The dynamics of the high-frequency internal gravity wave field radiated by a stratified turbulent momentum wake

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Objectives

- Quantify the spectral characteristics (length and time scales) of the most energetic internal gravity waves emitted by the turbulent wake of a sphere, towed in a linearly stratified fluid, using implicit large eddy simulation at oceanographically relevant Reynolds and internal Froude numbers (Re= UD/ν=5,000 and 100,000, Fr= 2U/ND=4, 16, 64 where U, D, N, ν are the towing speed, sphere diameter, Brunt Vaisala frequency and the kinematic viscosity of the fluid, respectively).
- Estimate the 3D propagation angles (polar and azimuth), the phase and group velocities of the internal gravity waves.
- Estimate the amplitude of the waves both close to the source (edge of the turbulent wake) and in the far field; and assess their potential for nonlinear steepening and breaking.
- Estimate the flux of wave energy and determine its importance for the volume-averaged turbulent kinetic energy budget of the wake.

Problem set-up

- Bottom Wall/Top Free Surface (and Lateral/Vertical Absorbing Sponge Layers).
- Domain Dimensions: 26 ⅔D × 26 ⅔D × 12D

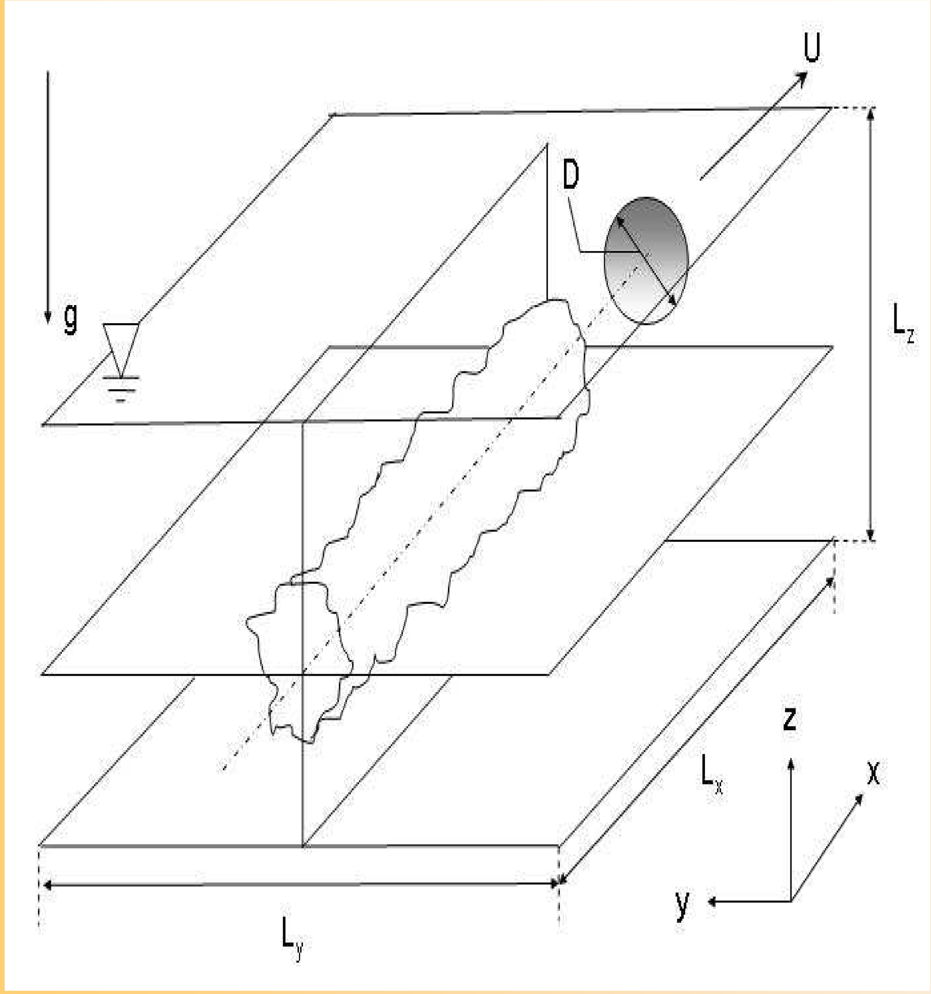

Fig.1 Problem setup: sphere is towed from left to right and the "observation window" is in a stationary reference frame in a "water tank".

