Simulating voltage collapse dynamics for power systems with constant power load models

S. G. Abhyankar, Student Member, *IEEE*, and A. J. Flueck, Member, *IEEE*

Abstract—**Electromechanical Transients are studied using Transient Stability Simulators with differential-algebraic equation (DAE) models employing phasor network variables. While these simulators can follow the electromechanical transients (e.g., voltage and power swings) due to large disturbances with loads modeled as constant impedance, they may fail to converge under voltage collapse conditions for systems with constant power load models. This non-convergence has been attributed to a voltage collapse during the transient in the literature. This paper aims at addressing this non-convergence issue of the phasor based transient stability simulators using constant power load models. It presents a methodology for examining the voltage collapse trajectories by modeling constant power loads as voltage dependent impedance loads. Simulations on a two bus and a nine bus system for line switching disturbances show that the voltage collapse trajectories can be examined via the proposed voltage dependent impedance load model.**

Index Terms—**Voltage collapse, transient stability simulation non-convergence, constant power loads, DAE model, voltage dependent impedance load.**

I. Introduction

The threat of voltage collapse is considered carefully by electric utilities worldwide. As power systems are operated at increasing loading levels, voltage collapse becomes more likely. Today's increased loading levels are due to the fact that the system load is increasing, yet transmission line capacity is stagnant. Several constraints such as economic, environmental, and territorial issues are the cause of such a mismatch. Voltage instability incidents have been reported in power systems around the world [1], [2]. Hence it becomes increasingly important to study the mechanism of voltage collapse in order to prevent it.

The traditional ways of analyzing transfer capability limits with respect to voltage collapse are based on the steady state power flow calculations. A variety of tools such as PV and QV curve analysis, modal analysis, and sensitivity methods have been proposed to estimate the steady state voltage collapse point [3]-[5]. For large disturbances like faults on the network, tripping of transmission lines, and generator outages, these steady state methods reveal the pre and post-contingency equilibrium points but do not have any information about the interconnecting transient. The transient is characterized by voltage and current swings before settling down to the post contingency operating point, assuming the system is stable.

Voltage collapse also has been explored using dynamic analysis with generator models based on differential equations, excitation limits, load dynamics, and tap changing transformers [6]-[8]. In [6], the voltage collapse trajectories at the saddle node bifurcation point are captured using dynamic analysis for small load variations. A constant power load in parallel with a dynamic induction motor load is used as the load model. The dynamics of voltage collapse for line tripping considering OLTC and overexcitation limits of a generator are discussed in [7]. The load model used is an aggregate load model based on voltage exponential. In [8], an index for detecting dynamic voltage collapse is proposed. However, the load models used in these dynamic analyses are not purely constant power.

This paper presents the simulation of voltage collapse dynamics by modeling constant power loads as voltage dependent impedance loads. It does not advocate modeling of entire bus loads as constant power loads as it is a rather conservative approach for voltage collapse studies during transients. However, the increased use of new power electronic drive systems may shift traditional motor loads towards constant power loads. Hence, load behavior identification is needed, but it is not the focus of this paper. The authors are presenting a tool that allows engineers to simulate the system dynamics leading to voltage collapse for the worst case scenario in which loads have a strong constant PQ component. Use of such a load model also resolves the premature numerical termination of the transient stability simulators.

The organization of this paper is as follows: Section II illustrates the motivation for this paper. Section III describes the system modeling along with the formulation for the voltage dependent impedance load model. Section IV discusses the solution methodology for the transient stability simulators. Results on a two bus system and a nine bus system are presented in section IV.

II. Motivation

Fig. 1 shows an example of PV curves for the normal case and line outage case from a continuation power flow. λ^*_{normal} is the loading limit for the system with all lines in service and

S. G. Abhyankar is with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, IL 60616 USA (e-mail: abhyshr@iit.edu).

A. J. Flueck is with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, IL 60616 USA (e-mail: flueck@iit.edu).

λ^{*}_{ctgc} is the loading limit with one of the transmission lines out of service.

