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A Game-Theoretical Dynamic Model for Electricity Markets

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***Abstract*—We present a game-theoretical dynamic model for competitive electricity markets. We demonstrate that the model can be used to systematically analyze the effects of ramp constraints, initial conditions, dynamic disturbances, forecast horizon, bidding frequency, and some other factors on the price signals. We illustrate the capabilities of the model using a numerical case study.**

***Index Terms*—dynamics, markets, bidding, ramps, forecast horizon, frequency, clearing**

NOMENCLATURE

Sets

$\mathcal{S}$	Set of suppliers
$\mathcal{C}$	Set of consumers
$\mathcal{T}$	Set of time steps

Variables

q_t^i	Production quantities (MW)
Δq_t^i	Incremental production quantities (MW)
p_t	Multiplier of Nash-Cournot ISO (price) ($/MW)
$\hat{p}_t$	Multiplier of supply function ISO ($/MW)
λ_t^i	Adjoints for dynamic system ($/MW)
$\underline{\pi}_t^i, \overline{\pi}_t^i$	Multipliers production ($/MW)
$\underline{\nu}_t^i, \overline{\nu}_t^i$	Multipliers ramps ($/MW)
γ_t^i	Multipliers supply function ($/MW)
κ_t^i, θ_t^i	Multipliers supply function parameters ($/MW)

Constants

S	Number of suppliers
C	Number of consumers
T	Number of time steps in horizon
d_t^j	Demand (MW)
a_t^i, b_t^i	Coefficients supply function (MW,MW/$)
h^i, g^i	Coefficients cost function ($/MW,$/MW^2)
$\underline{r}^i, \overline{r}^i$	Bounds ramps ($/MW)
$\underline{q}^i, \overline{q}^i$	Bounds production quantities ($/MW)
$\overline{q}_0^i$	Initial conditions suppliers (MW)

I. INTRODUCTION

Electricity market modeling has become an area of active research as a result of increasing levels of deregulation and restructuring. Diverse models have been proposed to analyze and predict the impact of different dynamic disturbances (e.g., weather, load, fuel prices, and wind supply), physical constraints (e.g., transmission congestion), and gaming behaviors (e.g., bidding strategies) on market efficiency and prices [30]. These models range from data-based time-series models [27], [11] to mechanistic models based on agent-based systems [9], [29] and game-theoretical formulations [7], [17].

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Game-theoretical models can be used to establish market properties in a systematic manner and thus provide more comprehensive predictive capabilities. Several models based on a range of market structure assumptions have been proposed, all of which are *static* in the sense that they assume some sort of steady-state behavior of the fundamental market drivers. These models can provide a reasonable representation of the market under stationarity or strong periodicity of dynamic disturbances. Consequently, they can be used to analyze long-term behavior and physical constraints such as transmission congestion in planning and market design exercises. However, static models are not capable of explaining the effect of other dynamic constraints and non-stationary behavior, which is the most common case in real-time operations. Consequently, their use in market monitoring and price forecasting is limited.

A widely used dynamic market model originally proposed in [1], [2] assumes that the players bid recursively in the direction that minimizes their marginal cost. Every bid can be interpreted as a steepest-descent step that converges to a steady-state equilibrium. While this model is useful for analyzing market stability properties, it is based on mathematical rather than mechanistic assumptions and thus has limited applicability. Recently, a dynamic market model based on predictive control concepts was proposed in [16], [15]. Here, supply functions and forecast horizon concepts are incorporated into the model. These provide a more natural representation of actual bidding procedures. This model has been used to analyze the effect of dynamic disturbances such as wind on prices under high penetration levels. A limitation of this framework, however, is that the dynamic model of the players is still based on the marginal-cost descent assumption.

The main observation motivating this work is that generator dynamics are a critical factor driving market behavior. In particular, ramping constraints restrict bidding procedures at subsequent time intervals (day-ahead and real-time markets) and thus affect short-term and long-term market stability and performance. In some sense, ramping constraints affect market performance much as transmission congestion does [13]. The effects of manipulation of ramp constraints on market behavior was studied in [23]. Ramp rates represent the maximum change that a generator can achieve in their power output level within a given time interval [32]. They implicitly represent the time that it will take the generator control system to move the power output level from the current level to the desired target. These ramp rates depend on multiple physical factors such as controller performance [3], [5], thermal stresses, and wall capacitances [28]. For instance, a given set-point power level signal sent by the independent system operator to the generator often cannot be reached by the control system because of a misspecification of ramp rates which can lead to the use of expensive reserves or load shedding [24]. Large generators such as those running on coal and nuclear fuel are operated at base load and are not ramped. Smaller generators and combined-cycle plants running on natural gas provide ramping capacity and are used to regulate real-time deviations of the loads from forecasts. The generation costs of these ramping units are much higher than those of base units due to higher prices of natural gas and capital costs. The effect of ramping units on electricity prices will become stronger in the presence of more volatile and dynamic environments, such as those expected under high wind-supply penetration and smart-grid programs.

