A new confirmation of voltage collapse via instantaneous time domain simulation

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***Abstract*—This paper aims at capturing the voltage collapse trajectories beyond the steady state loading limit for power systems immediately following a branch outage. Instantaneous Time Domain Simulations are done with two types of transmission line models: Bergeron Model, and Lumped Parameter Model. The constant power load is modeled as a voltage dependent impedance. The results are compared with the continuation power flow which assesses the steady state loadability of the system.**

***Index Terms*—Steady state voltage collapse, Instantaneous Time Domain Simulation, constant power loads, Continuation power flow.**

I. Introduction

VOLTAGE collapse is an important issue for utilities as power systems are operating at increasing loading levels. Today's heavy loading levels are due to the fact that the system load is increasing, yet transmission line capacity is stagnant. Several constraints such as economic, environmental, and territorial issues are the cause of such a mismatch. Voltage instability incidents have been reported in power systems around the world [1], [2] and hence it becomes increasingly important to study the mechanism of voltage collapse.

The work on finding the steady state voltage collapse point has been traditionally done using static analysis methods which assume slow variations of loading or generation. Various static methods such as PV and QV curve analysis, modal analysis, and sensitivity analysis methods have been proposed to estimate the steady state voltage collapse point[3]-[5]. Voltage collapse also has been explored using the system dynamics (generator models based on differential equations, excitation limits, load dynamics, tap changing transformers) using time domain integration [6]-[8]. In [6], the voltage collapse trajectories at the saddle node bifurcation point are captured using dynamic analysis for small load variations. A constant power load in parallel with a dynamic induction motor load is used as the load model. The dynamics of voltage collapse for line tripping considering OLTC and over-excitation limits of the generator are discussed in [7]. The load model used is an aggregate load model based on voltage exponential. In [8], an index for detecting dynamic voltage collapse is proposed.

While the static and dynamic methods can detect the collapse point, the voltage collapse trajectories cannot be examined in detail for large disturbances due to the non-convergence of the algebraic system for constant power loads. The examining of voltage collapse trajectories is important from the point of view of differentiating local collapse from widespread collapse, setting accurate transfer capability limits, studying protective device operation, and mitigating the sequence of events leading to a voltage collapse cascade.

In this paper, we approach the capturing of voltage collapse trajectories beyond steady state stability limit with the system variables represented in instantaneous form. The network is modeled in differential form while the load maintains its constant power characteristic. The modeling of the constant power load characteristic is done using a proposed voltage dependent impedance load model. No generator or transformer dynamics are modeled.

The organization of this paper is as follows: Section II describes the continuation power flow for the steady state voltage collapse analysis. The network and the load modeling for instantaneous time domain simulations is discussed in Section III. Section IV presents the simulations done on the two bus test system.

II. Steady state voltage collapse Analysis using continuation power flow

Steady state voltage collapse analysis is concerned with finding the maximum power that can be transferred without causing a collapse for slow variation of parameters. An indication of voltage collapse is that the standard power flow solution fails to converge. This is due to the fact that the power flow Jacobian becomes singular at the collapse point.

A. Continuation Power Flow

Continuation power flow analysis overcomes the Jacobian singularity problem by reformulating the power flow equations so that they remain well-conditioned at all possible loading conditions. The power flow equations are modeled as

$$g(x) = \begin{bmatrix} P(x) - P_{inj} \\ Q(x) - Q_{inj} \end{bmatrix} = 0 \qquad (1)$$

where, $x \equiv (V, \theta) \in R^{2N}$, with N = number of buses.

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The behavior of the power system for slowly varying loading and generator can be examined by introducing a vector of target injections $[\hat{P}_{inj}, \hat{Q}_{inj}]$ into the power flow equations as

$$f(x,\lambda) = g(x) - \lambda b \tag{2}$$

where, λ is a scalar variable and

$$b \equiv \begin{bmatrix} \hat{P}_{inj} - P_{inj} \\ \hat{Q}_{inj} - Q_{inj} \end{bmatrix} \tag{3}$$

An example of typical continuation power flow curves for a system with all lines in service and one line out is shown in Fig. 1.

