An Optimization Approach for Identifying and Prioritizing Critical Components in a Power System

Kibaek Kim, *Member, IEEE*

***Abstract*—We present an optimization-based approach for identifying and prioritizing critical components of a power system. We argue that this approach is more suitable for resilient power system planning than a traditional $N-k$ analysis. First, we formulate and solve a bilevel optimization problem that identifies the most critical grid component. Second, we use a cutting-plane method to exclude the most critical component from a set of components. Our approach repeats these steps until identifying a given number of critical components. The order of the components identified corresponds to their priority for protection. Numerical results are reported using the Western Electricity Coordinating Council test data with thousands of wind generation scenarios and load profiles. We compare our approach with $N-k$ analysis and discuss why our approach is more suitable for hardening and restoring the system from disruptions. Moreover, we perform a sensitivity analysis on seasonal net-load profiles and uncertain wind generations, which can identify *hidden critical components* of the system.**

***Index Terms*—resilient power system, critical components, bilevel optimization, cutting-plane method**

NOMENCLATURE

Sets:

$\mathcal{D}; \mathcal{D}_n$	Demand loads; demand loads at bus n
$\mathcal{G}; \mathcal{G}_n$	Generators; generators at bus n
$\mathcal{I}; \mathcal{I}_n$	Import points; import points at bus n
$\mathcal{L}$	Transmission lines
$\mathcal{L}_n^+; \mathcal{L}_n^-$	Transmission lines to bus n; lines from bus n
$\mathcal{N}$	Buses
$\mathcal{R}; \mathcal{R}_n$	Renewable generators; Renewable generators at bus n
$\mathcal{T}$	Time periods
$\mathcal{W}$	Wind-farm locations

Parameters:

B_l	Susceptance of transmission line l
C_i	Generation cost of generator i
C_j^d	Load-shedding penalty at load j
C_i^w	Spillage penalty at wind farm i
C_i^m	Spillage penalty at import point i
C_i^r	Spillage penalty at renewable i
D_{jt}	Demand load of consumer j at time t
F_l^{max}	Maximum power flow of transmission line l
M_{it}	Power production of import i at time t
P_i^{max}	Maximum power output of generator i
R_{it}	Power production of renewable i at time t
RU_i	Ramp-up limit of generator i
RD_i	Ramp-down limit of generator i
W_{wt}	Power from wind farm w at time t
Θ_{nt}^{min}	Minimum phase angle at bus n at time t
Θ_{nt}^{max}	Maximum phase angle at bus n at time t

Decision variables:

d_{jt}	Load shedding at load j at time t
f_{lt}	Power flow of line l at time t
m_{it}	Spillage at import i at time t
p_{it}	Power from generator i at time t
r_{it}	Spillage at renewable i at time t
w_{it}	Spillage at wind farm i at time t
z_i^B	1 if bus i is disrupted; 0 otherwise
z_l^L	1 if line l is disrupted; 0 otherwise
θ_{nt}	Phase angle at bus n at time t

K. Kim is with the Mathematics and Computer Science (MCS) Division, Argonne National Laboratory, Lemont, IL 60439 and also with the Computation Institute, University of Chicago, Chicago, IL 60637 USA, Email: kimk@anl.gov.

I. INTRODUCTION

Identifying critical components is an important task in designing resilient grid systems and in developing suitable mitigation and restoration strategies. While rare and often unpredictable, failure in such components can be catastrophic and capable of disabling large portions of a grid system and associated connected systems. For example, a sequence of coordinated cyber and physical attacks can be designed to cascade and progressively take down large sections of the power grid. Similarly, combinations of weather and manmade disruptions or combinations of wind ramping events can create large disruptions.

This critical task is technically challenging for real-sized electric power systems. In particular, the number of possible individual and combined contingency events is astronomical. As a result, it is simply technically and financially prohibitive for system operators and government agencies to study and prepare for all possible disruptions. For example, an $N-k$ contingency analysis considers disruptions to k infrastructure assets of a total N assets; that is, "N chooses k." When $N-3$ contingencies are considered among 1,000 infrastructure assets, the number of possible contingencies is nearly one billion. As a result, complexity explodes in $N-k$ studies, as shown in Figure 1. Consequently, even if a single contingency event can be simulated within ten minutes, spanning the entire $N-3$ space would take over 2,000 years on a single-core computing machine.

Existing $N-k$ contingency analysis often considers only the most critical component (i.e., the worst case), which can be computed by solving a robust optimization problem (e.g., [1]–[5]). Extensive studies have been done for solving the robust optimization problem and often have provided computationally tractable reformulations [6]. Moreover, this robust optimization framework has been widely used in risk-averse

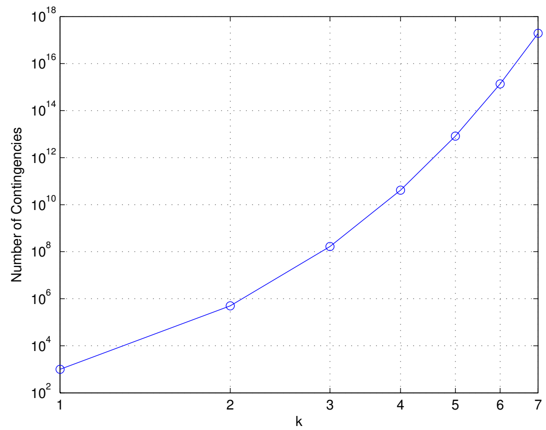


Fig. 1. Astronomical number of possible contingency events for 1,000 grid components.

