Fast Power Flow Analysis using a Hybrid Current-Power Balance Formulation in Rectangular Coordinates

Shrirang Abhyankar
Mathematics and Computer Science Division
Argonne National Laboratory
Argonne, IL
Email: abhyshr@mcs.anl.gov

Qiushi Cui
Department of Electrical and Computer Engineering
McGill University
Montreal, QC
Email: qiushi.cui@mail.mcgill.ca

Alexander J. Flueck
Department of Electrical and Computer Engineering
Illinois Institute of Technology
Chicago, IL
Email: flueck@iit.edu

***Abstract*—Power flow is the backbone of steady-state analysis of power systems. Various steady-state applications such as contingency analysis, transfer limit calculations, and initialization of transient stability simulation, etc., hinge on the solution of power flow analysis. Hence, any performance improvement in power flow will improve various steady-state applications. In this paper we present a fast power flow analysis using a hybrid current-power balance formulation with the variables expressed in rectangular form. The computational efficiency and robustness of the proposed algorithm is presented for several test systems ranging from 100 to 3000 buses.**

***Index Terms*—Power Flow, Load Flow, Current Balance, Power Balance**

ACRONYMS

PB Power Balance Form
HCPB Proposed Hybrid Current-Power Balance Form

I. INTRODUCTION

Power flow analysis, sometimes referred to as load flow analysis, is the linchpin of steady-state power systems analysis. Several power system applications ranging from planning to operation, and from economic scheduling to exchange of power between utilities, require the solution of power flow equations. Moreover, power flow analysis serves as a starting step for transient stability by providing an initial operating point. The power flow problem entails calculating complex bus voltages and line flows in a large sparse electrical network, for a given load and generation schedule. Mathematically, the problem is formulated as a set of nonlinear equations and solved by using an iterative scheme such as Newton's method.

Power flow formulation first appeared in the late 1960s [12]. In the early 1970s, a fast decoupled technique was introduced [10] based on the physical insight of weak coupling between real power-voltage magnitude (PV) and reactive power-voltage angles (Qθ). Since then several variations of power flow formulations and techniques have been introduced [2], [9], [8], [6]. Luo and Semylen [8] introduced active and reactive powers as flow variables rather than complex currents, thus simplifying the treatment of PV buses and reducing the related computational effort to half. Exposito and Ramos [7] presented a power flow solution using an augmented system with rectangular coordinates. In this augmented system, the bus current injections are introduced as additional variables. A comparison of load flow with optimal multipliers with rectangular and polar coordinates was given in [11]. DaCosta and Rosa [5] compared the convergence of polar, rectangular and current injection Newton-Raphson formulations on well-behaved and ill-conditioned systems. They observed that for the ill-conditioned test system the polar formulation may fail to converge but the rectangular and current injection approach converged for all tested cases.

This paper introduces a hybrid current-power balance formulation where each PQ bus is described by nodal current balance equations, while a real power balance equation and a voltage magnitude constraint equation are given for each PV bus. Moreover, we use rectangular coordinates (real, imaginary) for the variables instead of polar (magnitude, angle). This hybrid formulation results in an efficient evaluation of the Jacobian matrix, one of the computational bottlenecks of the power flow algorithm, and thereby provides significant time saving as seen in Sections III and V.

II. POWER BALANCE FORMULATION IN POLAR COORDINATES (PB)

The power balance formulation is the most widely used formulation for power flow analysis. Most commercial packages use a power balance formulation for the solution of load flow equations [11]. In the power balance form (PB), the set of nonlinear equations to be solved is described by complex power balances at each bus. In other words, the summation of the power injected at each bus and absorbed by the network must equate to zero. The resultant complex power balance equation for each bus, or network node, is given by (1).

$$\bar{S_i^{inj}} = \bar{V_i}(\sum_{k=1}^{n}(G_{ik} + jB_{ik})\bar{V_k})^* \tag{1}$$

