Getting Out More from the Power Grid--Mathematical Challenges of Energy Systems

Mihai Anitescu INFORMS 2015

M2ACS colleagues @ANL: Emil Constantinescu, Victor Zavala, Cosmin Petra Shri Abhyankar, Francois Gilbert (ANL)

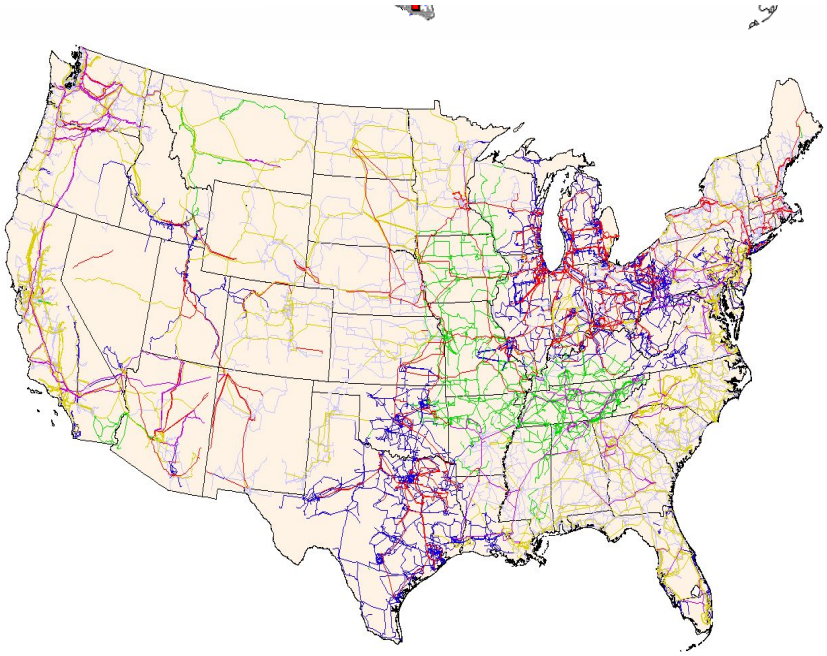
M2ACS colleagues: Henry Huang, Mahantesh Halappanavar, Alex Tartakovsky (PNNL) Chris De Marco, Bernie Lesieutre (Wisconsin), Jonathan Weare (UC), Jean-Paul Watson (Sandia)

Other Contributors: Miles Lubin (MIT), Dan Molzahn (Argonne)

1. The Problem

Complexity of the power grid

- **O**(10^**5**) transmission lines,
- **O**(10^**4**) generation nodes, and
- **O**(10^**4**) demand nodes.
- Distribution network:**O**(10^**8**) components.
- Reliability: – combinations of failures (2003 blackout)
- time scale ranges: 1e-6 in power electronics to decades (1e 10s) in transmission planning.



U.S. Electrical Transmission System

Physical Constraints

State Equations

- ~ 0.1 second dynamics: swing equations; DAEs.
- Steady-State: Power Flow Equations, Nonlinear, Quadratic (ACPF)
- Their Linear Approximation: DC Power Flow.
- Sometimes, Power Flow Only.

$$M\dot{x} = f(x, y, p)$$

$$0 = g(x, y, p)$$

Decisions

- Generation Levels
- Generation Schedules
- Asset Location
- System states (angles, inertia; for inverse problems)

