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Abstract

Platooning vehicles—connected and automated vehicles traveling with small intervehicle distances—use less fuel because of reduced aerodynamic drag. Given a network defined by vertex and edge sets and a set of vehicles with origin/destination nodes/times, we model and solve the combinatorial optimization problem of coordinated routing of vehicles in a manner that routes them to their destination on time while using the least amount of fuel. Common approaches decompose the platoon coordination and vehicle routing into separate problems. Our model addresses both problems simultaneously to obtain the best solution. We use modern modeling techniques and constraints implied from analyzing the platoon routing problem to address larger numbers of vehicles and larger networks than previously considered. While the numerical method used is unable to certify optimality for candidate solutions to all networks and parameters considered, we obtain excellent solutions in approximately one minute for much larger networks and vehicle sets than previously considered in the literature.

1 Introduction

In 2011, urban traffic congestion resulted in a loss of 5.5 billion man-hours and an extra 2.9 billion gallons of fuel being consumed, for an economic cost of \$121 billion and substantial environmental costs [17]. The U.S. population will grow by 70 million by 2045, and 75% of this population will live in a “megaregion.” Thus, new operational technologies are needed to make urban traffic networks sustainable. Many strategies can be applied to improve operational characteristics of a transportation network and to reduce congestion, including dynamic speed control, ramp metering, and lane management. We focus on platooning, in which sets of vehicles travel together with small intervehicle distances to conserve fuel and improve throughput.

Recent developments in connected and automated vehicles can improve traffic flow by improving the way vehicles are driven [15, 16]. Human risk-averse driving

behavior leads to inefficient use of roads. On average, the traffic flow changes from free to congested at a traffic density of 18%. In other words, 82% of the road space is wasted. Automated driving in a platoon reduces headways between vehicles while maintaining traffic flow speed and thus improves road throughput and avoids traffic flow breakdowns for high-density traffic flows. The goal of platooning is to improve three metrics associated with a transportation system: mobility (reduced congestion), sustainability (reduced fuel use), and safety.

Fuel use is reduced as a byproduct of the reduced drag forces experienced by trailing vehicles. Several field studies show fuel savings ranging from 5% to 15% from vehicle platoons in isolated test environments [4, 5, 6, 7, 18, 19, 20, 22]. Moreover, platoon driving improves the throughput of a road network. Several simulation studies show that platoon driving enables shorter following gaps, thereby increasing the road capacity from the typical 2,200 vehicles per hour to almost 4,000 vehicles per hour, assuming all vehicles have platooning capabilities [20, 27].

In this paper, we analyze a coordinated platooning model both mathematically and numerically. This model is a combinatorial problem that involves simultaneously routing vehicles through the network and determining when platoons should form or dissolve in order to minimize their collective fuel use. Our analysis strengthens the formulation, enabling us to produce near-optimal solutions for larger instances. We implement the mixed-integer programming model in the GAMS modeling language [8], a domain specific language for specifying such optimization problems and conveying them to numerical methods; and we apply Gurobi [10], a modern branch-and-bound method for mixed-integer linear and quadratic programs, to test how quickly we can produce near-optimal solutions. Our GAMS model and example problem data are available at

<http://www.mcs.anl.gov/~jlarrison/Platooning>

The routing of existing platoons in small road networks has been studied in the literature using approaches ranging over discretized optimal control [1, 2, 3], dynamic programming [9, 23], and graph-based al-

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gorithms [26]. In contrast to our coordinated model, the platoons in these models are not allowed to merge with other vehicles and consequently save additional fuel; they consider only the optimal routing for a given set of platoons.

Given routes for the platoons, several authors determine possible mergers to minimize fuel consumption [24, 25]. These authors include detailed fuel savings estimates based on the speed changes required to catch up or slow down to form a single platoon. In contrast to our coordinated model, they assume the vehicle routes are known a priori, typically based on the shortest path, and platoons do not deviate from that route for additional platooning opportunities.

Opportunistic, distributed platooning using distributed controllers is studied in [11, 12, 14]. As vehicles approach an intersection in the road network, information is transmitted to a controller, and platoons are formed when the vehicles share some subset of route edges and platooning would reduce fuel costs. In this paper, we consider a centralized controller that yields greater fuel savings than can be obtained from distributed, opportunistic control. Such centralized controllers can be used in practice by companies routing their commercial vehicle fleet through a metropolitan network. Moreover, the centralized controllers determine the maximum possible platooning savings and serves as an important baseline when studying and assessing the benefits of distributed controllers.

Our previous work [13] on coordinated platooning constructed a model that combines both routing and platoon formation and dissolution. To reduce fuel costs, vehicles can deviate from their shortest path provided they reach their destination in the required time. The problem is shown to be NP-complete, and heuristics are proposed to compute good solutions. In this paper, we formulate, mathematically analyze, and solve a similar model without resorting to heuristics. Our modeling approach produces a compact formulation with a limited number of variables and constraints, while our mathematical analysis enables us to solve larger problem instances that have previously been addressed in the literature, in terms of both the network and number of vehicles. In Section 2, we present our coordinated platooning model and the mathematical analysis required to tighten the formulation and make this combinatorial optimization problem tractable. Section 3 provides numerical results for two metropolitan networks, a 10×10 grid and the greater-Chicago-area highway network, and studies how long it takes to find optimal solutions as a function of key parameters. We finish with a discussion in Section 4.