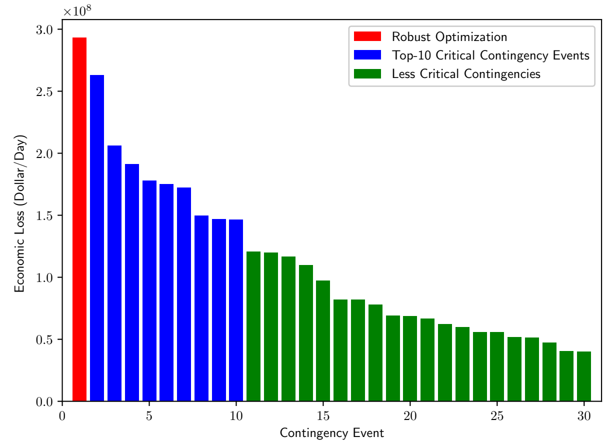


Fig. 2. Top-30 critical contingency events resulting from one bus failure for the Western Electricity Coordinating Council (WECC) test system. Each contingency event is quantified with respect to economic loss.

power system planning and operations to hedge against all the possible contingency scenarios (e.g., [5], [7]). For example in [5], a two-stage robust unit commitment problem models robust optimization as subproblems in order to operate power system under $N-k$ contingency events. The subproblems were formulated to equivalent mixed-integer programming (MIP) problems with big-M parameters, which can be handled in off-the-shelf MIP solvers (e.g., CPLEX). However, the model could not be solved in 5 hours for an IEEE 118-bus system when $k \geq 2$, as reported in [5].

Besides the computational complexity, the robust optimization approach is limited to the most critical contingency only and ignores all other contingencies (i.e., others with the same critical impact, the second and third most critical, and so on), as illustrated in Figure 2. Note that for given $k \geq 2$, $N - k$ analysis identifies k critical components that are not prioritized. More important, the critical components identified by $N-k$ analysis are not necessarily those identified by $N - (k + 1)$ analysis. The inconsistency of the critical components identified makes it difficult to effectively plan on hardening and protecting the system components. We discuss the numerical evidence in Section IV.

Such limitations of $N - k$ analysis necessitate prioritizing the critical components for viable protection planning for the grid system. In this paper, we develop a novel prioritization approach that aims to successively identify the critical components in the order of criticality. The approach first identifies the most critical component. Then, the second most critical component is identified by assuming that the most critical component has been protected. The approaches repeat that for the third most critical component and so on. Consequently, the prioritization is the order of the critical components identified by the approach. This prioritization approach would help government agencies and utility companies make investment decisions for hardening and protecting the critical components.

Our prioritization method differs from traditional $N - k$ analysis in several ways. $N - k$ analysis finds a *combination* of k components that maximizes the system damage, whereas the prioritization approach finds a *prioritized* list of k critical components in which the second most critical component is identified with the assumption that the most critical component has been protected. From a practical point of view, the prioritization approach provides the best defensive plan, whereas $N-k$ analysis provides the best tactical plan. In this work, we focus on how to protect the critical components, rather than effectively attacking them.

We developed an optimization-based approach that combines a detailed power system model and a cutting-plane method to identify and prioritize the critical components of power system. Our approach formulates and solves a bilevel optimization model to find the most critical component of the system with respect to a given criticality (e.g., total system cost). The second most critical component can be identified by excluding the most critical component from the component set and by solving the maximin optimization model again. Components can be excluded by using *no-good integer cuts*. This procedure can be applied recursively by identifying the kth most critical components.

Using a test system of California Independent System Operator (CAISO) interconnected with the Western Electricity Coordinating Council (WECC), we performed numerical studies to support the argument that our approach is more suitable than traditional $N - k$ analysis for planning resilient power system. In addition, we analyzed a number of scenarios for load profile and wind power generation that affect the identification and prioritization of critical components and can potentially identify *hidden critical components* of the system.

The rest of the paper is organized as follows. In Section II we present a maximin optimization model that identifies the most critical component of power system. Section III presents the prioritization algorithm that recursively identifies the critical components. Section IV presents extensive numerical studies. In Section V we summarize our results and briefly discuss future work.

II. Critical Grid Component Identification

The main idea of the critical component prioritization is successively identifying the most critical component by restricting the set of candidate components. This procedure is a key difference from traditional $N-k$ analysis that solves only one identification problem.

In this section we present an optimization-based model that identifies the most critical component of an electric grid system. In this paper, we focus on the economic loss resulting from disruption at the component. Note, however, that our method can be applied to other criticality metrics (e.g., economic loss, load shedding). We assume that system operator operates a power grid in order to minimize the total operational cost, including the economic lost by load shedding and excess power generation (denoted by $C_j^d, C_i^w, C_i^m, C_i^r$). We are interested in identifying one or more power grid assets whose removals maximize the minimal operational cost. This can be modeled by a bilevel optimization problem that aims to finding a contingency that causes the maximum damage to the system.

The bilevel optimization problem is given by

$$\mathbf{z}^{(1)} = \arg\max_{\mathbf{z}\in\mathcal{Z}} \mathcal{Q}(\mathbf{z}), \tag{1}$$

where $\mathcal{Q}(\mathbf{z})$ is the minimum operation cost for given disruptions $\mathbf{z}$ and $\mathcal{Z}$ is a set of feasible disturbances. We define the disturbance set as a set of binary vectors

$$\mathcal{Z} := \left\{ \mathbf{z} \in \{0,1\}^{|\mathcal{A}|} : \sum_{i\in\mathcal{A}} \alpha_i \mathbf{z}_i \le K \right\}, \tag{2}$$

where $\mathcal{A} := \mathcal{N} \cup \mathcal{L}$ is the set of grid components, the inequality in (2) represents the budget constraint, and α_i is the weighted cost of disrupting component i. For given disruption $\mathbf{z}$ the minimum operation cost is defined as follows:

$$\mathcal{Q}(\mathbf{z}) :=$$

$$\min \quad \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{G}} C_i p_{it} + \sum_{j\in\mathcal{D}} C_j^d d_{jt} + \sum_{i\in\mathcal{I}} C_i^m m_{it} + \sum_{i\in\mathcal{W}} C_i^w w_{it} + \sum_{i\in\mathcal{R}} C_i^r r_{it} \right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{l\in\mathcal{L}_n^+} f_{lt} - \sum_{l\in\mathcal{L}_n^-} f_{lt} + \sum_{i\in\mathcal{G}_n} p_{it} + \sum_{i\in\mathcal{I}_n} (M_{it} - m_{it}) + \sum_{i\in\mathcal{W}_n} (W_{it} - w_{it}) + \sum_{i\in\mathcal{R}_n} (R_{it} - r_{it}) = \sum_{j\in\mathcal{D}_n} (D_{jt} - d_{jt}), \quad \forall n \in \mathcal{N}, t\in\mathcal{T}, \tag{3b}$$

$$|f_{lt} - B_l(\theta_{nt} - \theta_{mt})| \le M[1-(1-z_l^L)(1-z_n^B)(1-z_m^B)], \quad \forall l=(m,n)\in\mathcal{L}, t\in\mathcal{T}, \tag{3c}$$

$$|f_{lt}| \le F_l^{max}(1-z_l^L)(1-z_n^B)(1-z_m^B), \quad \forall l=(m,n)\in\mathcal{L}, t\in\mathcal{T}, \tag{3d}$$

$$-RD_i \le p_{it} - p_{i,t-1} \le RU_i, \quad \forall i \in\mathcal{G}, t\in\mathcal{T}, \tag{3e}$$

$$\Theta_n^{min} \le \theta_{nt} \le \Theta_n^{max} \quad \forall n\in\mathcal{N}, t\in\mathcal{T}, \tag{3f}$$

$$0 \le p_{it} \le P_i^{max}, \quad \forall i\in\mathcal{G}, t\in\mathcal{T}, \tag{3g}$$

$$0 \le d_{jt} \le D_{jt}, \quad \forall j\in\mathcal{D}, t\in\mathcal{T}, \tag{3h}$$

$$0 \le m_{it} \le M_{it}, \quad \forall i\in\mathcal{I}, t\in\mathcal{T}, \tag{3i}$$

$$0 \le w_{it} \le W_{it}, \quad \forall i\in\mathcal{W}, t\in\mathcal{T}, \tag{3j}$$

$$0 \le r_{it} \le R_{it}, \quad \forall i\in\mathcal{R}, t\in\mathcal{T}. \tag{3k}$$

The objective function of the problem (3) is the total dispatch cost with penalty for load shedding and excess generation. Constraint (3b) is the flow balance equation ensuring that each demand load is satisfied by power supply, load shedding, and excess spillage over the grid network. Constraints (3c) and (3d) represent a linearized power flow equation based on Kirchhoff's law and the transmission line capacity limits, respectively, which are enabled or disabled by binary variables z_l^L, z_n^B, z_m^B. Specifically, the constraints are disabled if $z_l^L = 1, z_n^B = 1$, or $z_m^B = 1$ for any $l = (m,n) \in \mathcal{L}$. This also encodes that disruption at bus n inactivates the transmission lines connected to the bus. Constraint (3e) ensures the ramping capacity of generating units. Constraints (3f)–(3k) enforce the bounds for decision variables.

Note that line $l = (m,n)$ should be disrupted if either bus m or n is disrupted. This is equivalent to saying that both buses m and n are not disrupted if the line is not disrupted. Such a relation can be written as

$$z_l^L \ge z_m^B, \tag{4a}$$

$$z_l^L \ge z_n^B, \tag{4b}$$

$$z_l^L \le z_m^B + z_n^B. \tag{4c}$$

This allows us to replace constraints (3c) and (3d) by

$$|f_{lt} - B_l(\theta_{nt} - \theta_{mt})| \le M z_l^L, \ \forall l=(m,n)\in\mathcal{L}, t\in\mathcal{T}, \tag{5a}$$

$$|f_{lt}| \le F_l^{max}(1-z_l^L), \ \forall l=(m,n)\in\mathcal{L}, t\in\mathcal{T}, \tag{5b}$$

where z_l^L is subject to (4).

The bilevel optimization problem (1) is a standard robust optimization problem. The problem is challenging to solve because $\mathcal{Q}(\mathbf{z})$ is piecewise-linear convex in $\mathbf{z}$, and thus the problem (1) is a convex maximization problem. The bilevel problem can be reformulated to a single-level mixed-integer linear programming (MILP) problem by using arbitrarily large "big-M" parameters [2], [5]. The resulting MILP problem can be solved by off-the-shelf MILP solver (e.g., CPLEX). But, the reformulation is not scalable and also leads to a poor computational performance, because of the arbitrary choice of the big-M parameters. For more scalable solution, a Benders decomposition method has been used with mild assumptions to the model [1]. The method successively approximates the function $\mathcal{Q}$ by adding analytically derived linear inequalities (i.e., Benders cuts).

III. Prioritization Method

In this section we present an algorithm that can prioritize the critical grid components with respect to the criticality. The algorithm assumes that the power grid operator solves the economic dispatch model $\mathcal{Q}(\mathbf{z})$ for any given system disruption $\mathbf{z}$. Recall that the most damaging event $\mathbf{z}^{(1)}$ can be obtained by (1). The second most damaging event (denoted by $\mathbf{z}^{(2)}$) can be identified by restricting the event set as $\mathcal{Z}\backslash\{\mathbf{z}^{(1)}\}$ and by solving the problem $\mathbf{z}^{(2)} = \arg\max_{\mathbf{z}\in\mathcal{Z}\backslash\{\mathbf{z}^{(1)}\}} \mathcal{Q}(\mathbf{z})$. This procedure can be applied recursively to identify the kth most damaging disturbance. This is done by restricting the disturbance set as $\mathcal{Z}\backslash\{\mathbf{z}^{(1)}, \mathbf{z}^{(2)}, \ldots, \mathbf{z}^{(k-1)}\}$, iteratively adding the linear inequalities to the maximin optimization problem (1). The linear inequalities are of the form