Decision Making Structure and Optimization Tasks

Transmission/Generation Expansion: ISO, Yearly, MILP

Planning

↓

Unit Commitment: ISO, Daily, DC Flow, MILP

Day-Ahead Bidding: GENCOs/Utilities, Daily, LP/QP

Economic Dispatch: ISO, 5 Minutes, DC Flow, LP/QP

Real-Time Bidding: GENCOs/Utilities, 5 Minutes, LP/QP

Markets

↓

AC Power Flow: ISO, 1-2 Minutes, NLP

State Estimation: ISO, 1-2 Minutes, QP/NLP

Generation Control: GENCOs, Seconds, QP/NLP

Voltage and Dynamic Stability: ISO, MilliSeconds, No Optimization

Energy Management: Utilities/Consumers, Seconds/Minutes, LP/QP/NLP

Control

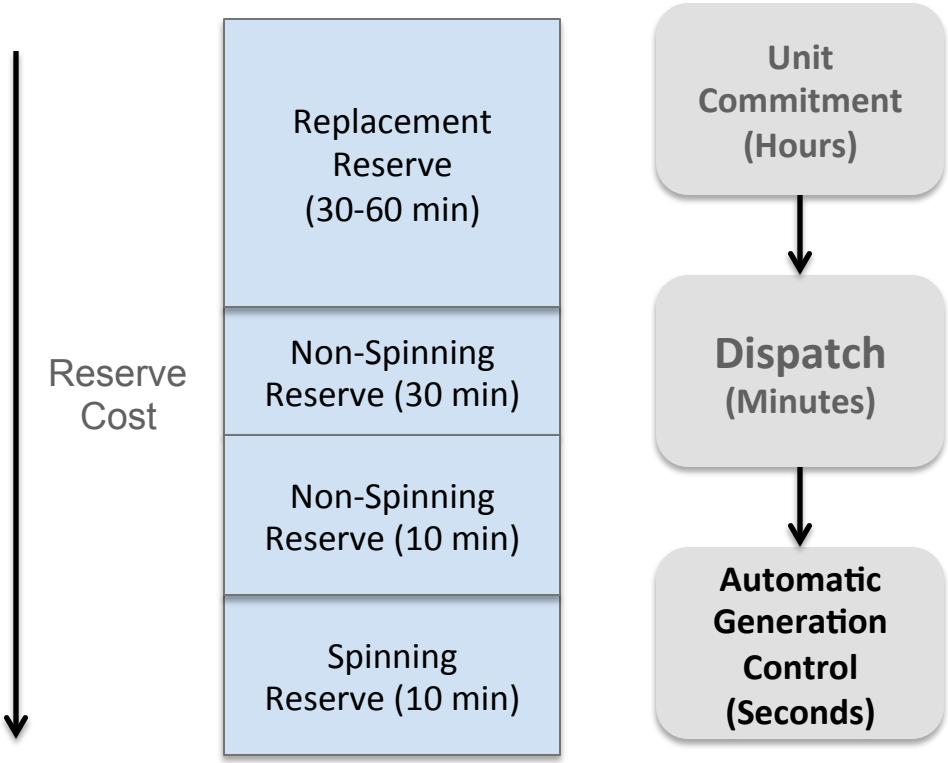
Current Features and State of the Analysis

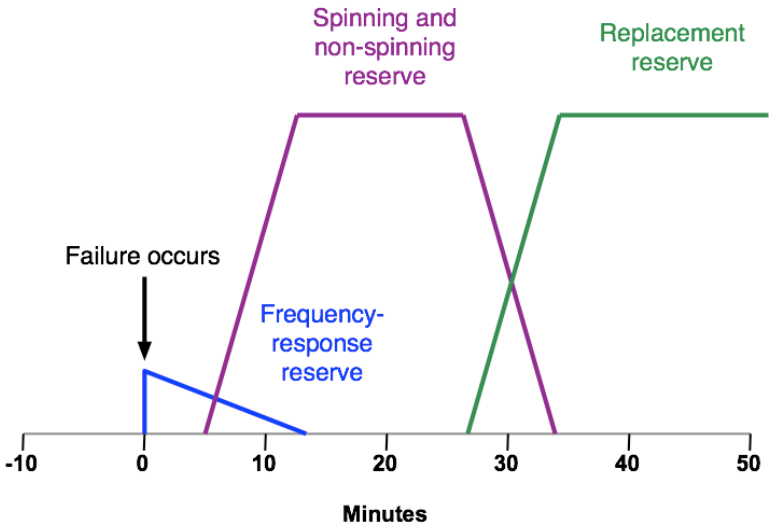
- Problem: grid operates cheaply and safely, including following acts-of-God
- Same physical model, but historical separation between analyses.
- Direct transcription is beyond any compute power in 2050.
- Engineering judgment is used offline for reduction. Eg.
  - Scenarios to be used in transmission expansion planning.
  - Static constraints for post-contingencies states.
  - Screening of the possible contingencies in security-constrained problems.
- Recourses are generally implemented with operator-in-the-loop.
- While perhaps a bit dated, it served us well. US has one of the lowest electricity prices and higher reliabilities.