Table 1: Sets and parameters defining the model.

Set	Meaning	GAMS
V	Vehicles to route	V,W
I	Network nodes	I, J, K
$E \subseteq I \times I$	Network edges	E(V, I, J)

Parameter	Meaning	GAMS
O_v	$v \in V$ origin node	O(V)
D_v	$v \in V$ destination node	D(V)
T_v^O	$v \in V$ origin time	T.O(V)
T_v^D	$v \in V$ destination time	T.D(V)
$C_{i,j}$	cost for taking $(i, j) \in E$	C(I, J)
$T_{i,j}$	time to take $(i, j) \in E$	T(I, J)
$M_{i,j}$	minimum time from i to j	M(I, J)

2 Coordinated Platooning Model

We first declare the model sets, parameters, and variables. We then declare the model constraints. We also highlight auxiliary parameters used to decrease the size of our GAMS formulation.

2.1 Model sets, parameters, and variables In Table 1, we declare the sets and parameters that are used to build the model. The values assigned to the sets and parameters define each instance of the platoon routing problem.

Note that the first five parameters are declared for all vehicles in V , while $C_{i,j}$ and $T_{i,j}$ are declared for all edges, and $M_{i,j}$ is declared for all pairs of nodes. Naturally, the nodes O_v and D_v must be in I , while the times and costs must be positive real numbers. For a problem to be feasible,

$$T_v^D \geq T_v^O + M_{T_v^O, T_v^D}$$

for all vehicles.

For any platoon routing problem instance, we can control the vehicles' routes and travel times. We list these decision variables in Table 2. The variables f and q are binary variables, while e and w are positive reals.

Table 2: Model variables.

Variable	Meaning	GAMS
$f_{v,i,j}$	1 if v travels on (i, j)	f(V, I, J)
$q_{v,w,i,j}$	1 if v follows w on (i, j)	q(V, W, I, J)
$e_{v,i,j}$	Time v enters (i, j)	time_e(V, I, J)
$w_{v,i}$	Time v waits at i	time_w(V, I)

We limit the declaration of some sets, decision variables, and constraints, in order to reduce the model

size in GAMS. For example, $E(V, I, J)$ need not contain all edges for a given vehicle but only those edges that the vehicle can travel on. To this end we use auxiliary GAMS parameters, listed in Table 3.

Table 3: Auxiliary parameters in the edge-based model.

GAMS Parameter	Meaning
$PE(V, I, J)$	1 if v can take (i, j)
$PQ(V, W, I, J)$	1 if v can follow w

The parameter $PE(V, I, J)$ is set to 1 only if $v \in V$ can potentially travel on edge $(i, j) \in E$. Then the GAMS declaration

$$E(V, I, J)\$(PE(V, I, J)) = \text{yes};$$

greatly reduces the size of the edge set E within GAMS. This reduces the model size by, for example, defining fewer constraints when making a declaration over all edges. Similarly, the parameter $PQ(V, W, I, J)$ is set to 1 only when vehicles V and W can platoon on (I, J) . That is, they satisfy

$$(2.1) \quad \max \{T_v^O + M_{O_v, i}, T_w^O + M_{O_w, i}\} + T_{i, j} \leq \min \{T_v^D - M_{D_v, j}, T_w^D - M_{D_w, j}\}.$$

In words, vehicles can platoon on an edge $(i, j) \in E$ only if the time the later-arriving vehicle reaches i plus the time it takes to traverse (i, j) allows for the most time-restricted vehicle to reach its destination on time. We therefore declare variables $q(V, W, I, J)$ only when $PQ(V, W, I, J) = 1$.

The use of PQ and PE to reduce the number of model variables/constraints may seem trivial or distracting. Their inclusion in the GAMS model, however, is critical to our ability to solve problems with more than a dozen vehicles.

2.2 Assumptions We assume that vehicles travel at free-flow speed on the various edges (therefore there is no speed consideration). We assume that vehicles are numbered in the order in which they arrive in the network, and we assume that vehicles with smaller indices are trailing vehicles with larger indices in platoons. These assumptions reduce some symmetry in the problem. In practice, vehicles cannot travel in a platoon in arbitrary order. Usually, vehicles are ordered by how quickly they can stop, with those that can stop quickest placed at the end of the platoon to avoid collisions if the platoon needs to unexpectedly stop. Our model does not take this ordering into consideration. During postprocessing, the vehicles in each platoon can simply be sorted by their stopping ability.

2.3 Free-flow speed model constraints We now declare the constraints that will accurately model the platoon routing problem.