We propose a game-theoretical dynamic model that uses a closer physical representation of generator dynamics and of bidding procedures based on supply functions. We demonstrate that the model can be used to systematically simulate the effects of ramping limits, initial conditions, intermittent supply, forecast horizons, bidding frequency, and some other factors on price stability. We provide numerical results under several operational scenarios in order to illustrate the consistency and analytical capabilities of the model. In addition, we discuss how the model can potentially be used for market monitoring and price forecasting.

The paper is structured as follows: In the following section we present the basic model, discuss underlying assumptions, and offer a solution strategy. In Section III we discuss closed-loop implementation details necessary to simulate day-ahead markets and evaluate market performance. In Section IV we present numerical results. The last section provides concluding remarks and directions for future work.

II. Game-Theoretical Dynamic Models

In this section, we present two basic dynamic game-theoretical models. The models are targeted to capture dynamic effects on market performance. Consequently, simplifications have been made to avoid unnecessary complexity. Potential extensions are discussed later on as part of future work.

A. Nash-Cournot Formulation

We consider a unilateral market model where the suppliers bid production quantities (power levels) to maximize their profit and a central entity such as the Independent System Operator (ISO) that clears the market by balancing supply and demand. The consumer demands are assumed to be fixed. Each supplier $i \in \mathcal{S} = \{1..S\}$ is assumed to solve the following problem:

$$
\begin{aligned}
\max_{q_t^i} \; & \sum_{t\in\mathcal{T}} \left(p_t q_t^i - c_t^i(q_t^i)\right) && \text{(1a)}\\
\text{s.t. } & q_{t+1}^i - q_t^i \le \overline{r}^i, \; t \in \mathcal{T}\setminus T && \text{(1b)}\\
& q_t^i - q_{t+1}^i \le \underline{r}^i, \; t \in \mathcal{T}\setminus T && \text{(1c)}\\
& \underline{q}^i \le q_t^i \le \overline{q}^i, \; t \in \mathcal{T} && \text{(1d)}\\
& q_0^i = \bar{q}_0^i, && \text{(1e)}
\end{aligned}
$$

where T is the forecast horizon and $\mathcal{T}$ is the set of time steps. The horizon T represents the bidding forecast of the duration of the length. Symbols $\underline{r}^i$ and $\overline{r}^i$ denote the down and up ramp rates. The bidding production quantities q_t^i are bounded by the down and up limits $\underline{q}_t^i$ and $\overline{q}_t^i$, respectively. The initial conditions for the production quantities are given by $\bar{q}_0^i$ and are fixed. These represent the current power output levels of the generators. The cost function, defined by $c_t^i(\cdot)$, is assumed to be any convex function (e.g., linear, quadratic, or piece-wise linear). Here, we consider quadratic costs of the form

$$
c_t^i(q_t^i) = h^i \cdot q_t^i + \frac{1}{2} g_t^i \cdot (q_t^i)^2. \tag{2}
$$

The price is defined by p_t and is given by by the market clearing condition

$$
\sum_{i\in\mathcal{S}} q_t^j = \sum_{j\in\mathcal{C}} d_t^j, \tag{3}
$$

where d_t^j are the consumer demands $j \in \mathcal{C} = \{1..C\}$. The market clearing condition amounts to assuming that the ISO minimizes the imbalance of supply and demand so the price can be seen as the Lagrange multiplier of the clearing condition. In other words, the ISO solves the problem

$$
\min_{p_t} \; -\sum_{t\in\mathcal{T}} p_t \left(\sum_{i\in\mathcal{S}} q_t^i - \sum_{j\in\mathcal{C}} d_t^j \right). \tag{4}
$$

We note that this game can be posed as a discrete-time dynamic game in state-space form

$$
\begin{aligned}
& \sum_{i\in\mathcal{S}} q_t^i = \sum_{j\in\mathcal{C}} d_t^j, \; t \in \mathcal{T} && \text{(5a)}\\
i \in \mathcal{S} \; \Bigg\{ & \max_{\Delta q_t^i} \; \sum_{t\in\mathcal{T}} \left(p_t q_t^i - c_t^i(q_t^i)\right) && \text{(5b)}\\
& \text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i, \; t \in \mathcal{T}\setminus T && \text{(5c)}\\
& \underline{r}^i \le \Delta q_t^i \le \overline{r}^i, \; t \in \mathcal{T}\setminus T && \text{(5d)}\\
& \underline{q}^i \le q_t^i \le \overline{q}^i, \; t \in \mathcal{T} && \text{(5e)}\\
& q_0^i = \bar{q}_0^i. && \text{(5f)}
\end{aligned}
$$

This representation is typical in the dynamic games literature [8]. Here, the production quantities q_t^i can be interpreted as differential *states* coupled in time, and the increments Δq_t^i can be interpreted as the *controls*. The price act as an algebraic *state* since it is not directly coupled in time. The initial conditions play a critical role in the performance of the overall system since they propagate the ramping constraints in time. The solution of this problem gives equilibrium *trajectories*, or a *dynamic equilibrium*, for the supply quantities that satisfy the demands at each point in time and the prices. Note that the trajectories depends on the initial conditions, forecast horizon, and on the ramp rates of the generators. In particular, depending on the ramp rates, the problem might be infeasible for certain initial conditions. If infeasibility occurs, the price will tend to infinity. We note that the supplier problem is parametric in p_t and is convex since the cost function is convex. The ISO's problem is convex as well for fixed quantities q_t^i.