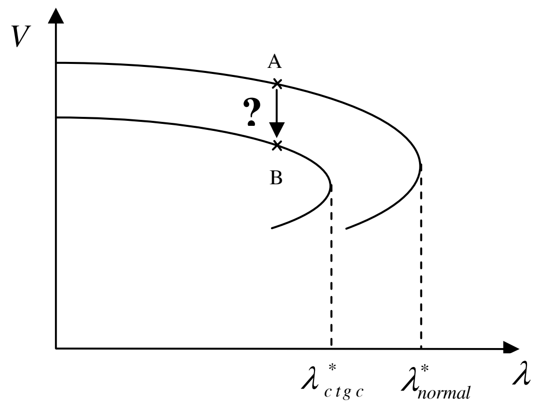

Fig. 1. Continuation Power Flow curves for a line outage.

Assume that for a given loading, the system is operating at point *A* with all the lines in service. The continuation power flow plots show that if a line is tripped then there exists a post-contingency operating point, *B*. The stability of the operating point can be found out using an eigenvalue analysis. As seen from Fig. 1, the system is operating at a high loading level and the goal is to investigate whether the system can go from point *A* to point *B* through the transient. This analysis is typically done using a transient stability simulator employing a DAE system model.

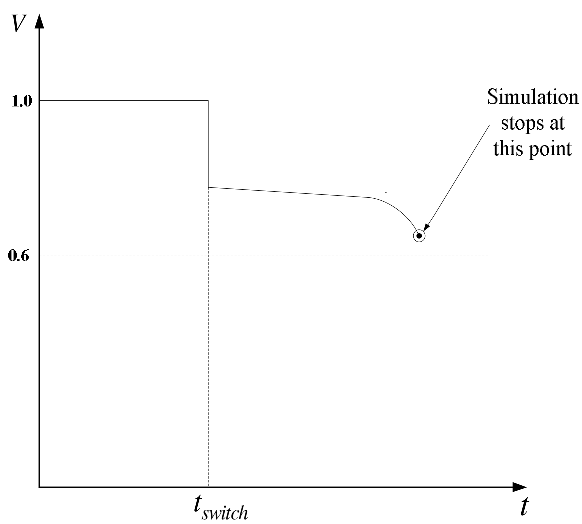

Fig. 2. Premature numerical termination of Transient Stability Simulation.

When the transient response is analyzed by a transient stability simulator, the simulation may stop before the system collapses. As seen in Fig. 2, the voltage at which the simulation stops, for this example case, is above 0.6 pu. While such a premature numerical termination of the simulation has been attributed to voltage collapse during the transient [8], it still does not confirm that it is a voltage collapse. Moreover, there are some important unanswered questions left, w*hat is the reason for having no solution for the system? How can the simulations be continued to follow the system trajectories?* Finding answers to these questions is the main motivation behind this paper. The first question may be a result of the violation of the complex voltage and current phasor assumption used in transient stability simulators [9] or singularity induced bifurcation as described in [10]. While this explanation may describe the reason for the numerical failure, it does not shed light on the associated voltage collapse dynamics. Therefore, the second question is more important for analyzing the events that may follow. The stability of the system, operation of protective devices, generator response, and possible cascading are some of the critical events that need to be studied. The voltage collapse cascade phenomenon is still a relatively unexplored domain in power systems. The potential for a cascade either is decided on heuristics or based on experience. However, to the knowledge of the authors, there is no efficient tool that can follow the sequence of events leading to a cascade or the cascade itself for large-scale power systems.

III. Problem Formulation

Electromechanical transient simulators use algebraic power flow equations for the transmission network and differential equations to represent the generator dynamics. Thus, the system model is a *differential-algebraic* or *DAE* model. The DAE model used in this paper is from [11] which has a detailed description on transient stability simulators for power systems.