- Node outflows must equal inflows.

$$(2.2) \quad \sum_{j:(i,j) \in E} f_{v,i,j} = \sum_{j:(j,i) \in E} f_{v,j,i} + B_{v,i} \\ \forall v \in V, i \in I,$$

where $B_{v,i}$ is 1 if $i = O_v$, -1 if $i = D_v$, and 0 otherwise.

- When platooning, enter times are equal.

$$(2.3) \quad -M(1 - q_{v,w,i,j}) \leq e_{v,i,j} - e_{w,i,j} \\ \leq M(1 - q_{v,w,i,j}) \\ \forall v, w \in V, (i, j) \in E, v > w$$

- Only one vehicle can follow.

$$(2.4) \quad \sum_v q_{v,w,i,j} \leq 1 \quad \forall w \in V, (i, j) \in E$$

- Platooning requires flow for the leader.

$$(2.5) \quad q_{v,w,i,j} \leq f_{w,i,j} \quad \forall w \in V, (i, j) \in E$$

- Platooning requires flow for the followers.

$$(2.6) \quad q_{v,w,i,j} \leq f_{v,i,j} \quad \forall v \in V, (i, j) \in E$$

- T_v^O plus waiting time is the origin enter time.

$$(2.7) \quad -M(1 - f_{v,O_v,j}) \leq e_{v,O_v,j} - T_v^O - w_{v,O_v} \\ \leq M(1 - f_{v,O_v,j}) \\ \forall v \in V, j \in I$$

- T_v^D is the final enter time plus the time required to travel the final edge plus waiting at the end.

$$(2.8) \quad -M(1 - f_{v,i,D_v}) \\ \leq T_v^D - e_{v,i,D_v} - w_{v,D_v} - T_{i,D_v} f_{v,i,D_v} \\ \leq M(1 - f_{v,i,D_v}) \\ \forall v \in V, i \in I$$

- Intermediate enter times are equal plus the travel and waiting times.

$$(2.9) \quad -M(2 - f_{v,i,j} - f_{v,k,i}) \\ \leq e_{v,i,j} - e_{v,k,i} - w_{v,i} - T_{k,i} f_{v,k,i} \\ \leq M(2 - f_{v,i,j} - f_{v,k,i}) \\ \forall v \in V, (i, j), (j, k) \in E, D_v \neq i \neq O_v$$

- If there is no flow, the enter time can not be nonzero.

$$(2.10) \quad e_{v,i,j} \leq M f_{v,i,j} \quad \forall v \in V, (i,j) \in E$$

- If there is no flow, the wait time can not be nonzero.

$$(2.11) \quad w_{v,i} \leq M \left(\sum_{i,j} f_{v,i,j} + f_{v,j,i} \right) \quad \forall v \in V, i \in I$$

A GAMS formulation of this model can be found at

<http://www.mcs.anl.gov/~jlarrison/Platooning>

The value we choose for M in equations (2.3), (2.7), (2.8), (2.9), (2.10), and (2.11) is

$$M = \max_v \{T_v^D\} - \min_v \{T_v^O\}.$$

We can tighten some Big- M values: for example, M in (2.3) needs to only be the largest time-gap between vehicles that can possibly platoon. GAMS efficiently removes the Big- M formulation for reasonable time values.

2.4 Objective function Since vehicles travel at free-flow speeds, the amount of fuel used is quantified by

$$(2.12) \quad \sum_{v,i,j} C_{i,j} \left(f_{v,i,j} - \eta \sum_w q_{v,w,i,j} \right).$$

$C_{i,j}$ is the gallons of fuel used by a vehicle to traverse edge (i,j) . One can consider $C_{i,j}$ to be a vehicle-dependent value, but we do not do so here. One also can include a cost of waiting for vehicles, but we do not do so in order to study the upper-bound on possible platoon savings.

2.5 Additional results and constraints Some constraints are not necessary for accurately modeling the platoon routing problem but their inclusion helps break symmetries in the problem (and therefore speed the time to solution). For example, without (2.4), if v_1, v_2 , and v_3 are platooning on an edge (i,j) , either $q_{v_3,v_1,i,j} = 1$ or $q_{v_3,v_2,i,j} = 1$ is a valid solution. When more vehicles are in a platoon, this combinatorial number of solutions is even more difficult for GAMS and Gurobi to account for.

The following results help us declare additional constraints that greatly reduce the problem size. First, [13, Theorem 2.2] proves the following lemma.

LEMMA 2.1. *There exists an optimal platoon routing in which no two vehicles split and then merge together.*

In practice, many vehicles travel from the same source to the same destination. It is therefore useful to prove that if a vehicle shares the same origin and destination with another vehicle and they do not platoon, then we do not need to consider platooning with later-arriving vehicles. First note that if M_{O_v,D_v} is the minimum time from O_v to D_v , then vehicles $v,w \in V$ sharing origin and destinations can platoon if their origin and destination times satisfy

$$(2.13) \quad \max \{T_v^O, T_w^O\} + M_{O_v,D_v} \leq \min \{T_v^D, T_w^D\}.$$

Also note that there is an upper bound on the length of a detour that a vehicle will take over its shortest path.