B. Supply-Function Based Formulation

An alternative formulation that represents actual market operations more accurately is based on supply functions [26],

[7], [6]. Here, we assume that the supplier decisions are the parameters of the supply function

$$q_t^i(p_t) = b_t^i p_t - a_t^i. \tag{6}$$

If we assume that the bids and the market are cleared simultaneously, we get the following game

$$\sum_{i\in\mathcal{S}} \left(b_t^i p_t - a_t^i\right) = \sum_{j\in\mathcal{C}} d_t^j,\ t \in \mathcal{T} \tag{7a}$$

$$i \in \mathcal{S} \left\{ \begin{aligned} &\max_{a_t^i, b_t^i, \Delta q_t^i} \sum_{t\in\mathcal{T}} \left(p_t q_t^i - c_t^i(q_t^i)\right) && \text{(7b)}\\ &\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i,\ t \in \mathcal{T}\setminus T && \text{(7c)}\\ &q_t = b_t^i p_t - a_t^i,\ t\in\mathcal{T} && \text{(7d)}\\ &\underline{r}^i \le \Delta q_t^i \le \overline{r}^i,\ t \in \mathcal{T}\setminus T && \text{(7e)}\\ &\underline{q}^i \le q_t^i \le \overline{q}^i,\ t\in\mathcal{T} && \text{(7f)}\\ &a_t^i, b_t^i \ge 0,\ t\in\mathcal{T} && \text{(7g)}\\ &q_0^i = \bar{q}_0^i. && \text{(7h)} \end{aligned}\right.$$

The ISO's problem becomes

$$\min_{p_t,\hat{p}_t} - \sum_{t\in\mathcal{T}} \hat{p}_t \left(\sum_{i\in\mathcal{S}} (b_t^i p_t - a_t^i) - \sum_{j\in\mathcal{C}} d_t^j \right), \tag{8}$$

where $\hat{p}_t$ is an additional (dummy) multiplier. Here, the price p_t is given by the intercept between the supply and demand curves. We note that the supply function $q_t = b_t^i p_t - a_t^i$ is an affine transformation between the states. Thus we have that, for any p_t, there exists a pair $a_t^i, b_t^i \ge 0$ yielding the optimal q_t^i obtained from the Nash-Cournot formulation (which must be nonnegative as well). Consequently, both the Nash-Cournot and the supply-function based models are equivalent and will reach the same solution. The difference between the two is in the implementation. Note that the players problems are convex for fixed price p_t but the ISO's problem is non-convex. In the following section, however, we will see that the dummy multipliers $\hat{p}_t$ can be eliminated making the ISO's problem convex.

One of the main advantages of game-theoretical models is that they can be systematically extended to capture more detailed effects arising in power markets. In particular, the proposed models can be extended to capture transmission constraints, social welfare and transmission costs in the ISO's problems, and arbitrager decisions [22]. Note also that the models assume that bidding and market clearing occur simultaneously (single-shot game). A more realistic representation can be considered where the bids and market clearing occur sequentially [18]. This, however, gives rise to much more challenging models from both a theoretical and a computational point of view. In this work, we set this complexity aside and focus on dynamic effects.

C. Complementarity Formulations

To solve the dynamic game problems, we can formulate them as coupled complementarity systems [17]. For the Nash-Cournot game we define the Lagrange function for supplier i as follows:

$$\begin{aligned} \mathcal{L}^i = &\sum_{t\in\mathcal{T}} -\left(p_t q_t^i - c_t^i(q_t^i)\right) + \sum_{t\in\mathcal{T}\setminus T} \lambda_{t+1}^i \left(q_{t+1}^i - q_t^i - \Delta q_t^i\right)\\ &+ \lambda_0^i (q_0^i - \bar{q}_0^i)\\ &- \sum_{t\in\mathcal{T}\setminus T} \underline{\nu}_t^i \left(\Delta q_t^i - \underline{r}^i\right) - \sum_{t\in\mathcal{T}\setminus T} \overline{\nu}_t^i \left(\overline{r}^i - \Delta q_t^i\right)\\ &- \sum_{t\in\mathcal{T}} \underline{\pi}_t^i \left(q_t^i - \underline{q}^i\right) - \sum_{t\in\mathcal{T}} \overline{\pi}_t^i \left(\overline{q}^i - q_t^i\right),\ i \in \mathcal{S}. \end{aligned} \tag{9}$$