$$\sum_{i\in\mathcal{A}|\mathbf{z}_i^{(\kappa)}=1} \mathbf{z}_i + \sum_{i\in\mathcal{A}|\mathbf{z}_i^{(\kappa)}=0} (1-\mathbf{z}_i) \leq |\mathcal{A}| - 1, \tag{6}$$

where $\mathbf{z}_i^{(\kappa)}$ represents whether asset $i \in \mathcal{A}$ is disrupted or not for given critical component solution $\mathbf{z}^{(\kappa)}$. We note that the linear inequality (6) is violated at $\mathbf{z}^{(\kappa)}$ and thus excludes $\mathbf{z}^{(\kappa)}$ only from $\mathcal{Z}$. We add the linear inequalities of the form (6) at $\mathbf{z}^{(1)}, \mathbf{z}^{(2)}, \ldots, \mathbf{z}^{(k-1)}$ successively, which identifies the critical components by priority. We emphasize that only a finite number of linear inequalities can be generated. The linear inequality (6) has been used for accelerating MILP solution time in the literature (e.g., [1], [8], [9]). However, none of these has used the linear inequality (6) to prioritize the solutions.

The steps to identify and prioritize the k most damaging disturbances are summarized in Algorithm 1.

Algorithm 1 Identifying k Most Damaging Disturbances

1: Create the initial disturbance set $\mathcal{Z}^{(1)}$, and set $\kappa \leftarrow 1$.
2: **while** $\kappa \leq k$ **do**
3:     Find $\mathbf{z}^{(\kappa)} = \arg\max_{\mathbf{z}\in\mathcal{Z}^{(\kappa)}} \mathcal{Q}(\mathbf{z})$.
4:     Update the disturbance set $\mathcal{Z}^{(\kappa+1)} = \mathcal{Z}^{(\kappa)} \cap \{\mathbf{z} : (6)\}$.
5:     Set $\kappa = \kappa + 1$.
6: **end while**
7: **return** $\mathbf{z}^{(1)}, \ldots, \mathbf{z}^{(k)}$

The disturbance set $\mathcal{Z}^{(1)}$ can be initialized as defined in (2) with the weight α_i and budget K (line 1). We note that the initial set can embed more logics on disruptions; for example,

- a subset of components can be excluded from the algorithm by adding the linear inequalities (6);
- disruption at bus i causes disruption at bus j for $i \neq j$ ($\mathbf{z}_i \leq \mathbf{z}_j$);
- either bus i of j should be disrupted ($\mathbf{z}_i + \mathbf{z}_j \geq 1$);
- at most one of buses i and j should be disrupted ($\mathbf{z}_i + \mathbf{z}_j \leq 1$).

These can be achieved by additional constraints to $\mathcal{Z}^{(1)}$. The problem in line 3 of Algorithm 1 finds the most critical component in $\mathcal{Z}^{(\kappa)}$, which can be solved by existing approaches. In line 4, we exclude the critical component $\mathbf{z}^{(\kappa)}$ from the disturbance set by adding the linear inequality (6). The algorithm terminates after k iterations, returning the k most damaging disturbances $\mathbf{z}^{(1)}, \ldots, \mathbf{z}^{(k)}$.

To complete the description of Algorithm 1, we present the big-M approach [2] and the generalized Benders method [1] in the following subsections. Note, however, that Algorithm 1 can use any method for solving $\mathcal{Q}(\mathbf{z})$ in line 3.

A. Big-M Approach

The big-M approach considers the dual of the inner minimization function $\mathcal{Q}(\mathbf{z})$. We define the dual problem as

$$\mathcal{Q}(\mathbf{z}) = \max_{\lambda\in\Lambda} \ H\mathbf{z}^T\lambda, \tag{7}$$

where λ represents the Lagrangian dual variable vector for the problem (3), and H represents the objective function coefficients for the dual. This reformulates the bilevel problem to a single-level problem,

$$\max_{\mathbf{z}\in\bar{\mathcal{Z}},\ \lambda\in\Lambda} H\mathbf{z}^T\lambda, \tag{8}$$

where $\bar{\mathcal{Z}} := \mathcal{Z} \cap \{\mathbf{z} : (4)\}$. The objective function is bilinear in $\mathbf{z}$ and λ. The set Λ of feasible solutions is independent of $\mathbf{z}$. Note that by strong duality, the optimal objective values of (3) and (7) are the same for any given $\mathbf{z}$.

The bilinear objective function can be linearized by introducing binary variables with big-M parameters. The linearized formulation of the dual (7) is given by

$$\max_{\mathbf{z}\in\bar{\mathcal{Z}},\ \lambda\in\Lambda,\ w} \quad Hw \tag{9a}$$

$$\text{s.t. } w \leq \lambda,\ w \leq Mz,\ w \geq \lambda + M(z-1),\ w \geq 0, \tag{9b}$$

where M is a sufficiently large number. Note that the set of constraints (9b) is a special form of McCormick envelopes [10]. While resulting in a single-level MILP formulation, the problem (9) requires introducing $2|\mathcal{L}||\mathcal{T}|$ additional binary variables. Moreover, in general, the big-M formulation results in weak lower bounds and leads to long computation time in a branch-and-bound method. We also note that another equivalent big-M reformulation can be obtained by using Karush-Kuhn-Tucker conditions, which performs worse than the dual-based one (9) with respect to computational time, as reported in [2].