LEMMA 2.2. *If vehicles use a fraction η less fuel when trailing in a platooning and t_s is the shortest time for a vehicle to travel from its origin to destination, it will never travel a path longer than $\frac{1}{1-\eta}t_s$.*

Proof. Suppose a vehicle v travels more than $t' > \frac{1}{1-\eta}t_s$ in an optimal routing. Then keeping all other routes fixed and switching v to the shortest path route will remove more than $(1-\eta)t'$ but add t_s to the objective resulting in a net improvement in fuel use.

We now prove that there exist optimal solutions with many $q_{v,w,i,j} = 0$. Therefore, we can enforce this in our GAMS model and reduce the search time without degrading the solution quality.

LEMMA 2.3. *Let $v,w \in V$ satisfy $O_v = O_w, D_v = D_w$, and (2.13). If an optimal solution has $q_{v,w,i,j} = 0$ for all $(i,j) \in E$, there exists an optimal solution with $q_{v,w',i,j} = 0$ for all (i,j) and w' such that $O_v = O_{w'}, D_v = D_{w'}, T_w^O < T_{w'}^O$, and v,w' satisfy (2.13).*

Proof. Assume for contradiction that there are only optimal solutions where w' arrives in the network later than w and follows v but no optimal solution where w follows v .

For a given optimal routing, let t_v, t_w be the vehicles' respective travel times and let t_v^p, t_w^p be the total time each is leading a platoon somewhere along their routes.

If $t_w > t_v$, having w platoon with v along v 's route will change the objective by

$$\eta t_w^p - t_w + (1-\eta)t_v \leq (1-\eta)(t_v - t_w) < 0,$$

since if w is leading some platoon for its entire route, the vehicle it is leading will incur and additional ηt_w^p units of fuel. Similarly, if $t_w < t_v$, having v platoon with w along w 's route will change the objective by

$$\eta t_v^p - t_v + (1-\eta)t_w \leq (1-\eta)(t_w - t_v) < 0.$$

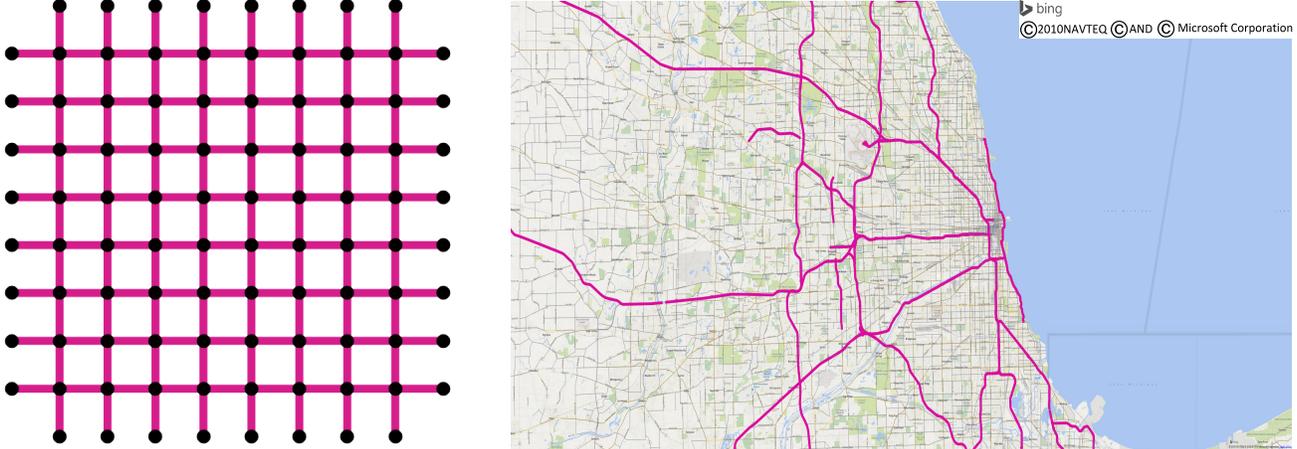


Figure 1: Networks under consideration: a 10×10 grid (left) and the Chicago-area highways (right).

Both of these cases contradict the platoon routing being optimal.

If $t_w = t_v$, having w follow v and w' follow w changes the objective by

$$\eta t_w^p - t_w + (1 - \eta)t_v \leq \eta t_w - t_w + (1 - \eta)t_v = 0$$

If the above is a strict inequality, this contradicts the optimality of the platoon routing. If it holds with equality, this contradicts the assumption that there are no optimal solutions where w follows v .