In (1), $\bar{S_i^{inj}}$ denotes the complex power injection, that is, $\bar{S_i^{inj}} = \bar{S_{Gi}} - \bar{S_{Di}}$ where $\bar{S_{Gi}}$ is the complex power injected by generators and $\bar{S_{Di}}$ is the complex power absorbed at bus i. Decomposing (1) into real and imaginary parts, we get the real and reactive power balance equations as follows:

$$P_i^{inj} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}cos(\theta_{ik}) + B_{ik}sin(\theta_{ik})) \quad (2)$$

$$Q_i^{inj} = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}sin(\theta_{ik}) - B_{ik}cos(\theta_{ik})), \quad (3)$$

where $\theta_{ik} = \theta_i - \theta_k$. In the power balance form, the variables are expressed in polar coordinates; that is, the variables are the magnitudes and angles of the complex voltage $\bar{V}$ at the buses. This coordinate system is convenient for representating the power balance equations as compared with expressing the equations in rectangular coordinates. In the power balance form, PQ buses (constant load, uncontrolled voltage magnitude) are given by Equations (2) and (3) while PV buses (controlled voltage magnitude) are described only by Equation 2. For PV buses (voltage-controlled buses), it is assumed that generators incident on these buses can produce adequate reactive power to regulate the terminal voltage and hence (3) can be omitted. As a result of the assumption of controlled-voltage magnitude for the PV buses, the total number of equations to be solved in the power balance form is $2npq + npv$, where npq is the number of PQ buses and npv is the number of PV buses.

One of the computational bottlenecks of the power balance form is in the evaluation of the Jacobian matrix. From (2) and (3) one can see that the real and reactive power balance equations are nonlinear functions of voltage magnitudes and angles of every bus k connected to bus i. This relation results in nonlinear terms in the off-diagonal elements of the Jacobian, along with the diagonal elements, as shown in the example Jacobian structure in Fig. 1. Notice that the entire Jacobian is nonlinear, and hence all the Jacobian elements need to be updated during each Newton iteration.

III. Proposed Hybrid Current-Power Balance Formulation Using Rectangular Coordinates (HCPB)

Instead of the power balance form as given in (1), one can use the current balance form, which is essentially the Kirchoff's current law at each bus. Dividing (1) by the complex bus voltage and taking the conjugate, one obtains the current balance equations.

$$(\frac{\bar{S_i^{inj}}}{\bar{V_i}})^* = \sum_{k=1}^{n} (G_{ik} + jB_{ik})\bar{V_k} \quad (4)$$

By using rectangular coordinates, namely, $V_i = e_i + jf_i$, (4) can be expressed in real and imaginary current balance

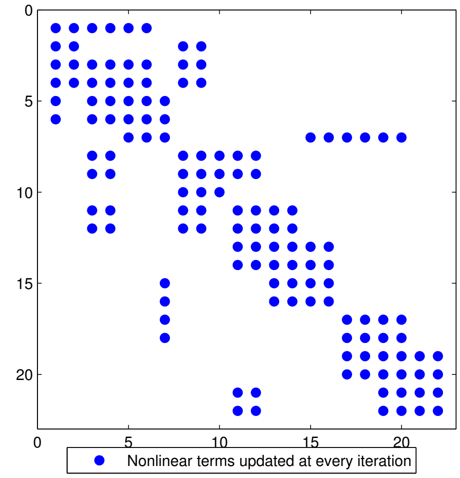

Fig. 1. Jacobian matrix structure for the IEEE 14-bus system using PB Formulation

equations as given in (5) and (6), respectively.

$$\frac{P_i^{inj}e_i + Q_i^{inj}f_i}{|V_i|^2} = \sum_{k=1}^{n} (G_{ik}e_k - B_{ik}f_k) \quad (5)$$

$$\frac{-Q_i^{inj}e_i + P_i^{inj}f_i}{|V_i|^2} = \sum_{k=1}^{n} (B_{ik}e_k + G_{ik}f_k) \quad (6)$$