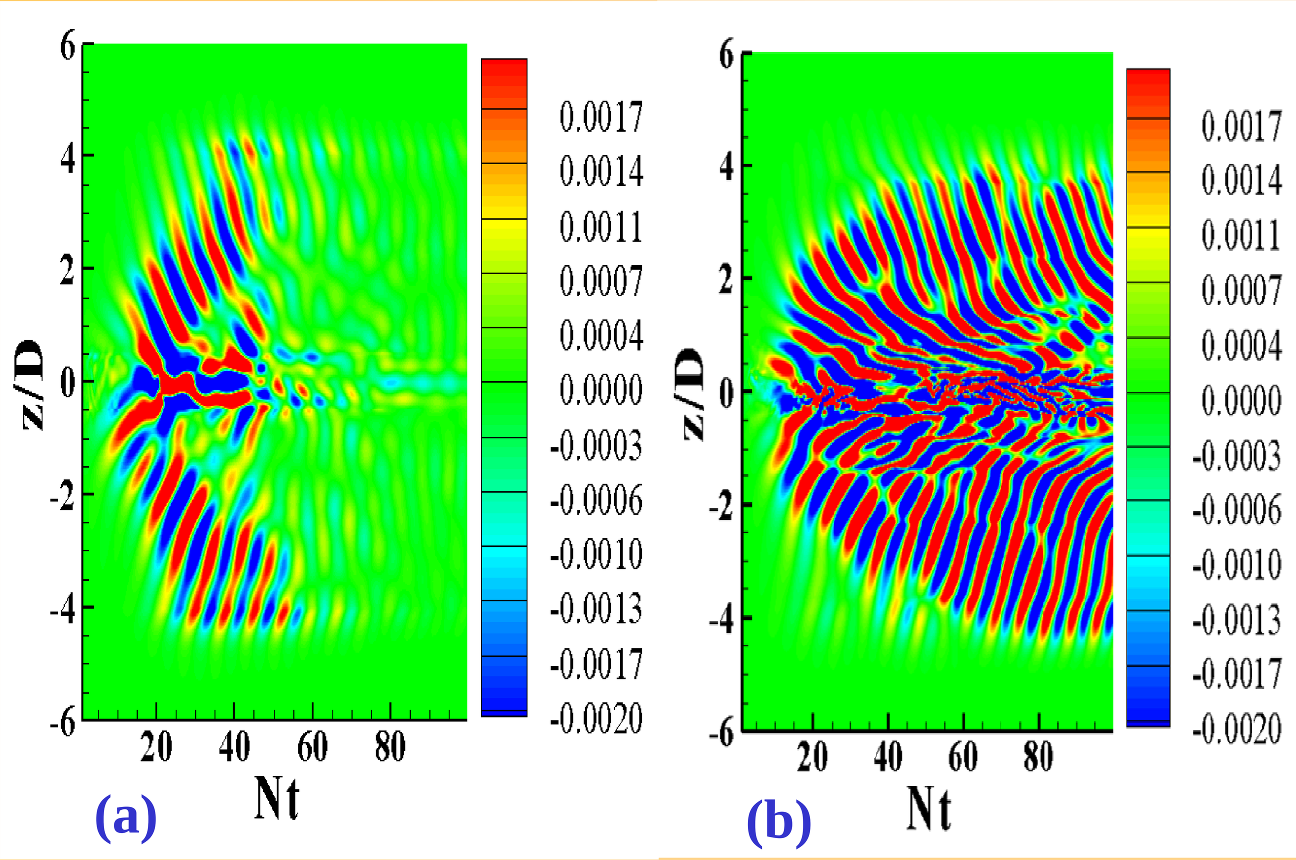

Fig.2 Hövmoller (Depth-time) diagram of the horizontal divergence $\nabla_H = \partial u/\partial x + \partial v/\partial y$ field, at x/D=15 for (a) Re=5K, Fr=4; (b) Re=100K, Fr=4. Note that the center of the coordinate system is at the geometric center of the "water tank" and that time is normalized by the Brunt Vaisala period (1/N).