However, is it really a solved problem?

- The 2003 blackout affected 50 million people
  - Task Force Report: "At the completion of this report, the modeling …. was continuing.."
- The 2013 Superbowl blackout (CNN)
  - "Manufacturer blames Super Bowl outage on incorrect setting "
  - "System operators essentially put the relay's trip setting too low,"
- The 2014 February Gas Shortage/Curtailment (Forbes) … In TEXAS (and California, Mass)
- The 2015 White House Blackout … (CBSDC)
  - "Explosion at Power Plant Responsible for D.C. Area Outages " -- grid operations should ensure that no one event to an asset results in blackout .

Why do these things not happen more often? Hierarchy of Reserves





- Margins and off-market actions probably cost Bs of Dollars
- And, the same reserve level may not hold for the NEW grid

Getting out more of the power grid: converging optimization and transient stability

- You say: “give me the best energy portfolio-”dispatch”-- but make sure that I survive transient from any one contingency “.
- Computer sees:

$$\min \quad C(p)$$

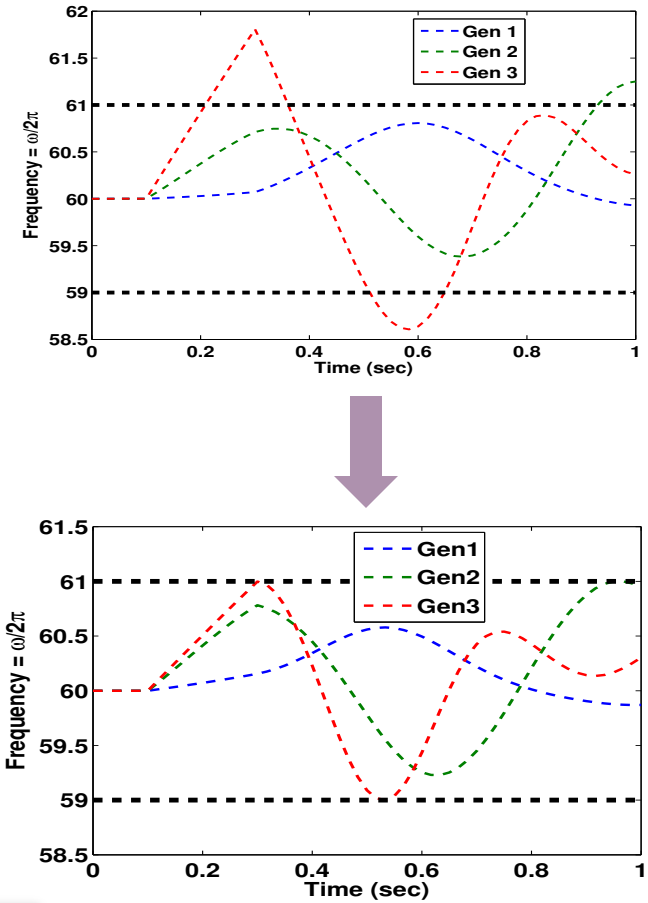
s.t.

$$g_s(p) = 0$$
$$h_s(p) \leq h^+$$
$$p^- < p < p^+$$

Optimal Power Flow

$$M\dot{x} = f(t, x, y, p), \quad x(t_0) = I_{x0}(p)$$
$$0 = g(t, x, y, p), \quad y(t_0) = I_{y0}(p)$$
$$h(x(t), y(t)) = 0, \quad \forall(t) \quad \text{path constraints}$$