Equations (5) and (6) can be used for PQ buses but not for PV buses because the reactive power injection from the generators is unknown. For a PV bus however, the real power injection is known and hence a power balance form can be used instead. Therefore, we use a real power balance equation for PV buses with the variables expressed in rectangular form. Modifying (2) by using rectangular coordinates instead of polar, we get the following

$$P_i^{inj} = e_i \sum_{k=1}^{n} (G_{ik}e_k - B_{ik}f_k) + f_i \sum_{k=1}^{n} (B_{ik}e_k + G_{ik}f_k) \quad (7)$$

Along with the real power balance equation given in (7), equation (8) enforces the fixed voltage magnitude constraint for PV buses.

$$e_i^2 + f_i^2 = |V_i^{sp}|^2 \quad (8)$$

Thus, our proposed approach for power flow analysis consists of the current balance equations (5), and (6) for PQ buses and power balance equation (7) and voltage magnitude constraint equation (8) for the PV buses, with the variables expressed in rectangular form. Since two equations are needed for each PV bus, $2n$ equations must be solved for the proposed HCPB formulation. Although the size of the system to be solved is greater than the power balance formulation, the structure of the Jacobian, as explained in the following paragraph, imparts the desired computational efficiency for HCPB. Extended details on the proposed formulation can be found in [4].

The proposed HCPB formulation results in the Jacobian structure shown in Fig. 2. The Jacobian rows corresponding to PQ bus equations (5) and (6) have constant off-diagonal terms. Thus, there is a decrease in the computation of Jacobian terms and this is especially attractive for load flow analysis of large systems.

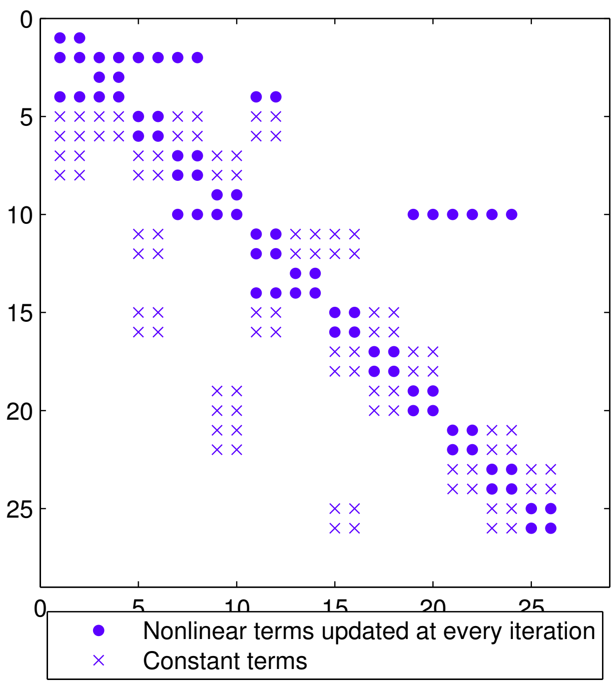

Fig. 2. Jacobian matrix structure for the IEEE 14-bus system using proposed HCPB formulation

IV. Implementation Details

The code for the power balance and proposed HCPB power flow simulators was written in C by using the high-performance library PETSc [3]. A brief introduction to the PETSc library for developing power system applications is given in [1]. A Newton's method with a full step line search scheme was used as the nonlinear solver. The linearized Newton equations at each Newton iteration are solved by using LU factorization with a quotient minimum degree reordering scheme to reduce the number of fill-ins in the factored matrices. All the test cases were run on a quad-core 2.3 GHz MacOS machine and compiled with a GNU compiler with -O3 optimization.