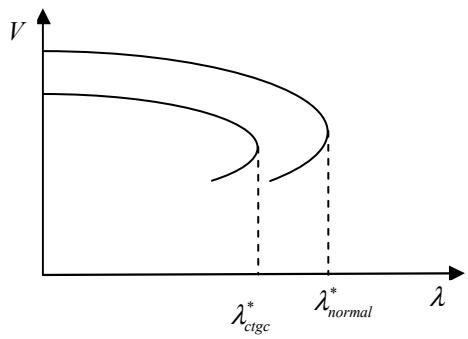


Fig. 1. Continuation Power Flow Curve for a system with all lines in service and with one line out.

In Fig.1, λ^*_{normal} is the steady state voltage collapse point with all lines in service while λ^*_{ctgc} is the collapse point with one line out. The collapse point is also the maximum power transfer limit or the steady state loadability for the system.

B. System Modeling for Static Analysis

The modeling of the network and the loads for continuation power flow analysis follows that for power flow. The network is modeled using algebraic equations while the loads are typically modeled as constant power. Generators are modeled as either swing generators or producing constant active power output. Swing generators are assumed to have a fixed voltage magnitude and angle at their terminals while the generators producing constant active power output are assumed to maintain constant voltage magnitude at their terminals (assuming the over-excitation limits are not reached).

III. Instantaneous Time Domain Simulation

All electrical circuits exhibit electromagnetic transients during switching. The power system with long transmission lines and various electromagnetic components exhibits very complex behavior during switching or lightning strikes [9]. The study of this behavior is done using electromagnetic or instantaneous time domain simulators. In such study, instantaneous voltage and current waveforms are used instead of phasors. It is a time domain analysis with non-constant frequency hence the network elements are modeled in terms of resistances, inductances and capacitances instead of impedances and admittances. The network equations are expressed in differential form along with the differential equations for the machines. The time step ranges from 10 to 1000 microseconds. While instantaneous time domain simulation could be used to capture the voltage collapse dynamics, the very detailed modeling required for such studies and the time frame of interest (microseconds) are such that the computation is extremely high. In case of widespread blackouts due to cascading outages, the modeling of very large regions of interconnected power systems is required which is difficult with instantaneous time domain simulation even with today's powerful computers.

A. Network Model

For modeling the network, two types of transmission line models were used: Bergeron Model and Lumped Parameter Model.

1) *Bergeron Model*:

Bergeron Model is a type of distributed parameter line model based on the traveling wave theory. It is an extension of the lossless line model with the distributed resistance added in lumped form [9]. Half of the line resistance is added at the mid-point and ¼th at the line ends.

The equivalent two-port network for the Bergeron Line Model is shown in Fig. 2.

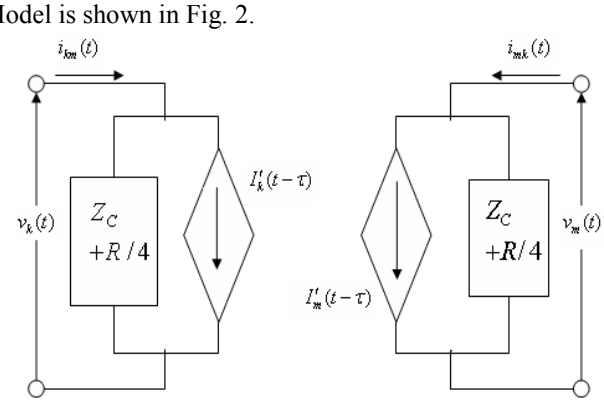


Fig. 2. Equivalent two-port network for Bergeron Line Model

The instantaneous currents at the line ends are given as

$$\begin{aligned} i_{km}(t) &= \frac{1}{Z_C + \frac{R}{4}} v_k(t) + I'_k(t-\tau) \\ i_{mk}(t) &= \frac{1}{Z_C + \frac{R}{4}} v_m(t) + I'_m(t-\tau) \end{aligned} \tag{4}$$

