Mihai Anitescu, 01/20/2011

Stat 310, Part II, Optimization. Homework 2.

Problem 1: (computation; trust region Steihaug CG)

Implement the trust-region CG-Steihaug Algorithm 7.2 from the textbook (in conjunction with the trust-region management algorithm 4.1). Choose *Bk* to be the exact Hessian. Use the same ε_k rule from Algorithm 7.1 Apply the code to solve Fenton's function starting at both [3 2] and [3 4]. Report the total number of matrix-vector multiplications and the total number of function evaluations.

(Optional). Apply the algorithm to the "cute" function from the previous homework. Experiment with increasing values of the size of the problem (start with 10). Choose the stopping tolerance on the size of the gradient to be relatively large (say 1e-3 to 1e-4, but experiment with it as well).

Problem 2: (theory, linear conjugated gradient)

Use the properties of the conjugated gradients methods for linear positive definite systems proved in class (or in the textbook) to prove that

$$\alpha_k = \frac{r_k^T r_k}{p_k^T A p_k}; \quad \beta_{k+1} = \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}$$

In turn, this proves the correctness of the Algorithm 5.2 (the "definitive" version of conjugated gradients method). Hint: the most important properties will likely be (with the labeling from my slides).

$$p_{k+1} = -r_{k+1} + \beta_{k+1} p_k, \quad k = 0,1,\ldots (K3)$$

$$r_k^T p_i = 0, \quad i = 0,1,\ldots,k-1, k = 1,2,\ldots (C3)$$

$$r_k = r_{k-1} + \alpha_{k-1} A p_{k-1}, \quad k = 0,1,\ldots (C2)$$

$$r_k^T r_i = 0, \quad 0 \le i \ne k (K1)$$

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Problem 3: (theory, linear conjugated gradient). Problem 5.7 from the textbook.

Let $\{\lambda_i, v_i\}, i = 1,2,\ldots n$, be the eigenpairs of the symmetric matrix A. Show that the eigenvalues and the eigenvectors of the matrix $\left[I + P_k(A)A\right]^T A\left[I + P_k(A)A\right]$ are $\lambda_i\left[I + P_k(\lambda_i)\lambda_i\right]^2$ and v_i, respectively.