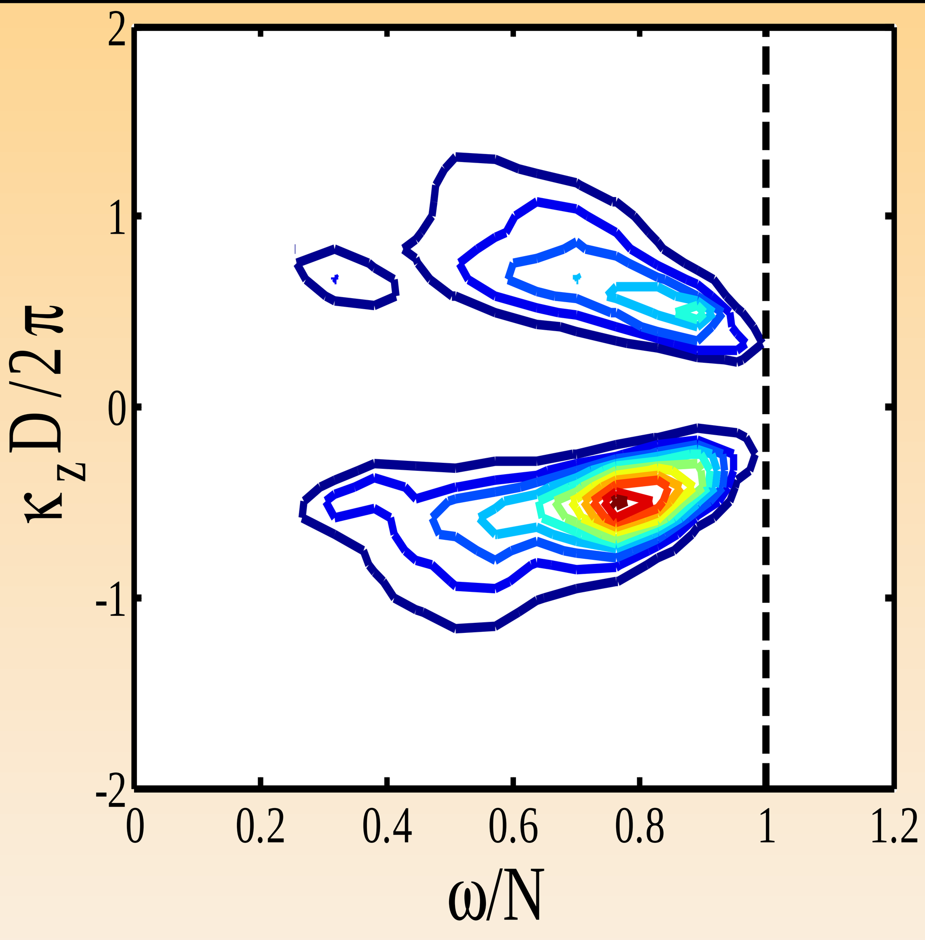

Fig.3 2D Fourier spectrum of depth-time diagrams extracted at thirteen equally spaced down-stream locations for the Re=5k, Fr=4 case. The peak of the spectrum occurs at ω/N=0.6995 (propagation angle θ =45.6°)and vertical wavelength λ_z=1.5 -2D.