$Z_C \equiv \sqrt{L'/C'}$ is the characteristic or surge impedance of the transmission line, $\tau \equiv d\sqrt{L'C'}$ is the travel time required for the traveling wave to traverse the line length d. There is no direct connection between the line ends and the conditions at one end are seen indirectly and with time delay (travel time) at the other through dependent current sources. The dependent current sources also termed as history terms are given as

$$I'_k(t-\tau) = \frac{-Z_C}{(Z_C+R/4)^2}(v_m(t-\tau)+(Z_C-R/4)i_{mk}(t-\tau)) + \frac{-R/4}{(Z_C+R/4)^2}(v_k(t-\tau)+(Z_C-R/4)i_{km}(t-\tau))$$
$$I'_m(t-\tau) = \frac{-Z_C}{(Z_C+R/4)^2}(v_k(t-\tau)+(Z_C-R/4)i_{km}(t-\tau)) + \frac{-R/4}{(Z_C+R/4)^2}(v_m(t-\tau)+(Z_C-R/4)i_{mk}(t-\tau)) \quad (5)$$

2) *Lumped Parameter Model*:

A lumped parameter line model is obtained by lumping the distributed parameters of the line. Such a model assumes instantaneous response of the network i.e. the conditions at one end are seen at the other end without any delay. Lumped parameter line models are widely used in simulations employing phasor representation of network variables. A lumped parameter π model for instantaneous time domain simulations is shown in Fig. 2.

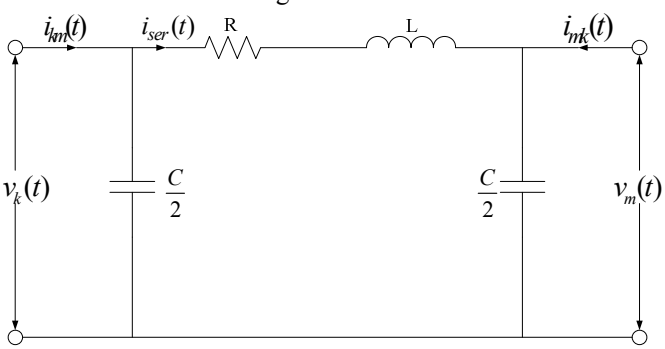


Fig. 2. Lumped Parameter π model

The relationship between the currents and voltages at the two ends is obtained by using a KCL and KVL and is given in (6)-(8)

$$L\frac{di_{ser}}{dt} = v_k(t) - v_m(t) - Ri_{ser}(t) \quad (6)$$

$$\frac{C}{2}\frac{dv_k}{dt} = i_{km}(t) - i_{ser}(t) \quad (7)$$

$$\frac{C}{2}\frac{dv_m}{dt} = i_{ser}(t) + i_{mk}(t) \quad (8)$$

Here, $R, L,$ and C represent the lumped line parameters.

B. Constant Power Loads

Constant power loads for steady state studies are modeled by fixed negative power injections into the network. There is no dependence on voltage for the constant power loads.

During a transient, any device can be thought of as sensing the stimulus first before reacting to it. Thus, a load having constant power characteristics reacts to a change in the voltage or current after sensing it first. The quicker it responds, the more closer it is to absorbing constant power during the transient state. Our modeling of loads trying to absorb constant power is based on this notion.

Constant power loads are described by real and reactive power absorbed which is not an instantaneous term but rather the observation over one cycle of the fundamental frequency. Thus, the modeling requires observation of the instantaneous values over one cycle.

Consider a constant impedance load drawing complex nominal power $P_0 + jQ_0$ at a voltage magnitude V_0 across its terminals. The resistance and inductance of the load is given by

$$R = V_0^2 / P_0$$
$$L = V_0^2 / \omega Q_0 \quad (9)$$

Here, $\omega = 2\pi f$ and f is the fundamental frequency. For a constant impedance load, if the voltage magnitude decreases, then the current drawn decreases and the load power decreases too since the load impedance is constant.