Lemma 2.3, along with the assumption that all vehicles have the same cost to traverse an edge, implies that if v and w share origin and destination nodes, $PQ(V, W, I, J)$ should be 1 only if w arrives directly after v , assuming that if $T_v^O < T_w^O < T_{w'}^O$ and (v, w') satisfy (2.13) then (w, w') do as well. (For our experiments, we study the case where all vehicles are willing to wait the same amount of time at the origin, so this assumption is valid.) In Section 3.3 we test the effect of using this fact.

Lemma 2.1 implies that vehicles that share the same source and destination either platoon the entire way or never platoon. That is, if $O_v = O_w$ and $D_v = D_w$ and (v, w) satisfy (2.13), then we need to consider platooning only with the next arriving vehicle. In Section 3.3 we test the effect of using this fact.

3 Numerical results

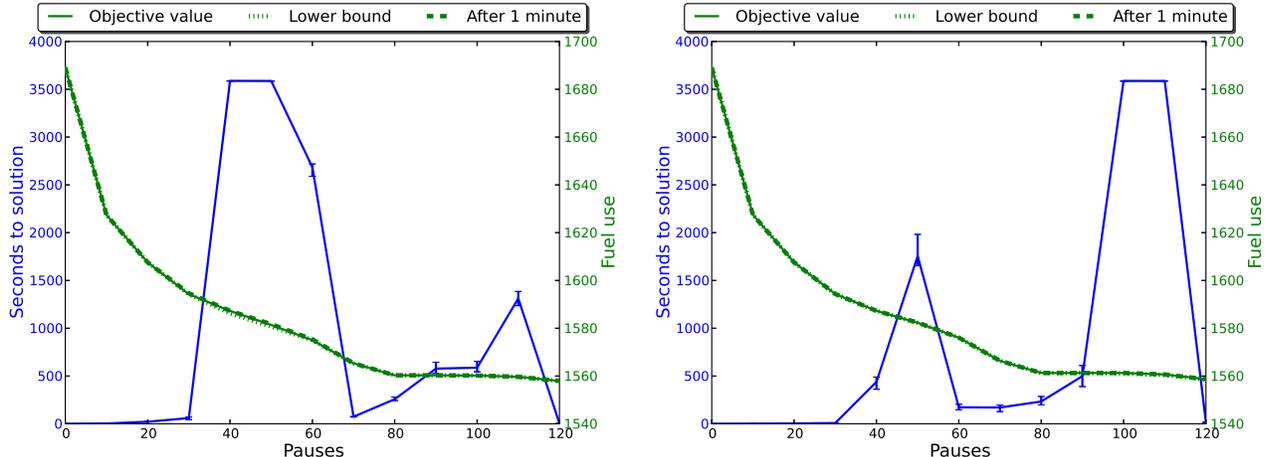
To test the capabilities of the proposed model, we perform experiments on a 10×10 grid and a 4553-node representation of the greater-Chicago highway network. The networks are shown in Figure 1. We believe both networks provide important tests for the platoon routing problem in a metropolitan network.

We test the ability of our model to quickly produce optimal solutions to the platoon routing problem with 25 vehicles, a number that is much greater than considered in the literature. (We are unaware of any paper that simultaneously coordinates the routes and platoons of more than 10 vehicles.) Specifically, 25 pairs of vehicle origin/destination nodes are drawn uniformly random for vehicles in the grid, and 25 common origin/destination nodes are uniformly drawn from the morning commute routes from a simulation of the Chicago highway system [21]. The costs of traversing edges in the Chicago network are taken from this same simulation; we assume a unit cost for traversing any edge in the grid network. Origin times T_O^v for each vehicle are drawn uniformly from $[0, 100]$, and destination times are set to

$$(3.14) \quad T_D^v = T_O^v + M_{O_v, D_v} + p,$$

where M_{O_v, D_v} is the minimum time between the vehicle's origin and destination and p is some pause time. We assume that trailing vehicles in a platoon use 10% less fuel do than vehicles leading a platoon or traveling alone on a given edge. That is, $\eta = 0.1$ in (2.12).

Although the regularity of the grid network may suggest simplicity, we find that the opposite is the case. Many different routes of the same length exist between most pairs of origin/destination nodes; the number of shortest paths between $(0, 0)$ and (m, n) in a grid is $\binom{m+n}{n}$. The Chicago highway network, on the other hand, has a unique shortest path between most origin/destination pairs. Also, we need not consider many alternative paths (longer than a vehicle's shortest path) because of the assumed 10% savings for platooning vehicles.



(a) Chicago highway network, waiting allowed at intermediate nodes (b) Chicago highway network, waiting allowed only at origin nodes

Figure 2: Solution time and objective function value as pausing time increases. Blue line is the mean time to solution of five replications, with the maximum and minimum times shown as error bars. Solid green line is the objective value at termination (optimality gap of zero or runtime more than 1 hour), dotted line is the objective lower bound for unsolved instances, and dashed line shows objective value after one minute.