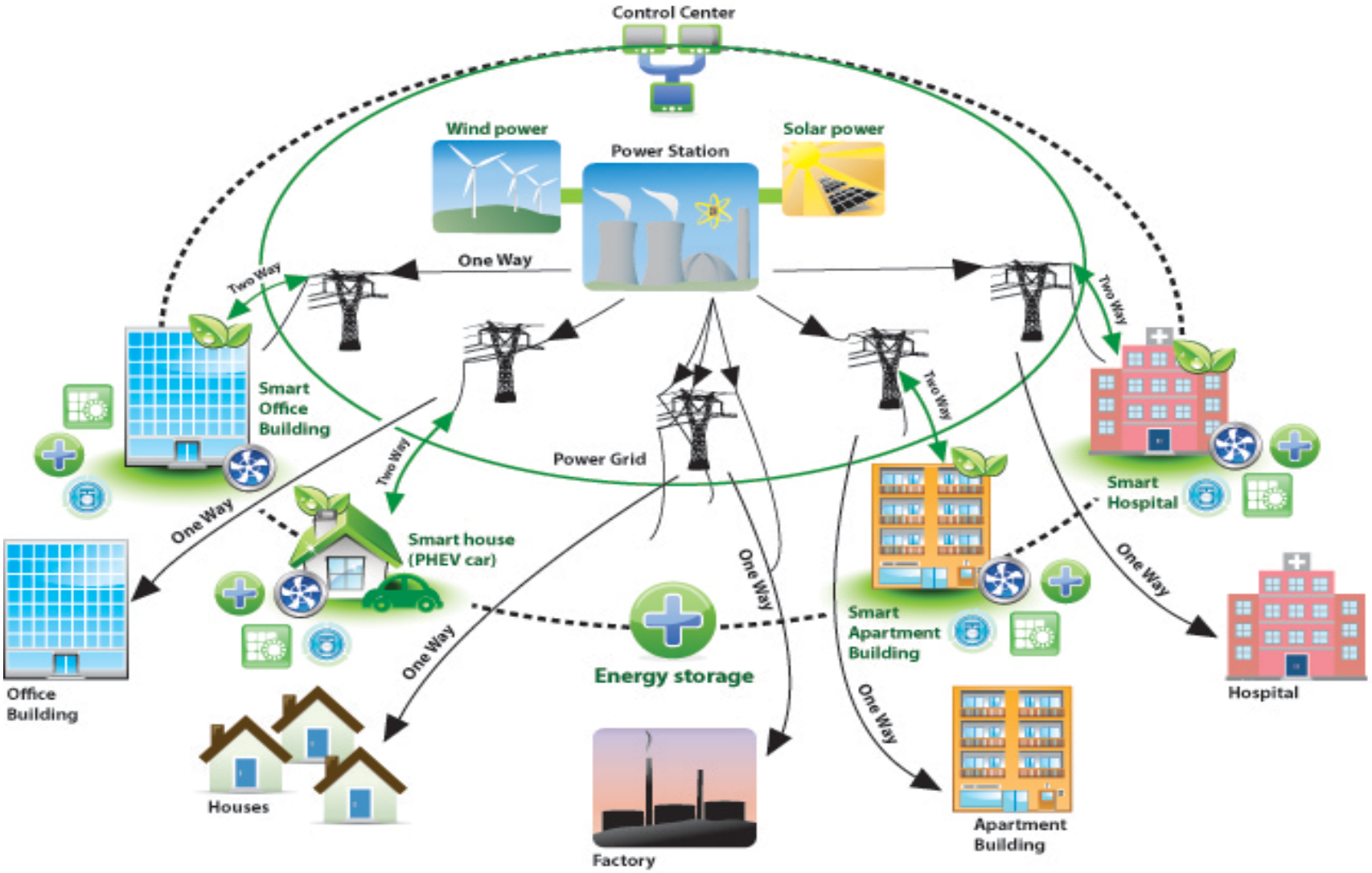
Transient Post-Contingency Stability



If the dynamic tool cannot compute derivatives, this becomes dim(p) time harder.