For a load absorbing constant power however, if the voltage magnitude decreases, then the current increases to maintain constant power. This increase in the current as the voltage decreases can be modeled by decreasing the load impedance as the voltage decreases. Using (9), loads trying to absorb constant power are modeled by changing the resistance and the inductance of the load at each time step. This requires the knowledge of the voltage magnitude V_0 at each time step. A *Fourier* analysis of the voltage waveform over one cycle of the fundamental frequency gives the voltage magnitude at each time instant.

If *n* and *n-1* represent the current (t_n) and previous (t_{n-1}) time instants, then the resistance and the inductance are modified at each time step as

$$R_n = V_{n-1}^2 / P_0$$
$$L_n = (V_{n-1}^2 / \omega Q_0) \quad (10)$$

Here, the voltage magnitude V_{n-1} is calculated by performing a Fourier analysis over a running window of one cycle of the fundamental frequency. The subscript, $n-1$, indicates that instantaneous voltage values over one cycle of fundamental frequency ending at the $n-1$ time instant are used for the calculation of the voltage magnitude V_{n-1}. Thus, the load responds to the voltage magnitude at the previous time instant.

IV. Test System Description and Results

A. Test System Description

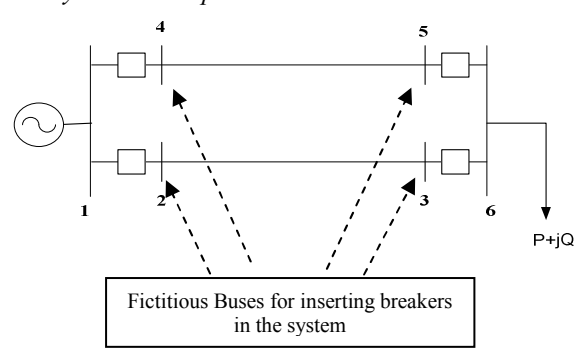


Fig. 3 Test system topology

Fig. 3 shows the test system topology used for continuation power flow analysis and instantaneous time domain simulations. It consists of two transmission lines in parallel,

Branches 2-3 and 4-5, with breakers at either ends. The load is a constant power factor PQ load and a constant voltage source is attached to Bus 1 assuming that Bus 1 is a swing bus.

B. Continuation Power Flow

Prior to running the instantaneous time domain simulations, a continuation power flow was run to determine the steady state loadability of the system. Fig. 2 shows the continuation power flow PV curves for the system with both lines in service and Branch 4-5 out. The steady state loading limit for the system with branch 4-5 out of service, $P^*_{d,4-5}$, was found to be approximately 3.53 pu and with both lines in service, $P^*_{d,normal}$, was around 7.05 pu. Note that the continuation power flow curves assume that the generator terminal voltage(not shown) is maintained constant at 1.0 pu.

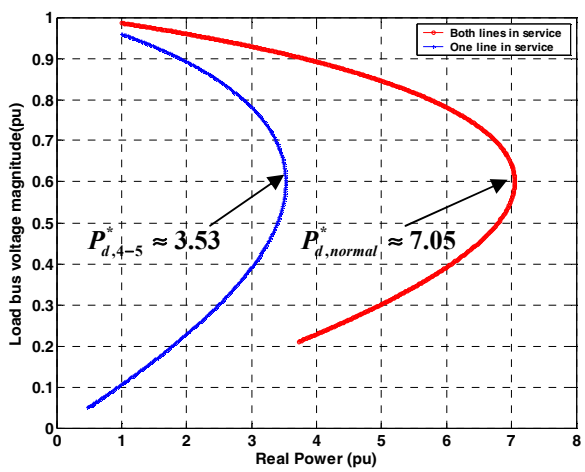


Fig. 4 Load Bus Voltage Magnitude variation from Continuation Power Flow

C. Instantaneous Time Domain Simulations- Bergeron Model

The instantaneous time domain simulations were done using Power System BlockSet which is a SIMULINK® based package for power system analysis. The breakers were modeled as resistances. The tripping of Branch 4-5 was used as a disturbance for the instantaneous time domain simulations. Numerous simulations were run by increasing the load on the system to see if it evolves from the pre-contingency operating point to post-contingency operating point following branch 4-5 tripping. The voltage dependent impedance load was configured in a parallel RL circuit as shown in Fig. 5. The load resistance and inductance were modified, as discussed earlier, to maintain the constant power load characteristic.