B. Benders Decomposition Method

The Benders decomposition method, first introduced in [11], has been widely used in mixed-integer programming, including deterministic, stochastic, linear, and convex programs. The key idea is to partition the problem into a master problem (usually with integer variables) and one or more subproblems (usually with continuous variables only). In the context of our model, the method solves the problem (1) by iteratively estimating the function $\mathcal{Q}(\mathbf{z})$ with a set of linear inequalities. That is, the master problem is given by

$$\max_{\mathbf{z}\in\mathcal{Z},\zeta\in\mathbb{R}} \ \zeta \tag{10a}$$

$$\text{s.t. } \zeta \leq \mathcal{Q}(\hat{\mathbf{z}}) + \sum_{i\in\mathcal{A}} q_i(\hat{\mathbf{z}})(\mathbf{z}_i - \hat{\mathbf{z}}_i),\ \forall \hat{\mathbf{z}} \in \hat{\mathcal{Z}}, \tag{10b}$$

where the linear inequalities (10b) outer-approximate the function $\mathcal{Q}(\mathbf{z})$ at each point $\hat{\mathbf{z}} \in \hat{\mathcal{Z}}$. The coefficients $q_i(\hat{\mathbf{z}})$ is the key for generating the linear inequalities (10b), which represents the subgradients of $\mathcal{Q}(\mathbf{z})$ at each $\hat{\mathbf{z}} \in \hat{\mathcal{Z}}$. However, generating the constraints (10b) is nontrivial, because the function $\mathcal{Q}(\mathbf{z})$ is convex in $\mathbf{z}$ for maximization.

Salmeron et al. [1] heuristically derive the coefficients $q_i(\hat{\mathbf{z}})$ based on the economic dispatch solution (denoted by $p_{it}(\hat{\mathbf{z}})$ and $f_{lt}(\hat{\mathbf{z}})$) of (3) for given disruption $\hat{\mathbf{z}}$. Although it is heuristic, optimal solutions were found for all the numerical results in [1]. The coefficients of the linear inequalities (10b) are derived from the following:

$$q_n(\hat{\mathbf{z}}) := (1-\hat{\mathbf{z}}_n)\bar{C}\sum_{t\in\mathcal{T}}\left[\sum_{i\in\mathcal{G}_n} p_{it}(\hat{\mathbf{z}}) + \sum_{l\in\mathcal{L}_n^+\cup\mathcal{L}_n^-}|f_{lt}(\hat{\mathbf{z}})|\right]$$
$$\forall n\in\mathcal{N},$$
$$q_l(\hat{\mathbf{z}}) := (1-\hat{\mathbf{z}}_l)\bar{C}\sum_{t\in\mathcal{T}}|f_{lt}(\hat{\mathbf{z}})| \quad \forall l\in\mathcal{L},$$

where $\bar{C} := \max_{j\in\mathcal{D}} C_j^d - \min_{i\in\mathcal{G}} C_i$. Using the coefficients, one can describe the Benders decomposition as in Algorithm 2. The algorithm requires an initial set $\mathcal{Z}$ and returns the most critical component $\hat{\mathbf{z}}$ after a finite number of iterations.

Algorithm 2 Benders Decomposition

Require: Initial set $\mathcal{Z}$

Find an initial feasible solution $\hat{\mathbf{z}}\in\mathcal{Z}$, and set $\hat{\zeta}\leftarrow\infty$.
Solve $\mathcal{Q}(\hat{\mathbf{z}})$, and find coefficients $q_i(\hat{\mathbf{z}})$ for all $i\in\mathcal{A}$.
while $\hat{\zeta} > \mathcal{Q}(\hat{\mathbf{z}})$ **do**
  Update $\hat{\mathcal{Z}} \leftarrow \hat{\mathcal{Z}} \cup \{\hat{\mathbf{z}}\}$.
  Find an optimal solution $(\hat{\mathbf{z}}, \hat{\zeta})$ by solving (10).
  Solve $\mathcal{Q}(\hat{\mathbf{z}})$, and find coefficients $q_i(\hat{\mathbf{z}})$ for all $i\in\mathcal{A}$.
end while
return $\hat{\mathbf{z}}$

IV. Computational Studies

We study a test system of CAISO interconnected with WECC. This test system consists of 225 buses, 375 transmission lines, 135 generation units, and 40 loads. We use the data from [12] for the test system, which consists of network topology, load profiles, import supplies, wind supplies, and renewable supplies. We study eight day types defined for each season (spring, summer, fall, and winter), weekday (WD), and weekend (WE). For each day type, different load profiles, import supplies, wind supplies, and renewable supplies are given. We calculate the net load demand as the load demand subtracted by the import and renewable generations. The net load profile for each day type is shown in Figure 3. We also consider uncertain wind power generation. To address the uncertainty, we use a thousand wind generation scenarios for each season at five wind farms of the test system. Figure 4 illustrates the wind generation scenarios for the four seasons.

Using the data, the economic dispatch model (3) considers a 24-hour horizon with hourly intervals. We assume that the excess energy from scheduled import, wind, and renewable sources is curtailed at large penalty cost. In particular, we use the penalty cost of \$2,000/MWh for curtailing the excess import and \$1,000/MWh for curtailing the excess wind and renewable energy. In addition, we penalize the amount of load lost at \$5,000/MWh.

In Sections IV-B and IV-C we present numerical analyses of the net-load profile of FallWD with 0% wind penetration level.

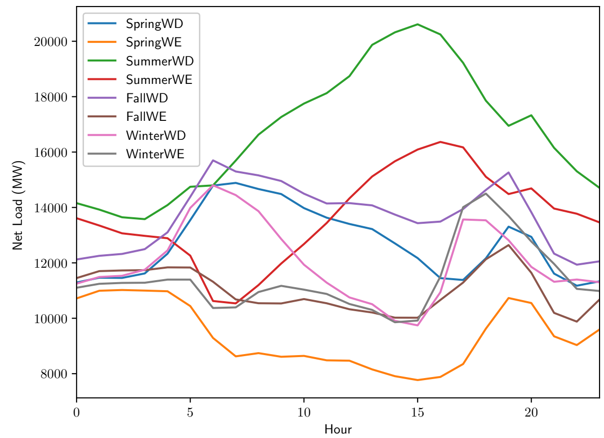
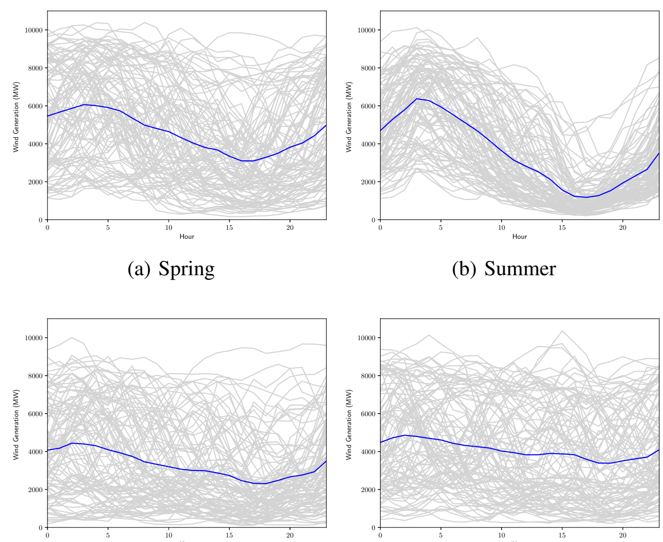