Here, λ_t^i are the Lagrange multipliers for the dynamic system (adjoints) and $\underline{\nu}_t^i, \overline{\nu}_t^i, \underline{\pi}_t^i$ and $\overline{\pi}_t^i$ are the bound multipliers. This gives the following mixed linear complementarity system:

$$\sum_{i\in\mathcal{S}} q_t^i - \sum_{j\in\mathcal{C}} d_t^j = 0,\ t\in\mathcal{T} \tag{10a}$$

$$\left.\begin{aligned} &\nabla_{q_t^i}\mathcal{L} = 0,\ t\in\mathcal{T}\\ &\nabla_{\Delta q_t^i}\mathcal{L} = 0,\ t\in\mathcal{T}\setminus T\\ &\lambda_{t+1}^i \perp q_{t+1}^i - \left(q_t^i + \Delta q_t^i\right) = 0,\ t\in\mathcal{T}\setminus T\\ &0 \le \underline{\nu}_t^i \perp \left(\Delta q_t^i - \underline{r}^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\ &0 \le \overline{\nu}_t^i \perp \left(\overline{r}^i - \Delta q_t^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\ &0 \le \underline{\pi}_t^i \perp \left(q_t^i - \underline{q}^i\right) \ge 0,\ t\in\mathcal{T}\\ &0 \le \overline{\pi}_t^i \perp \left(\overline{q}^i - q_t^i\right) \ge 0,\ t\in\mathcal{T}\\ &\lambda_0^i \perp \left(q_0^i - \bar{q}_0^i\right) = 0 \end{aligned}\right\} i\in\mathcal{S}, \tag{10b}$$

where

$$\nabla_{q_t^i}\mathcal{L}^i = -p_t + \frac{\partial c_t^i}{\partial q_t^i} + \lambda_t^i - \lambda_{t+1}^i - \underline{\pi}_t^i + \overline{\pi}_t^i = 0,\quad t\in\mathcal{T}\setminus T \tag{11a}$$

$$\nabla_{q_T^i}\mathcal{L}^i = -p_T + \frac{\partial c_T^i}{\partial q_T^i} + \lambda_T^i - \underline{\pi}_T^i + \overline{\pi}_T^i = 0 \tag{11b}$$

$$\nabla_{\Delta q_t^i}\mathcal{L}^i = -\lambda_{t+1}^i - \underline{\nu}_t^i + \overline{\nu}_t^i,\ t\in\mathcal{T}\setminus T, \tag{11c}$$

for $i \in \mathcal{S}$. Note that the optimality conditions for the ISO's problem are just the market clearing conditions (**??**). For the supply function formulation the Lagrange function for supplier i is given by

$$\begin{aligned} \mathcal{L}_s^i = &\sum_{t\in\mathcal{T}} -\left(p_t q_t^i - c_t^i(q_t^i)\right) + \sum_{t\in\mathcal{T}\setminus T} \lambda_{t+1}^i \left(q_{t+1}^i - q_t^i - \Delta q_t^i\right)\\ &+ \lambda_0^i (q_0^i - \bar{q}_0^i)\\ &+ \sum_{t\in\mathcal{T}} \gamma_t^i (q_t^i - b_t^i p_t + a_t^i)\\ &- \sum_{t\in\mathcal{T}\setminus T} \underline{\nu}_t^i \left(\Delta q_t^i - \underline{r}^i\right) - \sum_{t\in\mathcal{T}\setminus T} \overline{\nu}_t^i \left(\overline{r}^i - \Delta q_t^i\right)\\ &- \sum_{t\in\mathcal{T}} \underline{\pi}_t^i \left(q_t^i - \underline{q}^i\right) - \sum_{t\in\mathcal{T}} \overline{\pi}_t^i \left(\overline{q}^i - q_t^i\right)\\ &- \sum_{t\in\mathcal{T}} \kappa_t^i \cdot a_t^i - \sum_{t\in\mathcal{T}} \theta_t^i \cdot b_t^i,\ i\in\mathcal{S}. \end{aligned} \tag{12}$$

Here, κ_t^i and θ_t^i are bound multipliers for the supply function parameters, and γ_t^i are multipliers for the supply function constraint. This gives the following complementarity system:

$$\sum_{i\in\mathcal{S}}\left(b_t^i p_t - a_t^i\right) = \sum_{j\in\mathcal{C}} d_t^j,\ t\in\mathcal{T} \tag{13a}$$

$$\left.\begin{array}{l}
\nabla_{q_t^i}\mathcal{L}_s = 0,\ t\in\mathcal{T}\\
\nabla_{\Delta q_t^i}\mathcal{L}_s = 0,\ t\in\mathcal{T}\setminus T\\
\nabla_{a_t^i}\mathcal{L}_s = 0,\ t\in\mathcal{T}\\
\nabla_{b_t^i}\mathcal{L}_s = 0,\ t\in\mathcal{T}\\
\lambda_{t+1}^i \perp q_{t+1}^i - \left(q_t^i + \Delta q_t^i\right) = 0,\ t\in\mathcal{T}\setminus T\\
0 \le \underline{\nu}_t^i \perp \left(\Delta q_t^i - \underline{r}^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\
0 \le \overline{\nu}_t^i \perp \left(\overline{r}^i - \Delta q_t^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\
0 \le \underline{\pi}_t^i \perp \left(q_t^i - \underline{q}^i\right) \ge 0,\ t\in\mathcal{T}\\
0 \le \overline{\pi}_t^i \perp \left(\overline{q}^i - q_t^i\right) \ge 0,\ t\in\mathcal{T}\\
\gamma_t^i \perp q_t^i - b_t^i p_t + a_t^i = 0,\ t\in\mathcal{T}\\
0 \le \kappa_t^i \perp a_t^i \ge 0,\ t\in\mathcal{T}\\
0 \le \theta_t^i \perp b_t^i \ge 0,\ t\in\mathcal{T}\\
\lambda_0^i \perp \left(q_0^i - \bar{q}_0^i\right) = 0
\end{array}\right\}\ i\in\mathcal{S}, \tag{13b}$$

