factored $J_{TS,TS}$ and $J_{EMT,EMT}$ blocks.

TABLE III
COMPARISON OF MEMORY REQUIREMENT IN TERMS OF NONZEROS FOR LU FACTORIZATION AND MULTIPHYSICS SOLVERS

System size	Jacobian matrix nonzeros	LU factorization	Multiphysics solvers
9-bus	32103	197577	43228
118-bus	63276	794745	102103

From the results given in Tables I and II, it may seem that the computational speed with the multiphysics preconditioners seems to be independent of the size of the system. However, this conjecture in general may not be always true. For the results presented here, similar computational speed with the multiphysics solvers for the 9-bus and 118-bus system can be attributed to nearly same sizes of the EMT part, and the TS part being really small such that it fits in the cache entirely.

VI. Conclusions

This paper proposed several efficient multiphysics solvers for the implicitly coupled electromechanical and electromagnetic transients simulation. By exploiting the structure of the TSEMT Jacobian efficient solvers can be composed. Results presented on test 9-bus and 118-bus system show the numerical efficiency of these solvers.

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Appendix A
PETSc [5]: Portable Extensible Toolkit for Scientific Computation

The PETSc package consists of a set of libraries for creating parallel vectors, matrices, and distributed arrays, scalable linear, nonlinear, and time-stepping solvers. A review of PETSc and its use for developing scalable power system simulations can be found in [3]. The organization of PETSc is shown in Figure 4. In this work, we used the multiphysics solvers

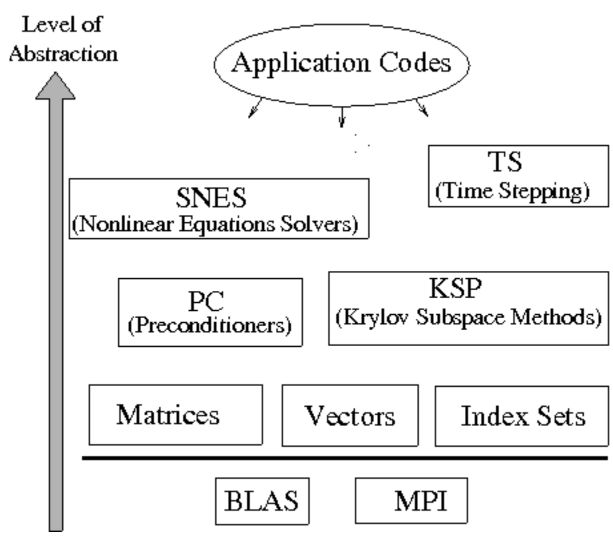

Fig. 4. Organization of the PETSc library [5]

available in PETSc through the *Fieldsplit* preconditioner class. The *Fieldsplit* preconditioner class is designed for solving coupled multi-physics problems and allows easy composition of solvers of individual physics. Any of PETSc's linear solvers/preconditioners can be used with *Fieldsplit* to construct solvers for individual physics.

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