Fig. 3. Net load profile of the test system for each day type.



(a) Spring (b) Summer

(c) Fall (d) Winter

Fig. 4. Wind generation scenarios at 15% penetration level for spring, summer, fall, and winter seasons. Only 100 out of 1,000 scenarios were plotted in grey, and the mean generation is plotted in blue.

In Section IV-D we perform a sensitivity analysis of the critical component results for the eight day types. In Section IV-E we introduce to the system 1,000 wind scenarios of 15% penetration level and perform the sensitivity analysis.

A. Computation Settings

We have implemented Algorithm 1 in the `Julia` script language. More specifically, we use the `JuMP` package [13] to model the critical component identification problem (1). For solving the bilevel optimization problem (1) in each iteration of Algorithm 1, we use Benders decomposition, described in Algorithm 2, which is scalable to the size of power system compared with the big-M approach (described in Section III-A). For example, the Benders decomposition solution for $k = 1$ takes less than a minute, whereas the big-M approach solution takes nearly an hour. However, we

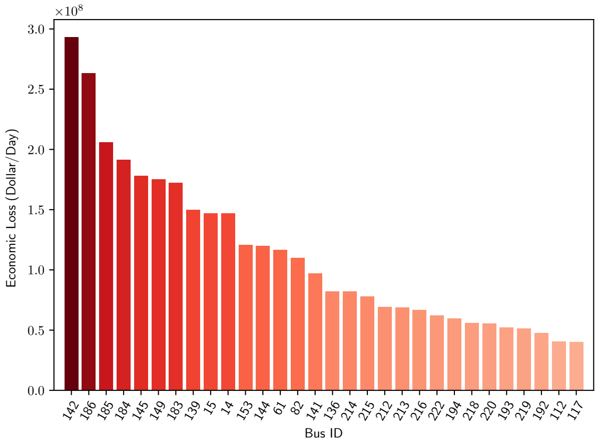


Fig. 5. Dispatch cost increase resulting from disruptions at the critical buses.

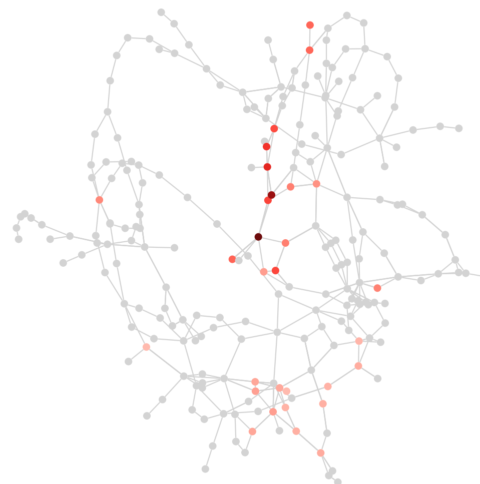


Fig. 6. Node-edge representation of the WECC test system: The critical nodes are identified and colored in red by the increase of dispatch cost.

acknowledge that the Benders decomposition method may find a suboptimal solution to problem (1), as discussed in [1].

Benders decomposition has been implemented by using `CPLEX` callback functions via the `JuMP` package. All the computations were run on Argonne's Blues cluster, consisting of 600 computing nodes with two octo-core 2.6 GHz Xeon processors and 64 GM or RAM on each node. The capabilities of running the algorithms on such clusters is key to rigorously perform the analysis particularly with a large number of scenarios. In particular, the contingency analysis with the wind scenarios was performed on the cluster by massive parallelization with the `Swift` script language [14]. The entire computational studies required nearly 8 hours of wall-clock time with 512 computing cores, equivalently 4,096 core-hours.

B. Critical Components of the WECC Test System

We apply Algorithm 1 to identify and prioritize the critical buses of the test system. We set $\alpha_i = K = 1$ for the budget constraint in (2). Hence, every iteration of Algorithm 1 identifies and excludes the most critical bus by adding the inequality (6). The algorithm was run to detect 30 most critical buses in the system. The criticality of substations is measured based on the amount of dispatch cost increased by the event that a bus is disabled.

The critical buses are prioritized in the amount of economic loss and reported in Figure 5. The topological locations of the critical buses are shown in Figure 6. Note that the results are based on the test system, not necessarily representing the actual WECC system. The dispatch cost increment ($294M) by disrupting bus 142 is more than seven times that ($40M) by bus 117. This suggests that failure in identifying the critical components can cost hundreds of millions of dollars more.

Among the 30 buses identified here, 25 buses have load demand; 2 buses (#14 and 82) have import points; 1 bus (#139) has both 47 generators and a renewable source; 1 bus (#117) has a generator, a renewable source, and a wind farm; and one bus (#15) has neither load demand nor any generating source. We found that most load buses are the critical components of the system. However, not all critical buses are loads. In particular, bus 15 is identified and prioritized as the 9th most critical bus, although it is not connected to any load or generation unit. We also note that the order of the critical buses is not consistent with the load capacity.