A. Generator Model

The differential equations for a detailed two-axis generator model at *m* generator buses are

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \tag{1}$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = T_{Mi} - E^{'}_{di} I_{di} - \left(X^{'}_{qi} - X^{'}_{di}\right) I_{di} I_{qi} - D_i\left(\omega_i - \omega_s\right) \tag{2}$$

$$T^{'}_{doi}\frac{dE^{'}_{qi}}{dt} = -E^{'}_{qi} - \left(X_{di} - X^{'}_{di}\right) I_{di} + E_{fdi} \tag{3}$$

$$T^{'}_{qoi}\frac{dE^{'}_{di}}{dt} = -E^{'}_{di} + \left(X_{qi} - X^{'}_{qi}\right) I_{qi} \tag{4}$$

$$T_{Ei}\frac{dE_{fdi}}{dt} = -\left(K_{Ei} + S_{Ei}\left(E_{fdi}\right)\right) E_{fdi} + V_{Ri} \tag{5}$$

$$T_{Fi}\frac{dR_{fi}}{dt} = -R_{fi} + \frac{K_{Fi}}{T_{Fi}} E_{fdi} \tag{6}$$

$$T_{Ai}\frac{dV_{Ri}}{dt} = -V_{Ri} + K_{Ai}R_{fi} - \frac{K_{Ai}K_{Fi}}{T_{Fi}} E_{fdi} + K_{Ai}\left(V_{refi} - V_i\right) \tag{7}$$

$i = 1, ..., m$

These equations are for a two-axis machine model without saturation, subtransient reactances nor the stator transients. The governor dynamics are also not modeled, resulting in constant mechanical torque input. The limit constraints on the voltage regulator output are also neglected. A linear damping term is assumed for the friction and windage torque. Equations (1)-(4) represent the generator dynamics while (5)-(7) represent the exciter dynamics. δ, ω are the rotor angle and generator frequency, $E_q^{'}, E_d^{'}$ are the transient voltages behind the transient reactances $X_q^{'}, X_d^{'}$ respectively. E_{fd}, R_F, and V_R are the field voltage, rate feedback and AVR output of the exciter respectively.

B. Stator Algebraic Equations

The stator algebraic equations are an interface between the generator and the network. The *d*, *q* axis generator currents are related to the network variables in polar form through (8).

$$\begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix} = \left[Z_{d-q,i} \right]^{-1} \begin{bmatrix} E_{di}^{'} - V_i \sin(\delta_i - \theta_i) \\ E_{qi}^{'} - V_i \cos(\delta_i - \theta_i) \end{bmatrix} \qquad i = 1,...,m \tag{8}$$

C. Network Equations

The network equations at *m* generator buses and *n-m* other buses are

$$\begin{aligned} & I_{di} V_i \sin(\delta_i - \theta_i) + I_{qi} V_i \cos(\delta_i - \theta_i) + P_{Li}(V_i) \\ & -\sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \qquad i = 1,...,m \end{aligned} \tag{9}$$

$$P_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad i = m+1,..,n \tag{10}$$

$$\begin{aligned} & I_{di} V_i \cos(\delta_i - \theta_i) - I_{qi} V_i \sin(\delta_i - \theta_i) + Q_{Li}(V_i) \\ & -\sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad i = 1,...,m \end{aligned} \tag{11}$$

$$Q_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad i = m+1,...,n \tag{12}$$

Equations (9), (11) represent the real and reactive power balance at the generator buses and (10), (12) represent the power balance at the other buses. The network variables are the bus voltage magnitude V and the phase angle θ. Y_{ik} represents the magnitude of the (*i*, *k*) element in the bus admittance matrix.

D. Constant power load modeled as voltage dependent impedance

A constant power load can be thought of as a load having a negative impedance characteristic. The load current increases as the voltage decreases. The modeling of a constant power load modeled as a voltage dependent impedance load is based on this concept.

If n represents the time instant t_n then a constant impedance load is modeled in transient stability simulations as

$$\begin{aligned} P_n &= k_p V_n^2 \\ Q_n &= k_q V_n^2 \end{aligned} \tag{13}$$

where,

$$\begin{aligned} k_p &= \frac{P_0}{V_0^2} \\ k_q &= \frac{Q_0}{V_0^2} \end{aligned} \tag{14}$$

P_n is the real power drawn by the load at the n^{th} time instant. Q_n is the reactive power drawn by the load at the n^{th} time instant. P_0 is the nominal load real power; Q_0 is the nominal reactive power; V_0 is the nominal voltage magnitude.