The instantaneous time domain simulation results with a distributed parameter Bergeron Model and a voltage dependent impedance load model are shown in Fig. 6-9. At 0.2 seconds, branch 4-5 tripping was initiated by sequential tripping of the breakers on either ends of the branch during their respective current zeros. The breaker on branch 5-6 was tripped first followed by the tripping of breaker on branch 1-4.

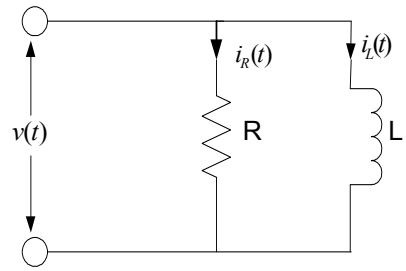


Fig. 5 Voltage Dependent Impedance Load modeled by a parallel RL circuit

The transition of the instantaneous load voltage from a pre-contingency steady state to post-contingency steady state, for a loading less than $P^*_{d,4-5}$, is shown in Fig. 6(a). It can be seen that the system evolves through the transient following branch 4-5 tripping and settles down at the post-contingency operating point. The post-contingency steady state voltage magnitude, shown in Fig. 7(a), matches with that from the continuation power flow.

Fig. 6(b) shows the load bus instantaneous voltage response when branch 4-5 is tripped beyond the post-contingency steady state loading limit. At this loading level, there is no power flow solution and the continuation power flow predicts a steady state voltage collapse. Following Branch 4-5 tripping, the voltages decay slowly over a span of 0.35 seconds before collapsing within half a second. The corresponding load bus voltage magnitude is shown in Fig. 7(b). The load bus phase angle response for line tripping beyond post-contingency steady state stability limit is shown in Fig. 8. The phase angle shows an oscillation near collapse conditions which suggests that the frequency does not remain constant. With the frequency not remaining constant, the phasor assumption loses its validity.

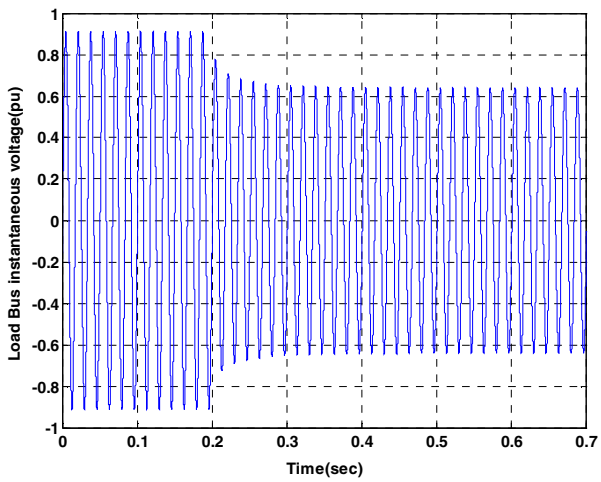
The instantaneous load power, shown in Fig. 9, indicates that beyond the post-contingency loading limit, if a line is tripped then the load power cannot be met due to a voltage collapse at the load bus.

D. Instantaneous Time Domain Simulations- Lumped Model

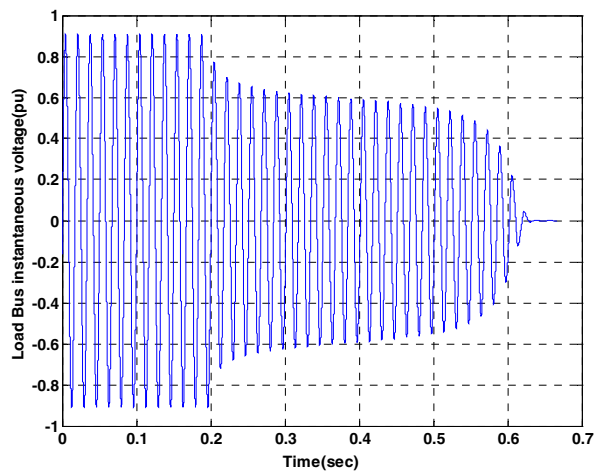
The response of the system using a lumped parameter model is shown in Fig. 10-11. These plots show that the qualitative nature of the response for this particular line is similar to the Bergeron Model. In fact, the two different models give nearly the exact same results.