C. Comparison with $N-k$ Analysis

We compare the critical components identified by Algorithm 1 with those resulting from a traditional $N-k$ analysis. Table I reports the 10 critical buses identified by Algorithm 1 and $N-k$ analysis for $k = 2,3,4,5$. For example, $N-2$ analysis found two critical buses (# 142 and 15), and $N-3$ analysis found three critical buses (#142, 15, and 107). Both Algortihm 1 and the $N-2$ analysis suggest protecting bus 142 first. Once bus 142 is protected, Table I suggests protecting either bus 186 (by Algorithm 1) or bus 15 (by the $N-2$ approach). Failure to protect buses 186 and 15 can cause $260M and $146M economic losses, respectively. These amounts clearly suggest protecting bus 186, rather than bus 15. Simultaneous disruptions at buses 142 and 15 are the most effective from the attacker's point of view (i.e., the best attacking combination of two components). In fact, simultaneous disruptions at buses 142 and 15 cause more economic loss than those at buses 142 and 186 ($575M vs. $520M). We note, however, that the disruptions can be significantly mitigated by defending bus 142 only.

We also compare two approaches with respect to average economic gain. Figure 7 shows the average economic gain obtained for each protection strategy. Top-k strategies for $k = 2,\dots,5$ protect the first k critical components suggested by Algorithm 1. $N-k$ strategies protect the critical components suggested by the $N-k$ approaches for $k = 2,\dots,5$. The economic gain is the amount of economic loss reduced by protecting the components suggested by the approaches. We calculated the average economic gain for the $N-k$ maximum disruptions for $k = 1,\dots,5$. In Figure 7 we found that the average economic gains obtained by the Top-k strategies are larger than those obtained by the $N-k$ strategies for all $k = 2,\dots,5$. The differences in the average economic

TABLE I
CRITICAL BUSES IDENTIFIED BY DIFFERENT ANALYSIS METHODS

Priority	Bus ID				
	Algorithm 1	N – 2	N – 3	N – 4	N – 5
1	142	142	142	142	142
2	186	·	·	·	·
3	185	·	·	185	185
4	184	·	·	·	·
5	145	·	·	·	·
6	149	·	·	·	·
7	183	·	·	·	·
8	139	·	·	·	·
9	15	15	15	15	15
10	14	·	·	·	·
> 30	107	·	107	107	·
	138	·	·	·	138
	152	·	·	·	152

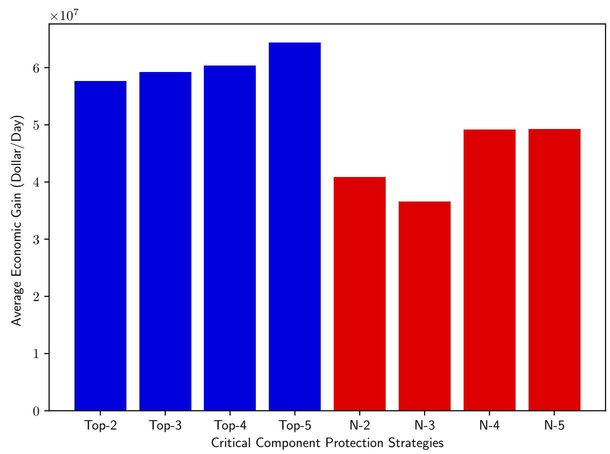


Fig. 7. Average economic gain obtained by different protection strategies for critical components.

gain range from \$11M (Top-4 vs. N-4) to \$22M (Top-3 vs. $N-3$) per day. Moreover, as we protect more components, the average economic gain by using the Top-k strategies increases, whereas the gain by using the $N-k$ strategies does not consistently increase. These results suggest that our prioritization approach provides larger and more consistent returns on investment for hardening and protecting the system that the $N-k$ approach does.

D. Sensitivity Analysis of Seasonal Net Load

We perform a sensitivity analysis identifying and prioritizing the critical buses for the eight day types (see Figure 3). Figure 8 is a heat map that shows the amount of dispatch cost increased by disrupting each bus for each day type. The critical buses identified in Figure 8 are ordered by the maximum dispatch cost increments over the day types. The dispatch cost is increased the most in summer and the least in winter, implying that the system is more vulnerable in summer than winter.

Most of the critical buses are ordered consistently for the different day types. However, bus 139 was identified as being critical for SummerWD, SummerWE, and FallWD only. In particular, the bus is the second most critical bus of the system. Disruption at bus 139 significantly increases the dispatch cost

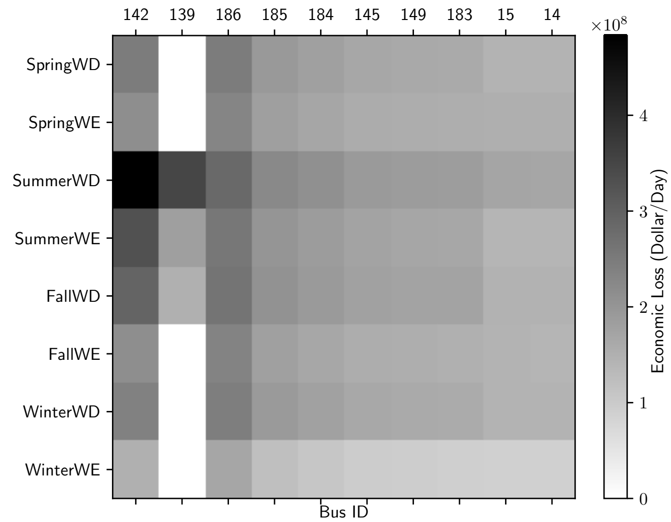


Fig. 8. Heat map representing the amount of dispatch cost increased by disrupting each bus (shown in x-axis) for each day type (shown in y-axis)

of the system, whereas the disruption is not critical for the other day types. These results suggest that a large number of scenario analyses for demand profiles are necessary in order to identify *hidden critical buses*. For example, bus 139 is a large source of power supply, which is connected to 47 generating units and 1 renewable source. Therefore, disruption at bus 139 results in significant damage when high demands are observed in summer and fall.