The variables and the equations can be ordered in several ways. In this work, the variables for each bus are grouped. Thus, for the PB formulation, the variables are ordered as $[\theta_i, |V_i|]$, while for HCPB formulation the ordering is $[e_i, f_i]$ for each bus. This ordering leads to an adjacency matrix form for the Jacobian with a 2X2 block structure, as shown in Figs. 1 and 2.

We note from the equations for the PQ buses in the HCPB formulation and the ordering of the variables that the diagonal terms in the Jacobian for buses with zero injection (no load or generator incident) would be G_{ii}. Since $G_{ii} << B_{ii}$, and for some cases $G_{ii} \equiv 0$, the Jacobian matrix factors can be ill-conditioned when not pivoting because of the presence of zeros on the diagonal. To avoid this ill-conditioning issue, the imaginary current balance equations are ordered first, followed by the real current balance equations for PQ buses in our implementation. This reordering of equations means that B_{ii}, which is nonzero, would be on the diagonal and hence would avoid having zeros on the diagonal for buses with zero injection.

V. Test Results

In this section we compare the PB and proposed HCPB formulations for several test cases. All the test cases are selected from the MatPower [13] package distribution (version 4.1). MatPower includes a variety of power flow test cases, with the smallest being a 4-bus network and the largest consisting of over 3000 buses. Since our goal is to test the efficiency and robustness of the proposed solver for large power systems, we select test cases with more than 100 buses. For all the test cases, the reactive power limits on the generators are not enforced. The inventory for the MatPower test cases used is given in Table I.

TABLE I
Inventory for test cases

MatPower casename	Buses	Gens	Lines
case118	118	54	186
case300	300	69	211
case2383wp	2383	327	2896
case2736sp	2736	420	3504
case2737sop	2737	399	3506
case2746wp	2746	520	3514
case2746wop	2746	514	3514
case3012wp	3012	502	3572
case3120sp	3120	505	3693

The results are presented for the following two initial guesses for the Newton method:

1) *Precomputed initial start*: This can be termed as a "good start" for the power flow since the initial guess, close to the solution, causes rapid convergence of the Newton method. The default initial guess in MatPower is a precomputed initial guess which is available in its test case data files. However, we note that such a "good start" may not be available in general.
2) *Flat initial start*: A typical starting method for the power flow is a "flat start", namely, initial voltage magnitudes set to 1.0 pu and initial angles set to 0.0. In our implementation, a slightly modified flat start is used where the PV bus initial voltage magnitudes are set to their set point values and all the angles are set equal to the angle of the reference bus.

Figures 3, 4, and 5 compare the PB and HCPB formulations, with a precomputed initial start for the load flow, in terms of

(a) the power flow execution time ratio ($t(PB)/t(HCPB)$) (b) the ratio for time taken per iteration, and (c) the number of iterations. The same comparison with a flat initial start for the load flow is shown in Figures 7, 8, and 9. Note that the execution times are measured for the Newton loop only and do not include any preprocessing steps such as data input, and admittance matrix assembly. The observations from these results can be summarized as follows:

1) *Precomputed initial start*: The proposed hybrid current-power balance formulation converges in almost the same number of iterations as does the power balance formulation. An aberration from this observation is seen for the test case *case3120sp*, where PB converges in 6 iterations while HCPB takes 12. A further analysis of the function residuals at each iteration showed that the voltage magnitude constraint equation for PV bus 2797 causes a slowdown in convergence. This slow convergence is due to the generator struggling to regulate its terminal voltage during several iterations. This slow convergence issue is also discussed in [11]. We plan to investigate this convergence slowdown in the future. The execution time for HCPB was lower than that of PB, with speedups of almost 2.5X for several test cases. For *case3120sp* the execution time with HCPB formulation is less than PB, even though HCPB takes twice the number of iterations, because the time per iteration for HCPB is significantly less than the PB.