Origin/destination nodes/times, network information, optimization model, and other necessary parameters for many problem instances can be found at the website

<http://www.mcs.anl.gov/~jl Larson/Platooning>

3.1 Increasing pauses in Chicago highway network One of the most important parameters for determining the possibility for saving fuel by platooning is the upper bound on the amount of time vehicles are willing to wait. In general, the longer vehicles are willing to wait, the more platooning possibilities exist, and more savings can occur. If the pause time p in (3.14) is zero, then every vehicle must travel from its origin to its destination along its shortest path, without allowing for any platooning. If p is larger, a vehicle can wait to lead/follow another vehicle, thereby decreasing the collective fuel use. In our experiments, increasing p past some point provides no additional savings.

Although we have fixed the randomly chosen origin/destination nodes, and origin times, each value of p induces a different destination time and therefore requires recalculating the $PQ(V,W,I,J)$ indicator variables for all pairs of vehicles and all shared edges. In other words, we must check whether vehicles v and w satisfy (2.1) for shared edges (i,j) when T_v^D and T_w^D increase.

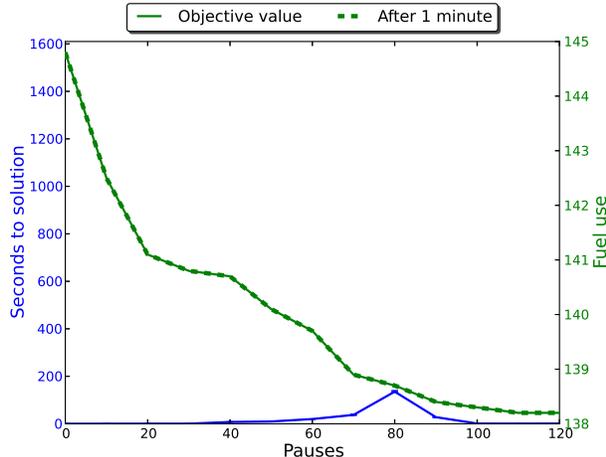
It is not known what capabilities will exist for

forming vehicle platoons on existing roadways. For example, vehicles may or may not be able to wait at intermediate nodes in their path to facilitate more platoon formation. We study the effect of waiting at intermediate nodes as well.

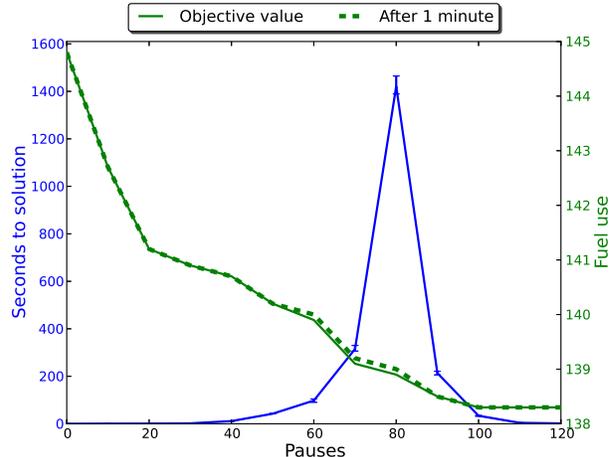
We are interested in studying the time required for Gurobi (with a single thread) to determine that a given solution is optimal. We consider a solution returned from Gurobi to be certified optimal if Gurobi’s relative optimality gap and absolute optimality gap parameters are set to zero. We limit the solution time given to Gurobi, and we record the best objective function value at termination. If Gurobi is stopped because it exhausted the time budget, we also report the lower bound on the best objective value. We are also interested in studying the objective function value after running Gurobi for one minute.

Figure 2 shows the fuel cost and Gurobi solution time (limited to one hour) for the Chicago highway network. In the few cases where Gurobi does not certify an optimal solution in under one hour, the lower bound on the objective is shown. It is nearly impossible to see the difference between the one-minute solution and the best-found solution for most pause values.

The variation in the time required for Gurobi to certify a solution is optimal may seem surprising, but we believe it can be explained by the following observations. When no pause is allowed, Gurobi quickly recognizes that all vehicles must take their shortest paths, and no



(a) Grid network, waiting allowed at intermediate nodes



(b) Grid network, waiting allowed only at origin nodes

Figure 3: Solution time and objective function value as pausing time increases. Blue line is the mean time to solution for five replications, with the maximum and minimum times shown as error bars. Solid green line is the objective value at termination (optimality gap of zero or runtime more than 1 hour), and dashed line shows objective value after one minute.

platooning can occur. (The fuel use when the pause is zero is the sum of the fuel required for each vehicle to travel from its origin to destination along a shortest path.)

When the pause time is sufficiently large, it takes relatively little effort for Gurobi to “greedily” wait until all platooning possibilities have been exploited for each vehicle. The problem becomes much more difficult when Gurobi must determine whether a given platoon routing is optimal when many alternatives exist.

Waiting up to one hour to find a platoon routing is not ideal for real-time routing. As is often the case with many combinatorial optimization problems, most of this time is spent certifying that a solution is optimal and not because improvement is found uniformly over the hour. For the Chicago highway network, the quality of Gurobi’s best solution after one minute is almost indistinguishable from the solution after one hour.