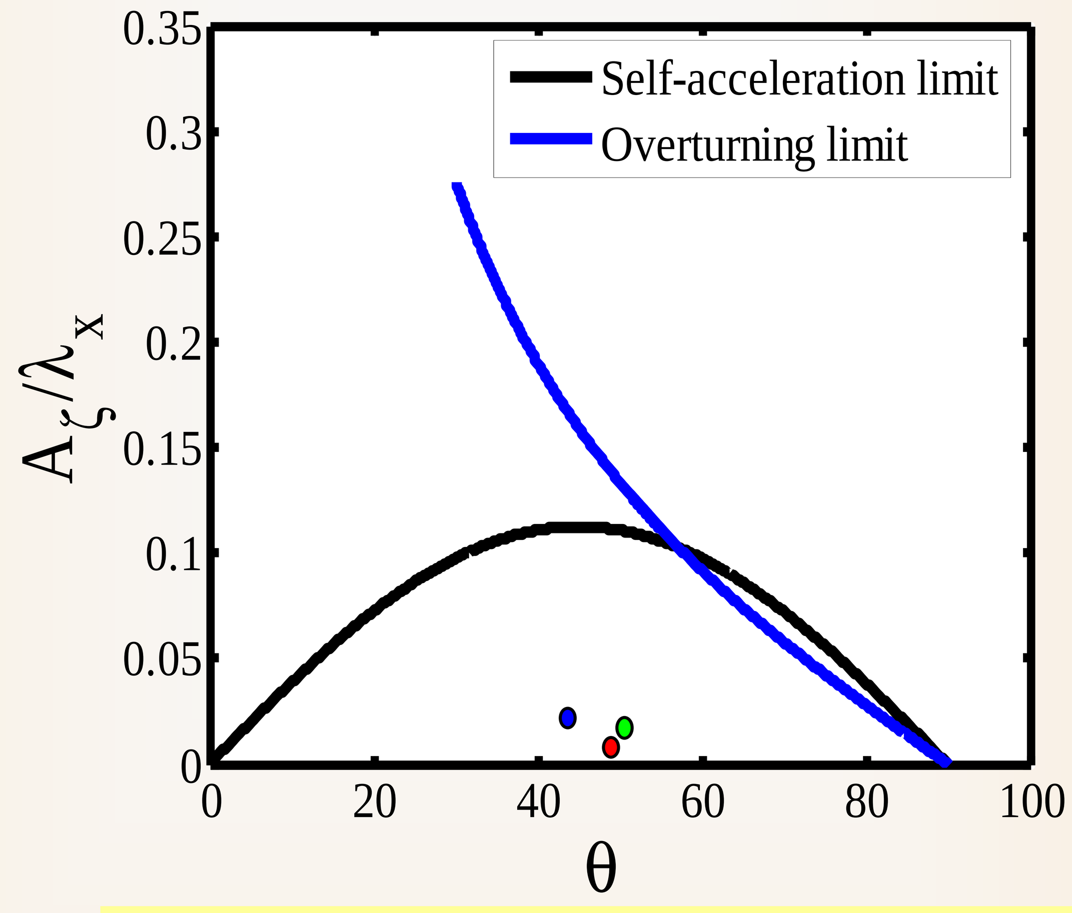

Fig.4 Comparison of the wave steepness *Aξ/λx* (isopycnal displacement divided by the horizontal wave length) to the theoretical instability limits of Sutherland et al. (Re=5K, Fr=4, Aξ/λx =0.005, 4%; Re=5K, Fr=16, Aξ/λx =0.01, 7%; Re=100K, Fr=4, *Aξ/λx* =0.0145, 8%)