S. Abhyankar, E. Constantinescu, and M.A.

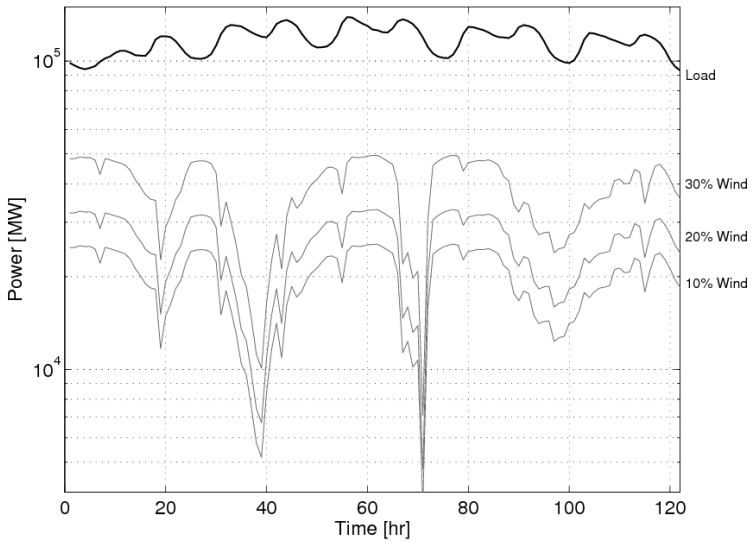
Changing nature of the power grid



- New usage drivers (PHEV, bidirectional, smart grid, smart buildings) are changing completely the response of the system

Increased Exposure to Ambient Conditions

- Plans are for US to have 20% renewable energy by 2030.
- To operate the grid, we need 1-24 hour resource forecasts.
- Weather is chaotic, and sparsely measured.
- We need to represent its effect probabilistically (at least, if we aim for 1-2% errors)