V. Conclusions

This paper presented the confirmation of the steady state voltage collapse using instantaneous time domain simulation. Results on the two bus test system show that before the steady state voltage collapse point, the system can handle the line tripping disturbance. It evolves through the transient and settles at the post-contingency steady state operating point. Beyond the steady state loading limit, a voltage collapse occurs which can be tracked using instantaneous time domain simulations.

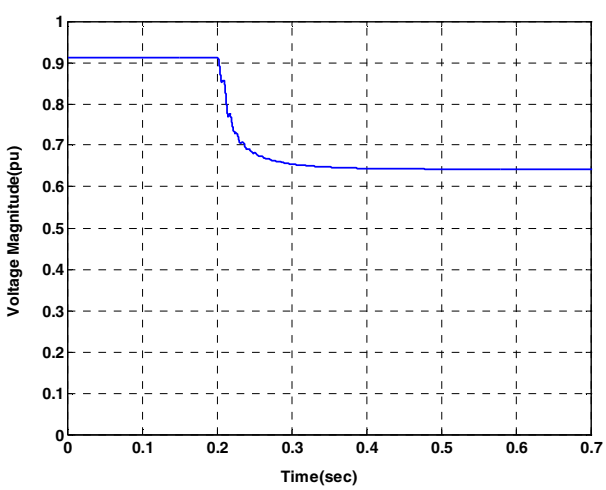


(a)

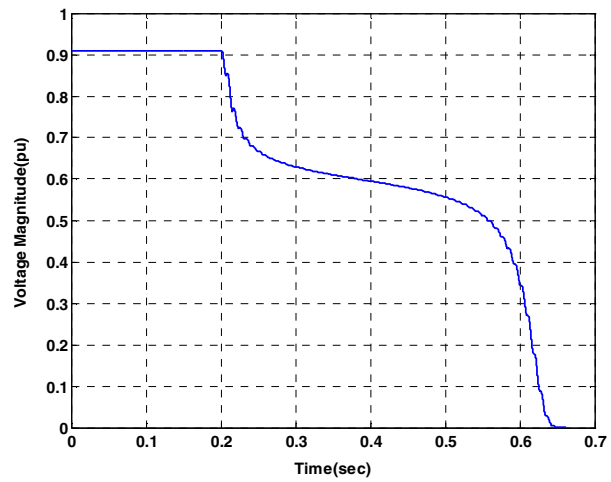


(b)

Fig. 6 Load Bus Instantaneous Voltage-Bergeron Model: (a) $P_d = 3.5pu$ (b) $P_d = 3.55pu$



(a)



(b)

Fig. 7 Load Bus Voltage Magnitude-Bergeron Model: (a) $P_d = 3.5pu$ (b) $P_d = 3.55pu$