k_p, k_q are the load conductance and susceptances respectively which remain fixed for a constant impedance load. For a voltage dependent impedance load trying to draw constant power, these parameters have to increase as the voltage decreases. Assuming voltage dependent impedance responds to a change in the voltage, k_p, k_q values were modified at each time instant as given in (15). In (15), *n-1* is the time instant t_{n-1} and V_{n-1} is the voltage magnitude at t_{n-1}. By using the previous time step for the computation of k_p, k_q, the load characteristic tries to maintain a constant power load characteristic. If measurements of actual events are available then a curve fit strategy could be used similar to [12] and the recovery time constant chosen to simulate the observed dynamic behavior.

$$\begin{aligned} k_{p,n} &= \frac{P_0}{V_{n-1}^2} \\ k_{q,n} &= \frac{Q_0}{V_{n-1}^2} \end{aligned} \tag{15}$$

This voltage dependent impedance is not a constant power load unless the delay in its update is zero. With non-zero delay, it behaves instantaneously as constant impedance, and then becomes voltage dependent impedance. In extreme case of low voltage, $V = 0$, the voltage dependent impedance looks like a short circuit since $Z_n = V_{n-1}^2 / P_0$.

IV. Solution Methodology

Along with the differential equations for the generator, the DAE model can be symbolically written as

$$\begin{aligned} \dot{x} &= f_o\left(x, I_{dq,} \bar{V}, u\right) \\ I_{dq} &= h\left(x, \bar{V}\right) \\ 0 &= g_o\left(x, I_{d-q,} \bar{V}\right) \end{aligned} \tag{16}$$

To find the solution of the DAE system, the differential equations are integrated from time t_n to t_{n+1} while solving the algebraic equations at time t_{n+1}. The resulting equations to be solved are given in (17).

$$\begin{aligned} & [x_{n+1} - \frac{\Delta t}{2} f_0(x_{n+1}, I_{dq,n+1}, \bar{V}_{n+1}, u_{n+1})] - [x_n + \frac{\Delta t}{2} f_0(x_n, I_{dq,n}, \bar{V}_n, u_n) = 0 \\ & I_{dq,n+1} - h(x_{n+1}, \bar{V}_{n+1}) = 0 \\ & g_0(x_{n+1}, I_{dq,n+1}, \bar{V}_{n+1}) = 0 \end{aligned} \tag{17}$$

At each time step, (17) is solved by Newton's method to obtain the state and the algebraic variables at time t_{n+1}.

Typical large disturbances include faults on the network, line trippings, generator outages, load outages etc. Such disturbances to the algebraic network equations are fast compared to the generator electromechanical dynamics which have large mechanical time constants. The stator equations are also algebraic equations and hence respond quickly to the disturbance. In the DAE model, "quickly" means "instantaneously". Hence, the network and the stator algebraic variables are solved at the disturbance time to reflect the post-disturbance values while keeping the generator dynamic variables fixed at their pre disturbance values. This one additional solution at the disturbance time involves the solution of the equations given in (18). The superscript f in (18) indicates that the algebraic equations correspond to the faulted state and t_d represents the disturbance time.

$$\begin{aligned} I_{dq}(t_d+) - h^f(x(t_d), \bar{V}(t_d+)) &= 0 \\ g_0(x(t_d), I_{dq}(t_d+), \bar{V}(t_d+)) &= 0 \end{aligned} \tag{18}$$

With the post-disturbance algebraic solution thus obtained, the trapezoidal integration process is resumed.

V. Simulation Results

All the simulations were done in Matlab using the equations in the previous section. The numerical integration of the DAE model was done using the implicit trapezoidal integration algorithm. Matlab based power system package MatPower [13] was used to solve the power flow to obtain the initial bus voltages. Using these initial voltages, the transient stability simulation was initialized to start in steady state. The objective of the simulations was to see if the system could survive the transient caused due to the tripping of a branch.

A. Transient simulation with constant power loads

Fig. 3 shows the one line diagram for a two bus test system. It consists of two transmission lines in parallel, Branches 2-3 and 4-5 with breakers at either ends. The load is a constant power factor PQ load.