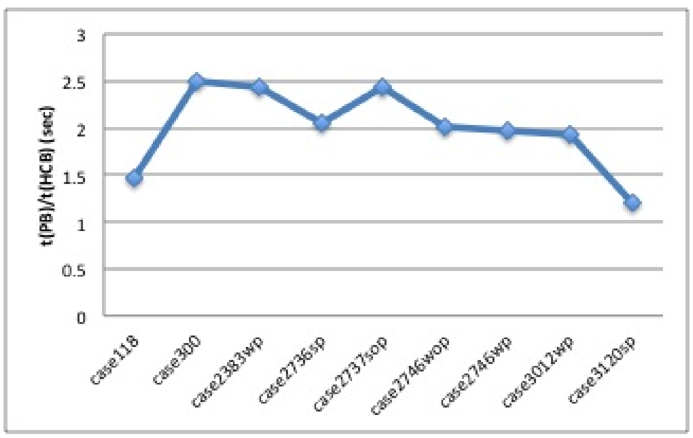

Fig. 3. Ratio of execution time, $t(PB)/t(HCPB)$, with precomputed initial start

2) *Flat initial start*: With a flat initial start the proposed HCPB formulation shows convergence characteristics and execution times similar to those seen with the precomputed initial start. The number of iterations for all the test cases was observed to be more for HCPB than PB, as seen in Fig. 9. Yet, the execution time of PB is greater because the average execution per iteration time for HCPB is significantly less. A maximum speedup of about 1.8X was measured for a couple of cases. We note that the comparison of number of iterations and execution times for *case2737sop* and *case3012wp* are not included because PB did not converge for these cases. On the other hand, the power flow converged with HCPB formulation in 6 and 15 iterations, respectively, for these cases.

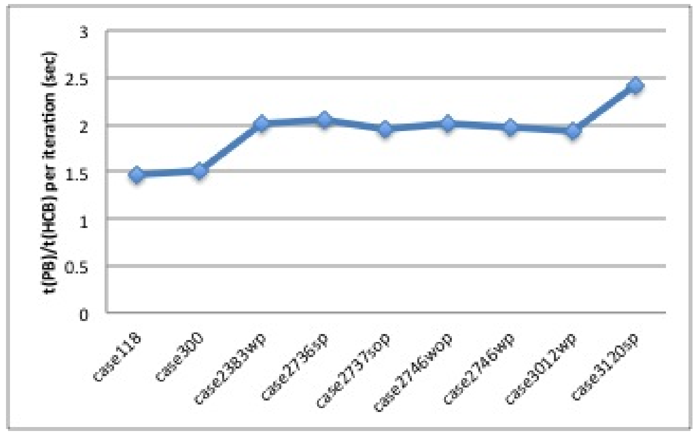

Fig. 4. Ratio of average execution time per iteration with precomputed initial start