Allowing vehicles to wait at intermediate nodes does not provide much improvement in objective function value. For example, a pause of 120 produces an objective function value of 1557.92 when waiting is allowed, but 1558.65 if no waiting at intermediate nodes is allowed. This relative difference (less than 0.04%) is surprisingly negligible. For the Chicago highway network, fuel savings when allowing vehicles to wait only at their origin nodes is very close to the possible savings when allowing vehicles to wait at any node in their path from their origins to their destinations. The total fuel

used for every vehicle taking its shortest path is 1689.05, which means that if vehicles are willing to wait, savings of 8% can be achieved. This is surprisingly close to the upper bound of 10% savings, given that $\eta = 0.1$ in (2.12).

3.2 Increasing pauses in grid network We now quickly discuss the results of an identical study where the pause length (3.14) is adjusted for vehicles traveling in the grid network, the results of which appear in Figure 3. In this case, Gurobi provided certified optimal solutions within one hour. It is surprising that the time required to find an optimal solution is significantly less when waiting is allowed at intermediate nodes. If waiting is allowed only at origin nodes, the solution after one minute is noticeably different for some pause values. This result highlights the difficulty of routing vehicles in a grid.

3.3 Clusters of vehicles with the same origin/destination One might consider the proposed model to be an “edge-based” model since edges are the units that define how a vehicle travels from its origin to destination. Another valid formulation would be to declare routes for each origin/destination pair and then restrict each vehicle to take one route. This may help Gurobi since a given route determines the edges a vehicle will traverse, leaving only the coordination of vehicles’ enter times on the edges. Routes also allow us