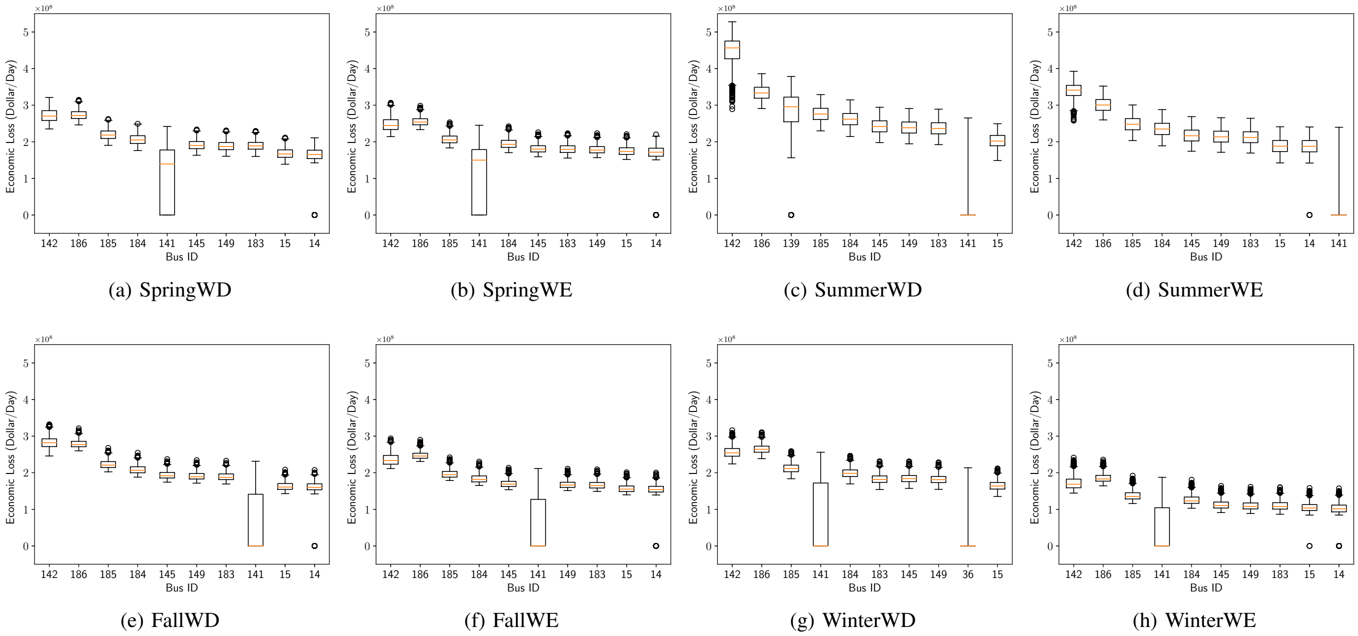
E. Sensitivity Analysis of Uncertain Wind Generation

In this section we present the impact of uncertain wind generation on the priority of critical buses. In this analysis, we identify and prioritize 10 critical buses for each day type with a thousand wind generation scenarios (see Figure 4). The maximum of the dispatch cost increase over the 1,000 scenarios is used as a prioritizing criterion of the critical buses. Results are reported as the box plots in Figure 9.

Most critical buses are consistently identified for every day type: buses 142, 186, 185, 184, 141, 145, 149, 183, and 15. Of the buses, however, we found that bus 141 resulted in variances and errors of the dispatch cost increase. Also bus 141 was not identified as one of the 10 most critical buses without wind generation (Figure 8). This result implies that bus 141 is not a critical bus for most days, but its failure can be significant to the system for a few days, which can be revealed only by extensive scenario analysis.

V. SUMMARY AND FUTURE WORK

We presented an optimization-based approach to identify and prioritize the critical components of an electric grid system. Identifying the critical buses is not a trivial task because any bus can be a critical bus even without any load or generating unit. Our approach and traditional $N-k$ analysis share the aim of identifying critical components. In our computational study on the WECC test system, however, we show that the prioritization method is necessary and suitable for planning



(a) SpringWD (b) SpringWE (c) SummerWD (d) SummerWE

(e) FallWD (f) FallWE (g) WinterWD (h) WinterWE

Fig. 9. Dispatch cost increased by each bus disruption for 1,000 wind generation scenarios for each day type

the system protection and mitigating disruptions. Moreover, a scenario analysis was performed by using thousands of wind generation scenarios and load profiles. The scenario analysis found that some critical buses can be hidden and not be identified if only a few scenarios are evaluated.

Our approach relies on algorithms for solving the bilevel optimization problem (1). The Benders method is scalable but limited to a heuristic approach, whereas the big-M approach is extremely inefficient. In future work, we will develop scalable exact methods for solving the bilevel optimization problem. Moreover, in order to perform more robust analysis, the inner minimization problem (3) is necessary to address physical laws with a number of nonlinear constraints. As a result, new solution approaches need to be developed for the nonlinear bilevel optimization problem. Stochastic variants of such bilevel problems are also necessary in order to address a number of scenarios in the analysis.

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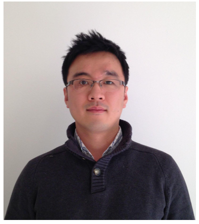

Kibaek Kim is an assistant computational mathematician in the Mathematics and Computer Science Division at Argonne National Laboratory. He holds Ph.D. and M.S. degrees from Northwestern University and a B.S. degree from Inha University in Korea, all in industrial engineering. He currently serves as a reviewer for several peer-reviewed journals, including Operations Research, Mathematical Programming, Computational Optimization and Applications, European Journal of Operational Research, and IEEE Transactions on Power Systems. His research interests are in modeling and parallel algorithms for large-scale optimization problems in applications to network design, planning, and operations.

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