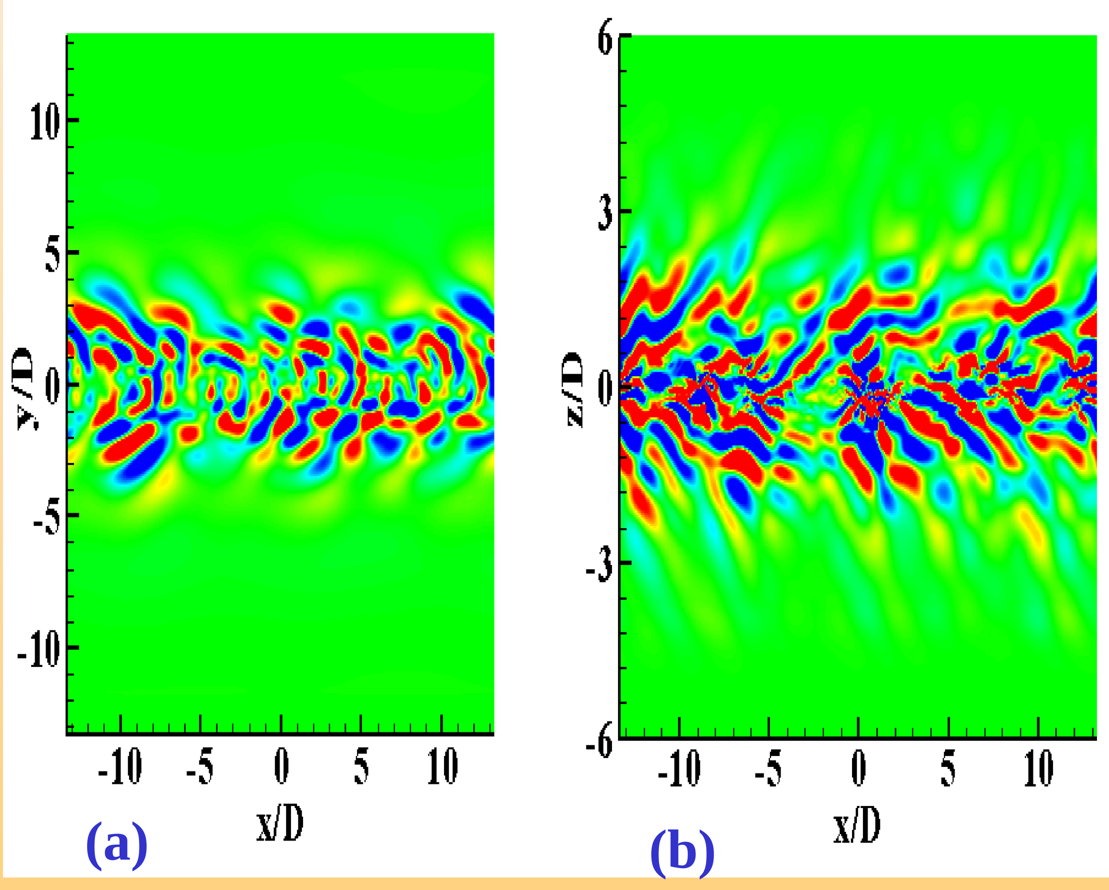

Fig.5 Horizontal divergence field for Re=100K, Fr=4 at Nt=20 on a (a) X(streamwise)-Y(spanwise) plane at z/D=2; (b) X(streamwize)-Z(depth) plane at y/D=2. Notice the intense wave emission of groups of spatially localized wave packets at high Re.