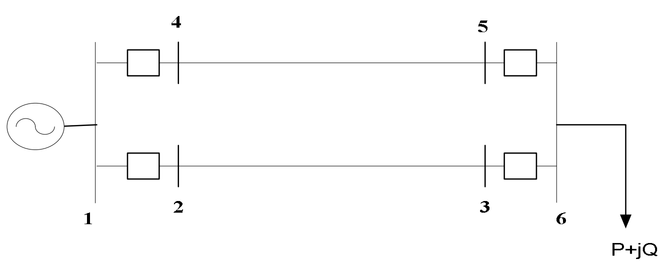

Fig. 3. Test two bus system topology.

Prior to running the transient stability simulations, a continuation power flow was run to determine the steady state loadability of the system. Figure 4 shows the continuation power flow PV curves for the system with both lines in service and one line out.

The steady state loading limit for the system with one line in service, $P^*_{d,ctgc}$, was found to be approximately 3.53 pu and with both lines in service, $P^*_{d,base}$, was around 7.15. Note that the continuation power flow curves assume that the generator terminal voltage is maintained constant at 1.0 pu, however during the transient this is not true.

With the continuation power flow as the basis for the transient stability studies, the transient stability simulations were carried out by tripping branch 4-5. The load was assumed to hold its constant PQ characteristic throughout the transient. The basic aim of the transient stability simulations was to determine whether, starting from the PV curve with both lines in service, the system can survive the transient and reach the steady state operating point corresponding to the PV curve with one line in service.

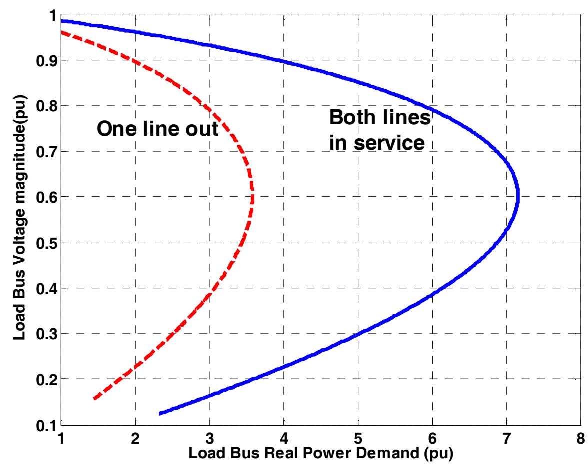

Fig. 4. PV curves from continuation power flow for two bus system.

It was observed that the system could survive the transient caused due to line tripping up to a loading level of $P_d = 2.31$ pu. Beyond this loading, the solution failed to converge during the transient as shown in the time domain plots of Fig. 4.

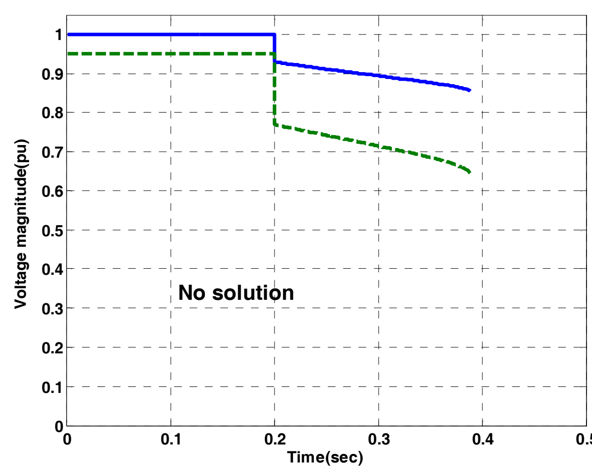

Fig. 5. Voltage magnitude plots for branch tripping at $P_d = 2.32$ pu.

Looking at Fig. 5, it can be seen that the load bus voltage magnitude has dropped below 0.65 pu. Intuitively, it can be perceived that the load bus voltage does collapse at the next step based on its slope and hence the system is not able to find the solution. However, the question arises then *Is the generator bus voltage collapsing too*? Seeing the generator bus voltage magnitude, this cannot be inferred.

B. *Transient simulation with voltage dependent impedance load model*

Fig. 6 shows the results obtained by implementing the voltage dependent impedance model in the DAE system instead of a constant power load model.