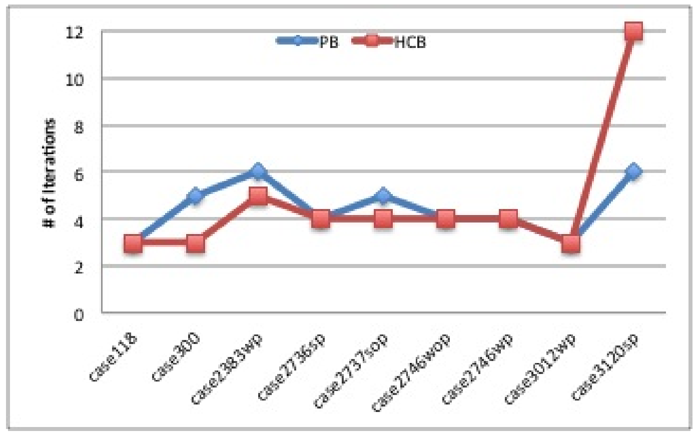

Fig. 5. Comparison of PB and HCPB iterations with precomputed initial start

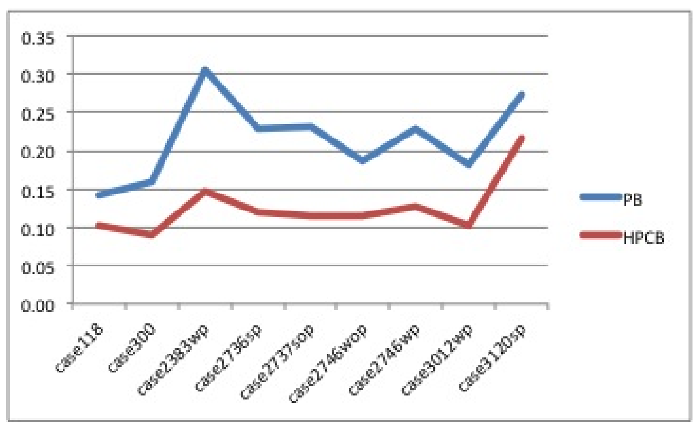

Fig. 6. Comparison of PB and HCPB relative Jacobian evaluation times as a fraction of the total execution time with precomputed initial start