where $\nabla_{q_t^i}\mathcal{L}_s$ and $\nabla_{\Delta q_t^i}\mathcal{L}_s$ are given by

$$\nabla_{q_t^i}\mathcal{L}_s^i = -p_t + \frac{\partial c_t^i}{\partial q_t^i} + \lambda_t^i - \lambda_{t+1}^i + \gamma_t^i - \underline{\pi}_t^i + \overline{\pi}_t^i = 0, \quad t\in\mathcal{T}\setminus T \tag{14a}$$

$$\nabla_{q_T^i}\mathcal{L}_s^i = -p_T + \frac{\partial c_T^i}{\partial q_T^i} + \lambda_T^i + \gamma_T^i - \underline{\pi}_T^i + \overline{\pi}_T^i = 0 \tag{14b}$$

$$\nabla_{\Delta q_t^i}\mathcal{L}^i = -\lambda_{t+1}^i - \underline{\nu}_t^i + \overline{\nu}_t^i,\ t\in\mathcal{T}\setminus T, \tag{14c}$$

and,

$$\nabla_{a_t^i}\mathcal{L}_s^i = \gamma_t^i - \kappa_t^i = 0,\ t\in\mathcal{T} \tag{15a}$$

$$\nabla_{b_t^i}\mathcal{L}_s^i = -\gamma_t^i \cdot p_t - \theta_t^i = 0,\ t\in\mathcal{T}, \tag{15b}$$

for $i\in\mathcal{S}$. Note that the optimality conditions for the ISO's problem are the market clearing conditions and the extra condition

$$\hat{p}_t \sum_{i\in\mathcal{S}} b_t^i = 0,\ t\in\mathcal{T}, \tag{16}$$

which implies that $\hat{p}_t = 0$, since b_t^i are fixed in the ISO's problem. Consequently, the dummy multipliers can be eliminated from the ISO's problem. We also note that the supply function-based formulation leads to the presence of bilinear terms in the complementarity system, equation (15b). This makes the problem computationally more difficult.

D. Solution Strategy

The resulting complementarity systems can be extremely large, depending on the length of the prediction horizon and the number of players. To solve these systems, one can use complementarity solvers such as PATH [12] or general nonlinear optimization solvers such as KNITRO [10] and IPOPT [31]. Nonlinear optimization solvers scale well with the problem size, which is important since dynamic games might have long horizons and short time steps thus leading to a large number of variables and constraints. To solve the complementarity systems (10) and (13) we apply a the nonlinear optimization approach using a ℓ_1 penalty formulation. Here, the objective is to minimize the complementarity products [21], [14], [4]. For (10), the problem takes the form:

$$\begin{aligned}\min\quad & \rho\sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}\setminus T}\left(\underline{\nu}_t^i\left(\Delta q_t^i - \underline{r}^i\right) + \overline{\nu}_t^i\left(\overline{r}^i - \Delta q_t^i\right)\right)\\ & + \rho\sum_{i\in\mathcal{S}}\sum_{t\in\mathcal{T}}\left(\underline{\pi}_t^i\left(q_t^i - \underline{q}^i\right) + \overline{\pi}_t^i\left(\overline{q}^i - q_t^i\right)\right)\end{aligned} \tag{17a}$$

$$\text{s.t.}\ \sum_{i\in\mathcal{S}} q_t^i - \sum_{j\in\mathcal{C}} d_t^j = 0,\ t\in\mathcal{T} \tag{17b}$$

$$\left.\begin{array}{l}
\nabla_{q_t^i}\mathcal{L} = 0,\ t\in\mathcal{T}\\
\nabla_{\Delta q_t^i}\mathcal{L} = 0,\ t\in\mathcal{T}\setminus T\\
q_{t+1}^i - \left(q_t^i + \Delta q_t^i\right) = 0,\ t\in\mathcal{T}\setminus T\\
0 \le \underline{\nu}_t^i,\ \left(\Delta q_t^i - \underline{r}^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\
0 \le \overline{\nu}_t^i,\ \left(\overline{r}^i - \Delta q_t^i\right) \ge 0,\ t\in\mathcal{T}\setminus T\\
0 \le \underline{\pi}_t^i,\ \left(q_t^i - \underline{q}^i\right) \ge 0,\ t\in\mathcal{T}\\
0 \le \overline{\pi}_t^i,\ \left(\overline{q}^i - q_t^i\right) \ge 0,\ t\in\mathcal{T}\\
\left(q_0^i - \bar{q}_0^i\right) = 0
\end{array}\right\}\ i\in\mathcal{S}, \tag{17c}$$

where $\rho > 0$ is a penalty parameter that is decreased sequentially.