As seen from Fig. 6, the voltage collapse is captured in transient stability simulations using a voltage dependent impedance load. A 1 millisecond integration time step was chosen for these simulations. Using such a small time step, the voltage dependent impedance is updated based on the voltages from the previous time step, i.e., a 1 millisecond delay. This allows the voltage dependent impedance to closely follow a constant power load characteristic. The update also can be delayed further, so that the voltage dependent impedance responds more slowly. If the delay is infinite, then the voltage dependent impedance acts a constant impedance load.

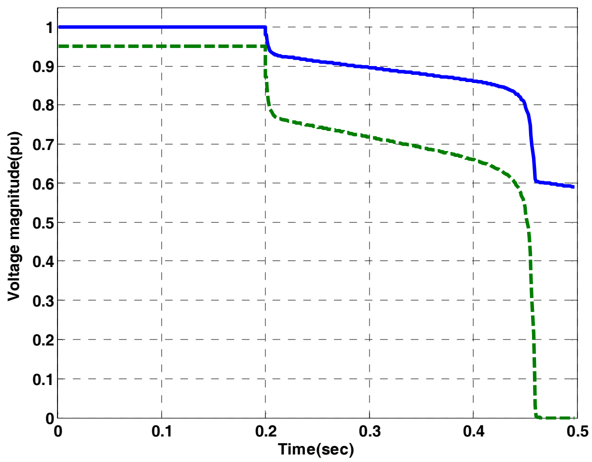

Fig. 6. Voltage collapse at the load bus captured in transient stability simulation using voltage dependent impedance load model. $P_d = 2.32$

Fig. 7 shows the load bus phase angle plot. It can be seen that the load bus phase angle has a large variation near the collapse point. This suggests that the load bus frequency is not constant near the collapse point, which implies that the constant frequency assumption behind complex voltage and current phasors is no longer valid.

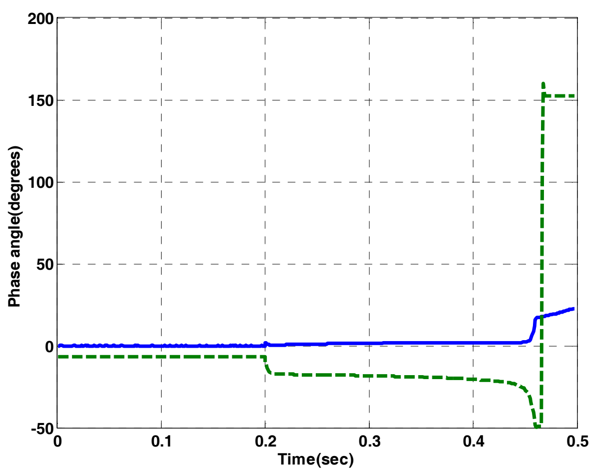

Fig. 7. Large deviation of the load bus phase angle near the collapse point. $P_d = 2.32$

C. *Nine bus system simulation*

The voltage dependent impedance load model was tested further on a nine bus system which consists of three generators, nine branches and three loads. The system data can be found in [11]. The generators are at buses 1, 2, and 3 while the loads are at buses 5, 6, and 8. All the loads are considered as constant power loads. The disturbance considered was the tripping of branch 8-9. The continuation plot for the nine bus system with and without branch 8-9 is shown in Fig. 9.

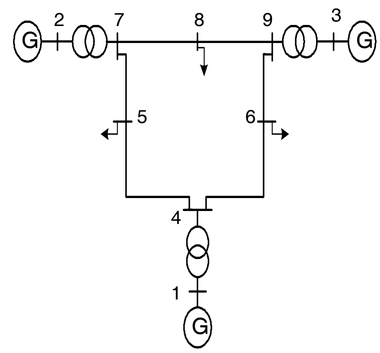

Fig. 8. Nine bus system topology

Scalar variable λ is a control parameter used for studying the slow variation of generation and loading on the system in steady state studies. $\lambda = 0$ represents the existing generation and loading while $\lambda = 1$ represents the new generation-loading condition for the given transfer schedule.

Fig. 10-11 show the transient simulation of the nine bus system with constant power load and voltage dependent impedance load model. As seen in Fig. 10, the transient stability simulation stops abruptly due to the non-convergence of the algebraic system at $\lambda = 0.42$.