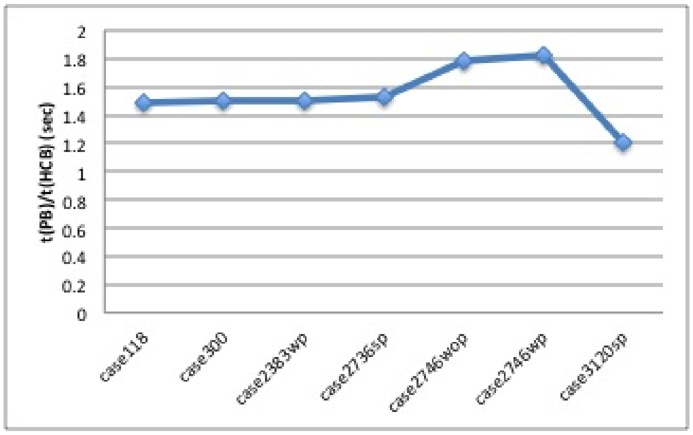

Fig. 7. Ratio of execution time, $t(PB)/t(HCPB)$, with flat initial start

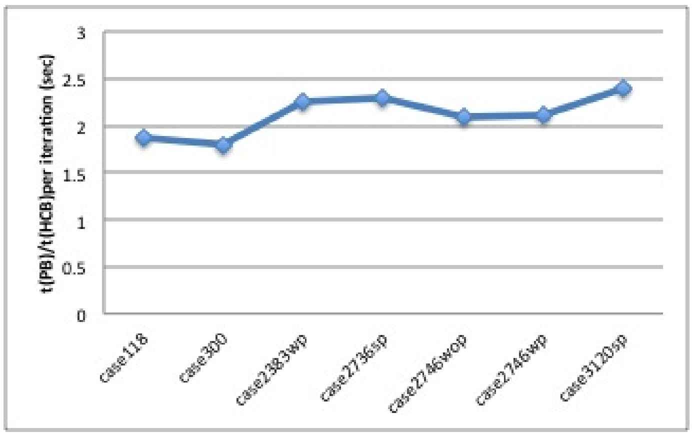

Fig. 8. Ratio of average execution time per iteration with flat initial start

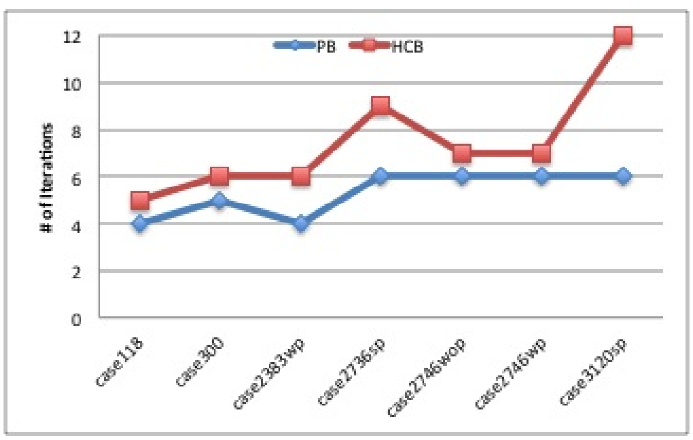

Fig. 9. Comparison of PB and HCPB iterations with flat initial start

VI. Conclusion

We have introduced a fast power flow analysis method using a hybrid current-power balance formulation in rectangular coordinates. The proposed formulation shows promising results for several large power system cases with a speedup of almost 2X as compared with the conventional power balance formulation. In the future we intend to analyze the convergence characteristics of this formulation for ill-conditioned systems and for parameterized load variations.

Acknowledgments

This work was supported by Korea Electrotechnology Research Institute (KERI) under and International Collaborative R&D project known as Local Area Monitoring System (LAMS) for micro grid.

This work was supported by the Office of Advanced Scientific Computing Research, Office of Science, U.S. Dept. of Energy, under Contract DE-AC02-06CH11357.

References

[1] S. Abhyankar, B. Smith, H. Zhang, and A. Flueck, "Using PETSc to develop scalable applications for next-generation power grid," in *Proceedings of the 1st International Workshop on High Performance Computing, Networking and Analytics for the Power Grid.* ACM, 2011. [Online]. Available: http://www.mcs.anl.gov/uploads/cels/papers/P1957-0911.pdf

[2] R. Bacher and W. F. Tinney, "Faster local power flow solutions: the zero mismatch approach," *IEEE Transactions on Power Systems*, vol. 4, no. 4, pp. 1345–1354, 1989.

[3] S. Balay, J. Brown, K. Buschelman, V. Eijkhout, W. D. Gropp, D. Kaushik, M. G. Knepley, L. C. McInnes, B. F. Smith, and H. Zhang, "PETSc Web page," http://www.mcs.anl.gov/petsc, 2011. [Online]. Available: http://www.mcs.anl.gov/petsc

[4] Q. Cui, "Rectangular hybrid current balance power flow and single phase induction motor model for dynamics simulation," Master's thesis, Illinois Institute of Technology, 2012.

[5] V. M. da Costa and A. L. S. Rosa, "A comparative analysis of different power flow methodologies," *IEEE Transactions on Power Systems*, no. 978-1-4244-2218-0, 2008.

[6] V. M. da Costa, N. Martins, and J. L. R. Pereira, "Developments in the newton raphson power flow formulation based on current injections," *IEEE Transactions on Power Systems*, vol. 14, no. 4, pp. 1320–1326, 1999.

[7] A. G. Exposito and E. R. Ramos, "Augmented rectangular load flow model," *IEEE Transactions on Power Systems*, vol. 17, no. 2, pp. 271–276, 2002.

[8] G. Luo and A. Semlyen, "Efficient load flow for large weakly meshed networks," *IEEE Transactions on Power Systems*, vol. 5, no. 4, pp. 1309–1316, 1990.

[9] A. Semlyen, "Fundamental concepts of a krylov subspace power flow methodology," *IEEE Transactions on Power Systems*, vol. 11, no. 3, pp. 1528–1537, 1996.

[10] B. Stott and O. Alsac, "Fast decoupled load flow," *IEEE Transactions on Power Systems*, vol. 17, no. no. 3: 859-869, 1974.

[11] J. Tate and T. Overbye, "A comparison of the optimal multiplier in polar and rectangular coordinates," *IEEE Transactions on Power Systems*, vol. 20, pp. 1667–1674, 2005.

[12] W. F. Tinney and C. E. Hart, "Power flow solution by newton's method," *IEEE Transactions on Power Systems*, vol. PAS-86: 1449-1456, 1967.

[13] R. Zimmerman and C. Murillo-Sanchez, "Matpower 4.0 users manual," Power Systems Engineering Research Center, Tech. Rep., 2011.