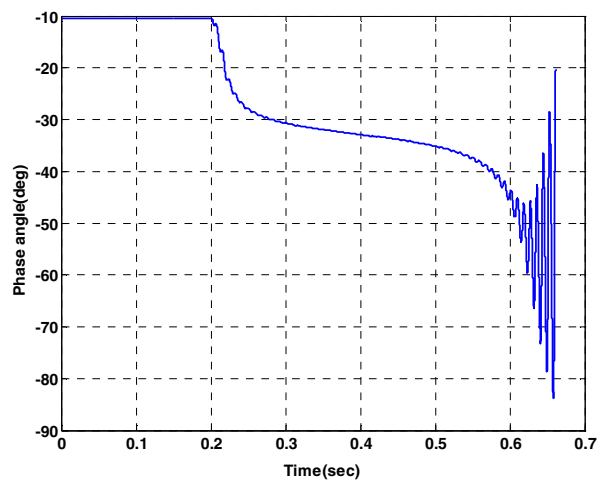


Fig. 8 Load Bus Phase Angle Oscillation at Collapse Conditions. $P_d = 3.55pu$

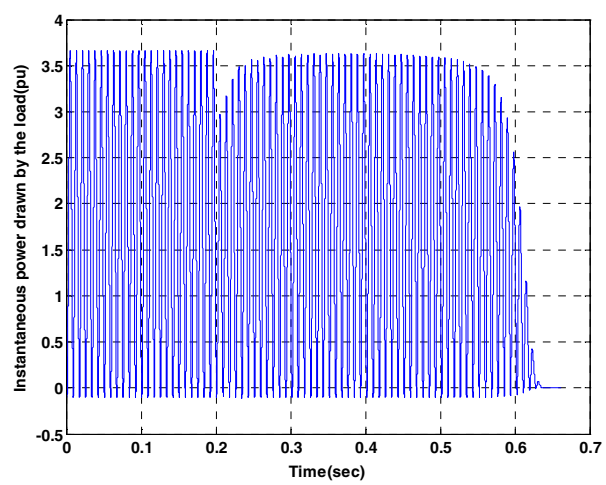


Fig. 9 Instantaneous Load Power: The load power collapses due to voltage collapse $P_d = 3.55pu$

The source model used for the simulations was a constant voltage source which is an unrealistic model. In the future, we intend to use three phase generator models.

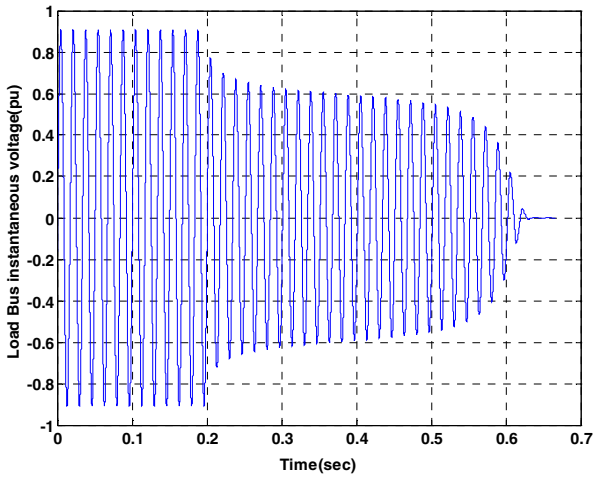


Fig. 10 Lumped parameter model: Load Bus Instantaneous Voltage $P_d = 3.55\,pu$

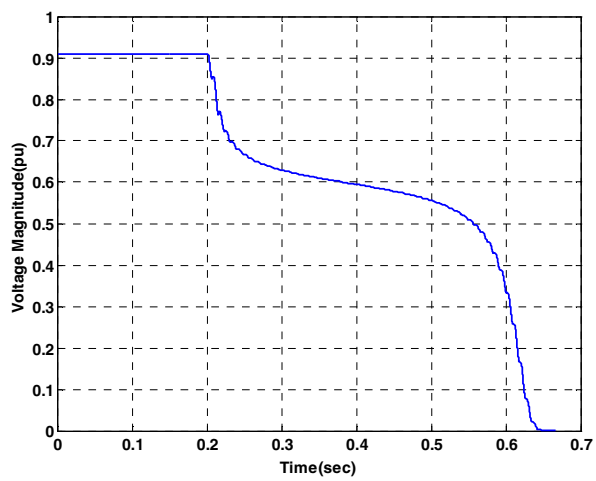


Fig. 11 Lumped Parameter Model: Load Bus Voltage Magnitude $P_d = 3.55\,pu$

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VII. Biographies

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Alexander J. Flueck received the B.S. degree (1991), the M.Eng. degree (1992) and the Ph.D. degree (1996) in electrical engineering from Cornell University. He is currently an Associate Professor at Illinois Institute of Technology in Chicago. His research interests include transfer capability of large-scale electric power systems, contingency screening of multiple branch or generator outages with respect to voltage collapse, continuation methods for parameterized nonlinear systems, transient stability and parallel simulation of power systems via message-passing on distributed-memory computer clusters.