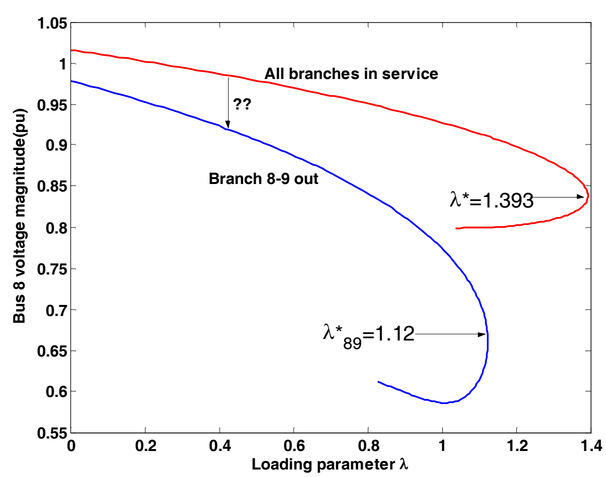

Fig. 9. PV curves for the nine bus system.

The voltage dependent impedance load model simulation is able to capture the voltage collapse dynamics at the load buses. The lowest bus voltage observed is at bus 8 since it is close to the branch outage and it has a load attached. As seen, the DAE simulation fails to converge with the constant power load model at $\lambda = 0.42$ but the voltage collapse is captured using the voltage dependent impedance load model. Load bus 8 suffers collapse first as it closest to the branch outage. This is followed by a collapse at buses 5 and 6. No protective devices, such as undervoltage relays for load shedding, are modeled in the system; hence the system does not recover. The modeling of protective devices is intended for future work.

VI. Conclusions

This paper presented a new strategy for simulating voltage collapse dynamics by using the proposed voltage dependent impedance load model. Results on a two bus system and nine bus system show that voltage collapse can be captured by the DAE model. It also illustrates the violation of the phasor assumption near the collapse point.

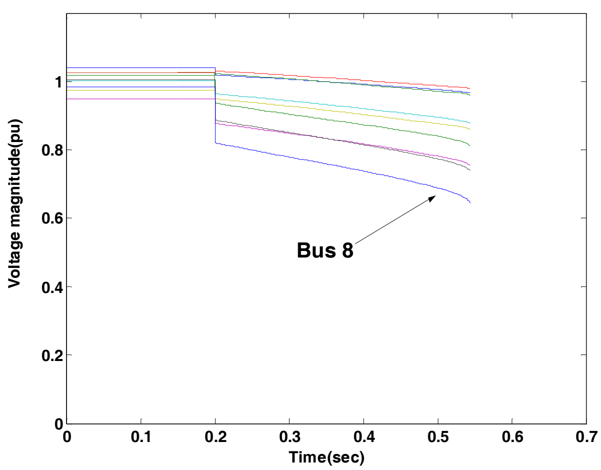

Fig. 10. Simulation fails to converge with constant power loads. $\lambda = 0.42$

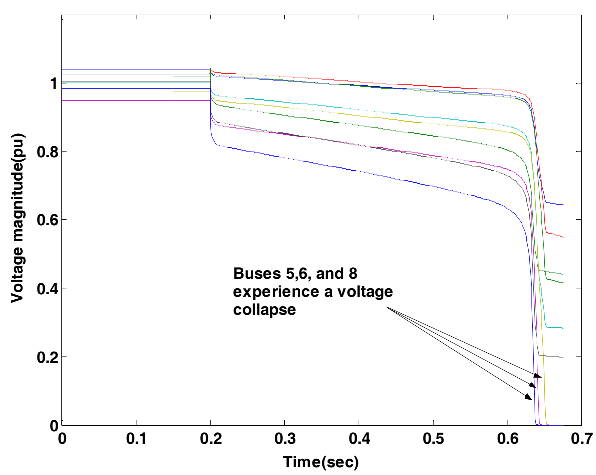

Fig. 11. Voltage collapse trajectories captured by the voltage dependent impedance load model. $\lambda = 0.42$

VII. References

[1] W. R. Lachs, and D. Sutanto, "Different types of voltage instability," *IEEE Trans. Power Systems*, vol. 9, pp. 1126-1134, May 1994.