III. Closed-Loop Considerations

To analyze real-time market operations, one can solve the dynamic game problem in a closed-loop manner with a predefined forecast horizon T. This can be used to account for changing conditions of dynamic disturbances such as weather, demands, model errors, and fuel prices.

The infinite horizon bidding game ($T=\infty$) gives the *optimal* equilibrium trajectory for a given disturbance forecast. For implementation, however, a moving horizon bidding strategy is used in practice. The idea is to define a finite horizon T to compute an equilibrium trajectory and to carry out the state to the next window. Typically, the day-ahead market is cleared with a prediction or forecast horizon of 24 to 36 hours. The idea is to start at a given bidding time t_ℓ with initial conditions $q_{t_\ell}^i$ and to use these as initial conditions $\bar{q}_0^i = q_\ell^i$ to compute the dynamic equilibrium trajectory over horizon $\mathcal{T} = \{t_\ell, ..., t_\ell + T\}$. The initial conditions are then updated to the generator states at the end of the horizon $\bar{q}_0^i = q_{t_\ell+T}^i$. In other words, the horizon is shifted forward by T steps so that the new bidding horizon is $\mathcal{T} = \{t_\ell + T, ..., t_\ell + 2T\}$. This is illustrated in Figure 1.

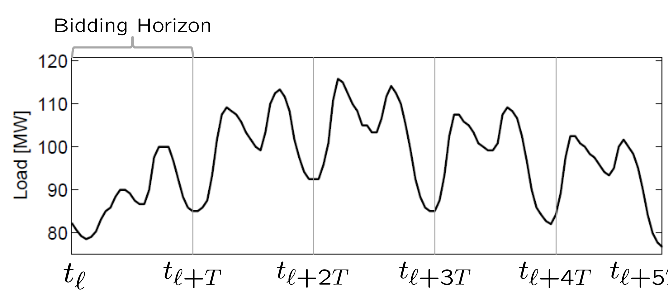

Fig. 1. Schematic representation of moving horizon implementation.

The moving horizon policy provides an *approximation* to the infinite horizon policy. One of the interesting questions that arise in this context is how long the horizon should be?. This is often problem dependent since it depends on the structure of the infinite horizon equilibrium policy. We also note that, in the absence of ramp constraints, the moving and infinite horizon policies coincide since the states q_t^i are no longer coupled in time.

We assume a deterministic setting using perfect forecast in the day-ahead market. With this, it is not necessary to model the real-time market, which takes care of high-frequency load imbalances resulting from day-ahead forecast errors. To model the real-time market, we would also need to model contractual agreements, which would lead to a more complex formulation.

IV. Numerical Results

In this section, we report on numerical simulations that we conducted under several operational scenarios. Our objective is to illustrate the effect of dynamic constraints on the price dynamics and to demonstrate the consistency of the model. In addition, we discuss the limitations of moving horizon bidding in the presence of strong disturbances such load and wind supply changes.

We consider a simple system with three suppliers and one demand. One of the suppliers has fast dynamics (high ramping capacity) but high cost such as natural gas generators, the second one has slow dynamics but also low cost, and the third one is used as a slack generator with infinite ramp limits (equal to generation capacity) and a large cost. This last supplier acts as a slack to avoid infeasibility. The nominal parameters used are $\underline{q} = [0, 0, 0]$, $\overline{q} = [50, 70, 120]$, $\underline{r} = -[5, 10, 120]$, $\overline{r} = [5, 10, 120]$, $h = [4, 2, 5]$, and $g = [2, 1, 5]$. The ramps were varied in some experiments from their nominal values. We used $\bar{q}_0 = [0, 40, 40]$ as initial conditions.

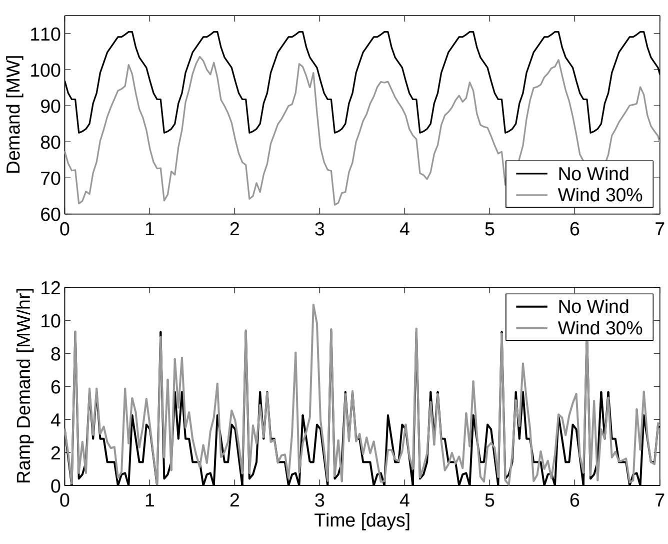

Fig. 2. Total (top) and ramp (bottom) demand for market system over 14 days of operation with 0% and 30% wind penetration.