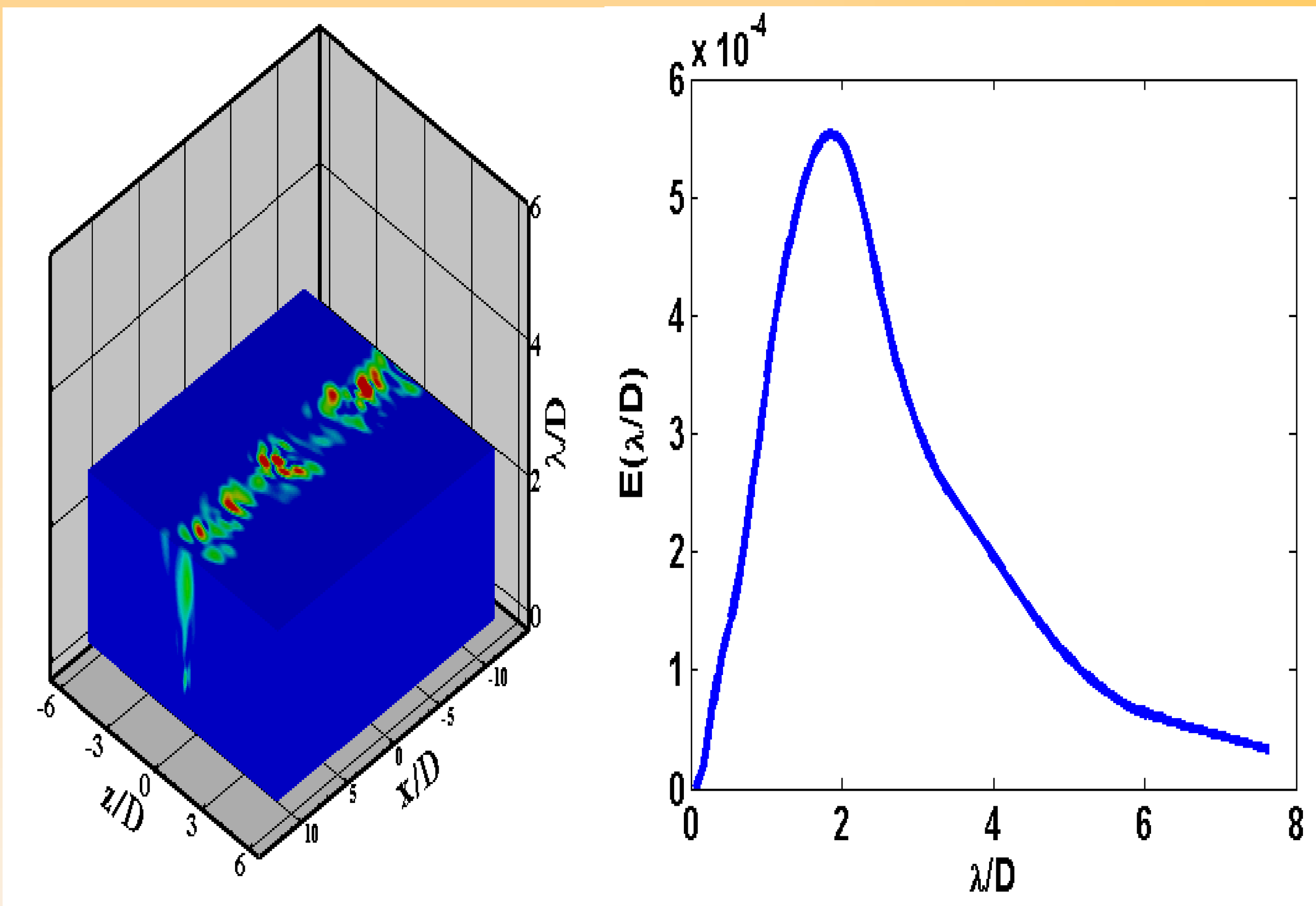

Fig.6 (a) Cutaway view in the modulus of the 2D non-directional Arc wavelet transform of fig.5-(b) at the scale containing the maximum energy (b) Plane-averaged energy of the transform as a function of scale (note that the scale is converted into an equivalent Fourier mode and the associated wave-length, normalized by the sphere diameter, is plotted).

Summary of the results

- Polar angles in the [θ=27-55°] range while azimuthal angles in the [φ=13-55°] range, in general agreement with studies of waves emitted from turbulent mixed regions.
- Steeper waves at higher Reynolds and Froude numbers.
- More complex and persistent wave field at high Reynolds numbers.
- Highly, spatially as well as temporally localized wave field.
- Spanwise wave-lengths, obtained from averaging the Fourier spectrum, are overestimated.
- Need for wavelet transform to accurately determine the wave spectra and to understand scale distribution in space and time.

Acknowledgment

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