[2] C. D. Vournas, G. A. Manos, J. Kabouris, and T. V. Cutsem, "Analysis of voltage instability incident in the Greek power system," *IEEE Power Engineering Society Winter Meeting*, vol. 2, pp. 1483-1488, 23-27 Jan 2000.

[3] P. Kundur, *Power system stability and control*, New York: McGraw-Hill Inc., 1994.

[4] V. Ajjarapu, C. Christy, " The continuation power flow: a tool for steady state voltage stability analysis," *IEEE Trans. on Power Systems*, vol. 7, pp. 416-423, Feb. 1992.

[5] H. D. Chiang, A. Flueck, K. Shah, and N. Balu, "CPFLOW: A practical tool for tracing system steady state stationary behavior due to load and generation variations," *IEEE Trans. on Power Systems*, vol. 10, pp. 623-634, May 1995.

[6] I. Dobson, H. -D. Chiang, J. S. Thorp, L. Fekih-Ahmed, "A model of voltage collapse in electric power systems," in *Proc. 1988 IEEE Decision and Control Conf.*, pp. 2104-2109.

[7] T. Van Cutsem, "Voltage instability: phenomena, countermeasures and analysis methods," in *Proc. IEEE* vol. 88, issue 2, pp. 208-227, Feb. 2000.

[8] G. M. Huang, N.-K.C. Nair, "Detection of dynamic voltage collapse," *IEEE Power Engineering Society Summer Meeting*, vol. 3, pp. 1284-1289, 2002.

[9] I. Dobson, T. V. Cutsem, C. Vournas, C. L. DeMarco, M. Venkatasubramaniam, T. Overbye, and C. A. Canizares, “Voltage stability assessment: concepts, practices, and tools,” Chapter 2, Power system stability subcomittee special publication, August 2003.

[10] V. Venkatasubramanian, H. Schattler, J. Zaborsky, "Dynamics of large constrained nonlinear systems – a taxonomy theory [power system stability]," in *Proc .Nov. 1995 IEEE,*. vol. 83 pp. 1530-1561.

[11] P. W. Sauer, and M. A. Pai, Power *System Dynamics and Stability*, New Jersey: Prentice Hall Inc., 1998.

[12] D. J. Hill, "Nonlinear dynamic load models with recovery for voltage stability studies," *IEEE Trans. on Power Systems*, vol. 8, pp. 166-176, Feb. 1993.

[13] R. D. Zimmerman, D. Gan, "Matpower user’s manual version 2.0," [Online]. Available: http://www.pserc.wisc.edu/ecow/get/publicatio/ 1998public/manual.pdf.

[14] I. Dobson, H. Glavitsch, C-C. Liu, Y. Tamura, K. Vu, "Voltage collapse in power systems," *IEEE Circuits and Devices Magazine*, vol. 8, issue 3, pp. 40-45, May 1992.

[15] B. H. Lee, K. Y. Lee, "A study on voltage collapse mechanism in electric power systems," *IEEE Trans. on Power Systems*, vol. 6, pp. 966-974, Aug. 1991.

[16] C. Ong, *Dynamic simulation of electric machinery using Matlab®/Simuink,* Taiwan; Prentice Hall, 2003.

VIII. Biographies

Shrirang Abhyankar received B.E. degree from University of Mumbai in 2003 and M.S degree from Illinois Institute of Technology in 2006. Currently, he is a Ph. D. candidate at the Illinois Institute of Technology. His research interests include voltage stability analysis, transient stability simulation electromagnetic transients simulation

Alexander J. Flueck received the B.S. degree (1991), the M.Eng. degree (1992) and the Ph.D. degree (1996) in electrical engineering from Cornell University. He is currently an Associate Professor at Illinois Institute of Technology in Chicago. His research interests include transfer capability of large-scale electric power systems, contingency screening of multiple branch or generator outages with respect to voltage collapse, continuation methods for parameterized nonlinear systems, transient stability and parallel simulation of power systems via message-passing on distributed-memory computer clusters.