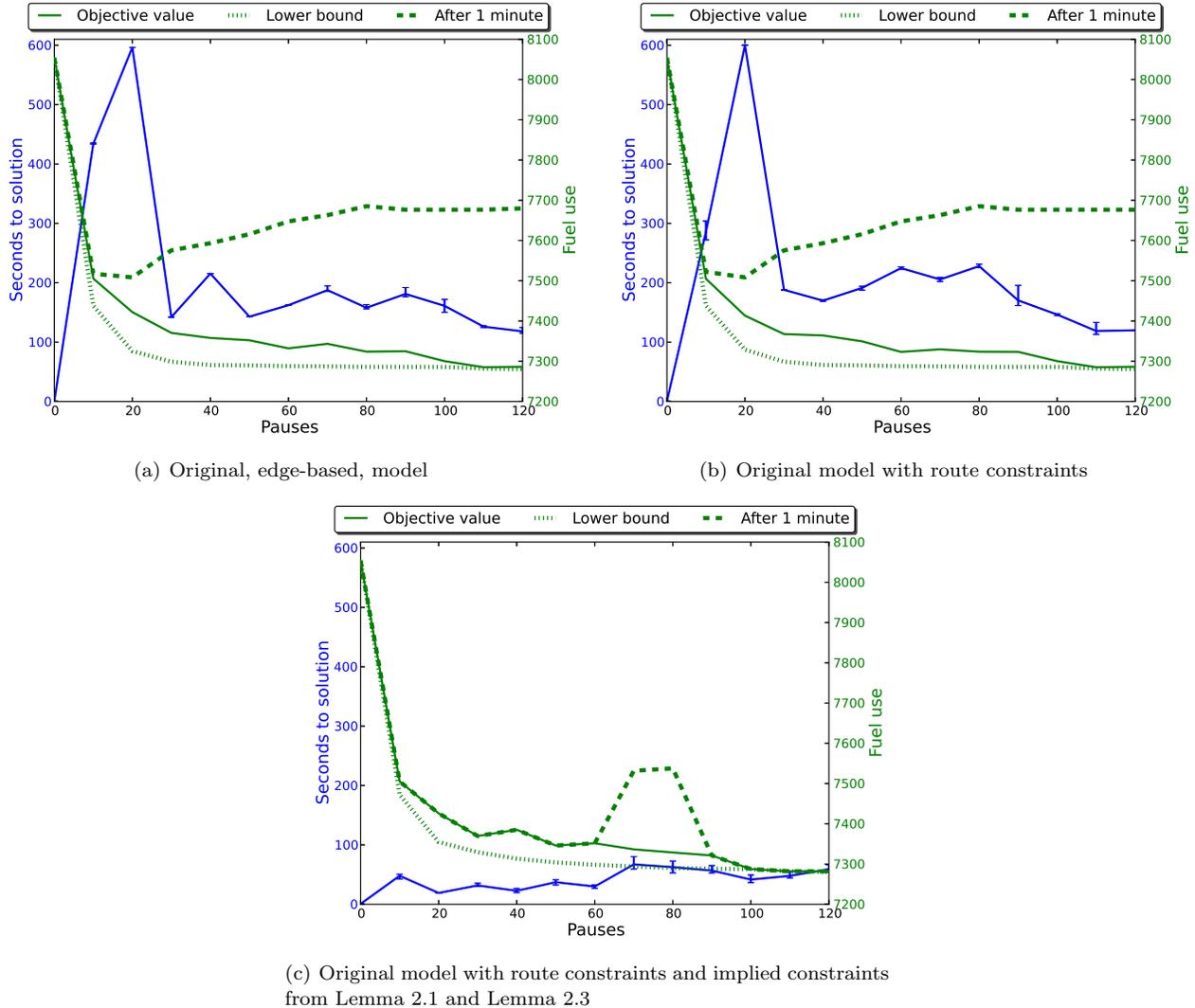


Figure 4: Solution time and objective function value as pausing time increases for 100 vehicles in the Chicago highway network. Blue line is the mean time to solution for five replications, with the maximum and minimum times shown as error bars. Solid green line is the objective value at termination (optimality gap less than 1% or runtime more than ten minutes), dotted line is the objective lower bound for unsolved instances, and dashed line shows objective value after one minute.

to describe additional constraints, for example, those implied by Lemma 2.1.

To include routes in the GAMS model, we first declare N routes for all vehicles, and we set $R_{n,i,j} = 1$ if route $n \in N$ contains edge $(i, j) \in E$. We then include a decision variable $u_{v,n}$ if vehicle $v \in V$ uses route $n \in N$. Although routes can share edges, a vehicle can take a route n only if the route's first vertex is T_v^O and its last vertex is T_v^D . Similar to before, we enforce this constraint with the indicator variable $\text{PR}(V, N)$, which is 1 if and only if V can take route N .

We then replace constraints (2.2) with

$$(3.15) \quad f_{v,i,j} = \sum_{R_{n,i,j}} u_{v,n} \quad \forall v \in V, (i, j) \in E,$$

and we constrain vehicles to take only one route by

$$(3.16) \quad \sum_n u_{v,n} = 1 \quad \forall v \in V.$$

Both of the summations in (3.15) and (3.16) occur only for routes such that $\text{PR}(V, N) = 1$.

We are interested in studying what effect (if any) routes have on the time required to solve larger platoon routing problems. We study the ability of our model to route 100 vehicles in the Chicago highway network. We take the five most common origins/destinations from the POLARIS simulation of the Chicago highway network and assign each pair to 20 different vehicles. Each of the 100 vehicles is given a random origin time drawn uniformly from $[0, 100]$, and destination times are (again) taken to be the origin time plus the shortest path time plus some pause as in (3.14). Duplicating origin/destination pairs is similar to reality where many vehicles enter the highway at the same on-ramp and leave the highway at the same exit.

We set Gurobi to stop when its relative optimality gap is less than 1%, and we limit the solution time to at most ten minutes. We compare the original model from Section 2, a model using the additional route variables/constraints described in (3.15) and (3.16), and a model constraining vehicles with the same origin/destination nodes to either platoon the entire way or not platoon at all (implied by Lemma 2.1). For vehicles sharing the same origin/destination, we set $PQ(V, W, I, J)=1$ only if W arrives at the origin after V (implied by Lemma 2.3).

Figure 4 shows the effects of including route constraints and these implied constraints in the model. We note that the inclusion of routes in the model does not have an appreciable effect on the time required to find a solution with an optimality gap less than 1%. Including the constraints implied by Lemma 2.1 and Lemma 2.3 significantly reduces the time required to solve this problem instance for all pause levels. Although the one-minute solution is not optimal when the pause is set to 70 or 80, we can see that a solution with a 1% optimality gap is found in less than 100 seconds.

For the original model (with or without the additional route variables/constraints), the one-minute solution is particularly poor. This may be due to the default Gurobi settings, which devote considerable time preprocessing the problem even when the time limit is relatively small. Such effort may be worthwhile, given that Gurobi does find an optimal solution in under 200 seconds for most pause values.

4 Discussion

By using parameters PQ , PE , and PR to limit the declaration of variables and constraints, our model is able to route vehicles and form/dissolve platoons throughout their routes in an optimal fashion. The inclusion of implied constraints dramatically decreases the time required to find optimal solutions for realistic problem instances. Often, the constraints apply to vehicles that

share the same origin and destination nodes (a common occurrence), but our declaration of PQ only for vehicles and edges that satisfy (2.1) is also effective at reducing the problem complexity. We believe that additional implied constraints that are inherent in the platoon routing problem can similarly increase the size of problems that can be solved optimally.

Current work involves better modeling of fuel consumption. While a general formulation likely requires a nonconvex objective, turning our mixed-integer program into a mixed-integer nonlinear program, we believe piecewise-linear models of fuel consumption can accurately approximate true fuel-usage rates while keeping the model linear. These models will also allow us to consider vehicles traveling at non-free-flow speeds.

Solving the platoon routing problem to optimality for millions of vehicles in real-world networks is likely intractable. Nevertheless, the models presented here can be used to solve subproblems within such larger platoon routing settings.

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