We consider the demand profiles for two cases, one with periodic demand (labeled "No Wind") and one where the periodic demand is shifted by a wind power profile representing 30% of wind penetration (labeled as "30% Wind"). In the top graph of Figure 2 we present total demands for the two cases. The periodic demand fluctuates between 80 MW and 110 MW. In the wind case, the net demand is shifted down (demand minus wind supply) but becomes more volatile. In particular, periodicity is partially destroyed.

In the bottom graph we present the ramp demands for the two demand cases. These have been obtained by computing the absolute differences $|d_{t+1} - d_t|,\ t \geq 0$. Note that even if the net demand is lower in the wind case, the ramp demands can increase significantly at particular points in time. This situation can be observed clearly at the beginning of the third day. This illustrates how ramping constraints can become more significant under more volatile environments.

A. Effect of Ramp Rates

We first analyze the effect of ramping constraints. For this analysis, we consider the case with periodic demand. In Figure 3 we present the dynamic equilibria for three ramp scenarios. The first scenario, "High Ramp", corresponds to the nominal ramp values, scenario "Low Ramp" corresponds to a 50% decrease in the nominal values of the suppliers, and "Infinite Ramp" corresponds to an unconstrained ramp case (ramps set to large value). Note that in the constrained cases the price signals reach a periodic steady-state after a couple of days. In the absence of ramp constraints, the periodic steady-state is reached immediately. Note also that the shape of the steady-state equilibrium is affected by the ramp rates. In particular, the prices are more volatile during high and low peaks when the ramps are lower. During peak hours, the prices reach values as high as 120$/MW during the first day and 90$/MW at steady-state. In the unconstrained case, the peak prices are 80$/MW. In Figure 4 we can observe that the bidding quantities saturate

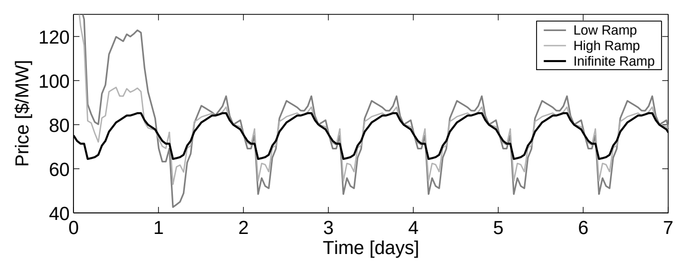

Fig. 3. Effect of ramp limits on price dynamics.

while reaching the periodic steady-state (dashed lines). In Figure 5 we plot the adjoint variables λ_t^i for the three suppliers. These multipliers reflect the sensitivity of the profit for each supplier to changes in the ramp rates. Note that the multipliers reach a steady-state after seven days and that the greater sensitivity is observed during peaking times, as expected. Supplier 2 is clearly the most sensitive since it has more limited ramping capacity. The adjoint of the slack supplier is zero since the ramps are never active. We have observed that the adjoints tend to diverge for extremely long horizons, introducing numerical problems in the solution. Divergence is attributed mainly to the lack of a terminal constraint in the suppliers problems. We have found that penalizing the profit in the last time step T by a large value stabilizes the adjoints.

B. Effect of Dynamic Disturbances

In Figure 6 we present the effect of ramps for the case of 30% wind power penetration. It is clear that the volatility of the prices increases significantly. During peak hours, the prices

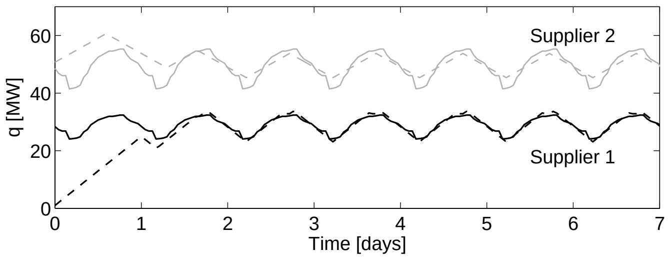

Fig. 4. Effect of ramp limits on bidding dynamics. Solid lines are profiles without ramp limits, and dashed lines are profiles with infinite ramp limits.

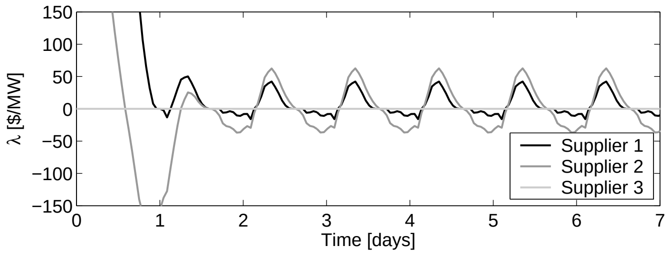

Fig. 5. Adjoint dynamics for suppliers in the presence of ramp limits.

reach values as high as 150$/MW during the first stabilization day and 140$/MW at steady-state. In the unconstrained case, the prices go down to 60 $/MW. This illustrates that, while the wind supply cost is very low, the increasing ramp demands can raise prices significantly because of the need of additional ramping capacity (natural gas).

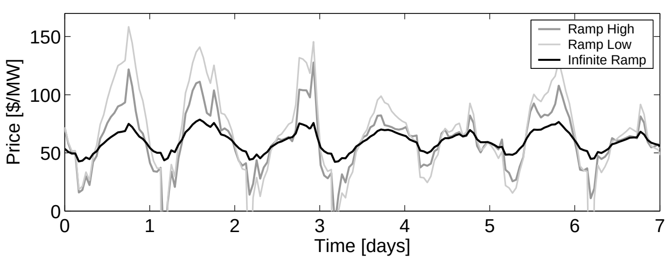

Fig. 6. Effect of ramp limits on bidding dynamics under 30% wind penetration case.

C. *Effect of Forecast Horizon*

In Figure 7 we illustrate the effect of the forecast horizon on the price signals. We compare the infinite horizon case and the one day-ahead forecast. Note that suboptimality is introduced during periods of strong dynamic variations. In the third day, the prices of the day-ahead case reach 120$/MW while the optimal ones are around 100$/MW. We have found that increasing the horizon to two days approximates well the infinite horizon policy. We have also found that, in the case of a perfectly periodic demand, the one day and infinite horizon policies are the same. A critical conclusion from this dynamic analysis is that *short horizons only work well under strong stationarity (periodicity) of the load*. In the presence of strong dynamic disturbances (wind supply and weather fronts), the horizon should be increased in order to keep prices more stable.

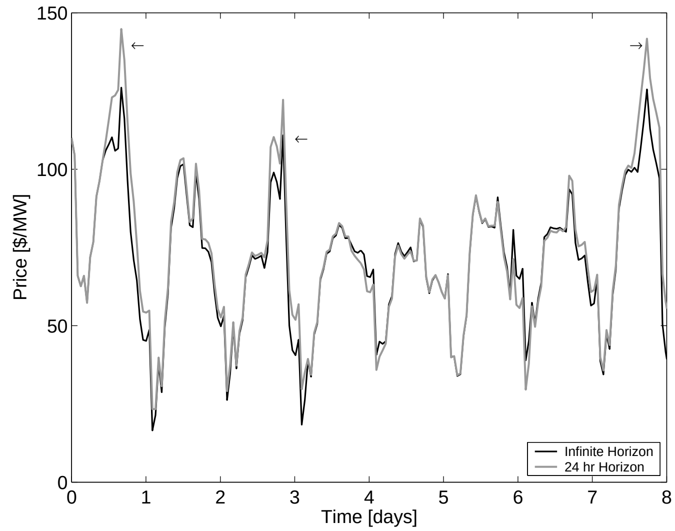

Fig. 7. Effect of horizon length on market performance under 30% wind penetration case.

D. *Game-Theoretic and Random Bids*

One of the main applications of game theoretical models is that they can be used to identify non-gaming behavior (e.g., market manipulation). In Figure 8, we present the price signals for a perfect game and for the case in which supplier 1 bids randomly (not trying to maximize its profit). From the profiles, it is clear that prices tend to become higher and more volatile in the presence of non-gaming behavior. With a market model, the degree of suboptimality (noise) in a player bid can be identified by solving an inverse problem. This capability can be used to monitor the market in real-time and forecast price signals. Another potential use is the design of optimal bidding strategies for the suppliers.

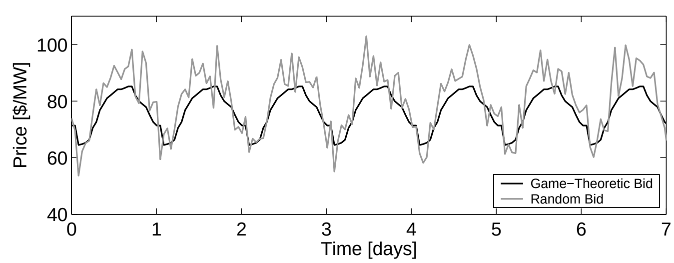

Fig. 8. Price signals under game-theoretic and random bids for supplier 1.

V. Conclusions and Future Work

We have presented a game-theoretical dynamic model for electricity markets. The model uses a dynamic game framework and optimal control concepts. In particular, we propose the incorporation of ramp constraints in the bidding formulation. This enables a more systematic analysis of the effects of dynamic constraints on market performance and price stability. This also enables the analysis of the effect of initial conditions and forecast horizons. We have presented numerical experiments to illustrate the consistency and analytical capabilities of the model.

The proposed model can be extended in a number of ways to consider more detailed physical effects and market design structures such as transmission constraints and coupled day-ahead and real-time markets (two-settlement markets) [33], [19]. In addition, it is possible to incorporate effects of uncertainty and risk aversion [20]. The model can also be constructed with more realistic set-ups where the suppliers bid their operational information (ramp and production limits, cost curves) [23] and the ISO clears the market by solving a unit commitment problem in the day-ahead market and an economic dispatch model in the real-time market [25], [34]. Other settings include information exchange, cooperation, and use of forecasting capabilities by the suppliers. These models, however, pose significant complexity from a theoretical and numerical point of view.

Acknowledgments

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