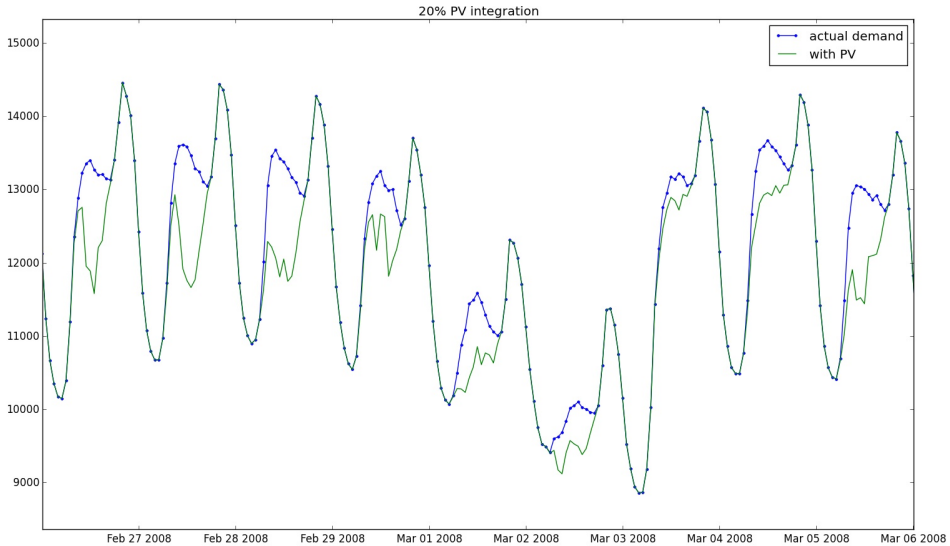


Wind Power and Demand

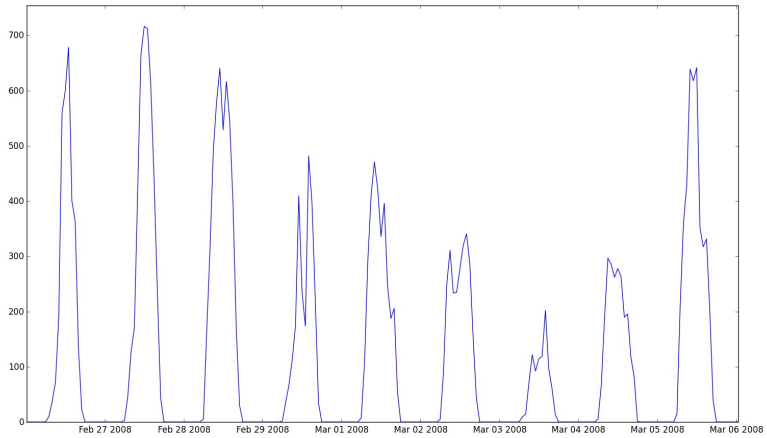
Wind Power Profiles

Changing Demand Patterns

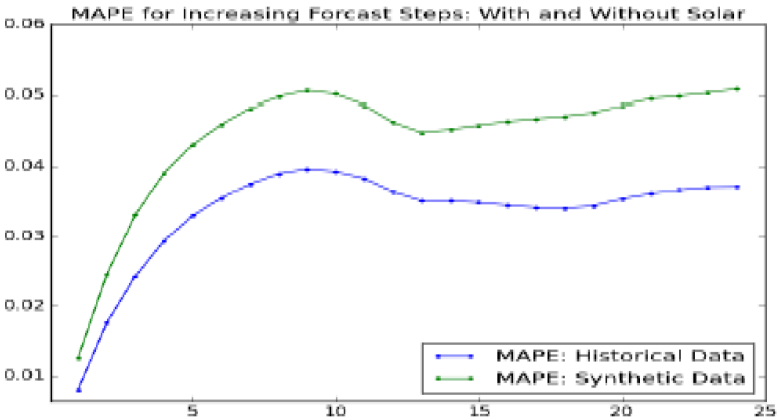
- Assuming 50 % solar penetration.
- More dynamic content.
- The error would have to increase – needing larger reserves.



Change in demand (above)
Forecast Error (below)

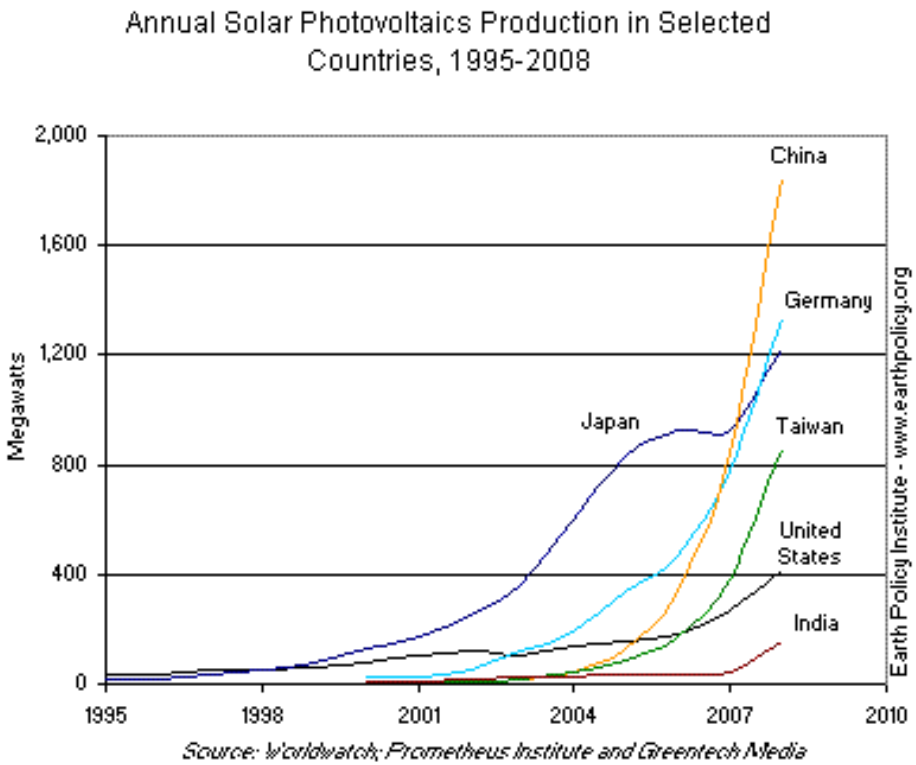


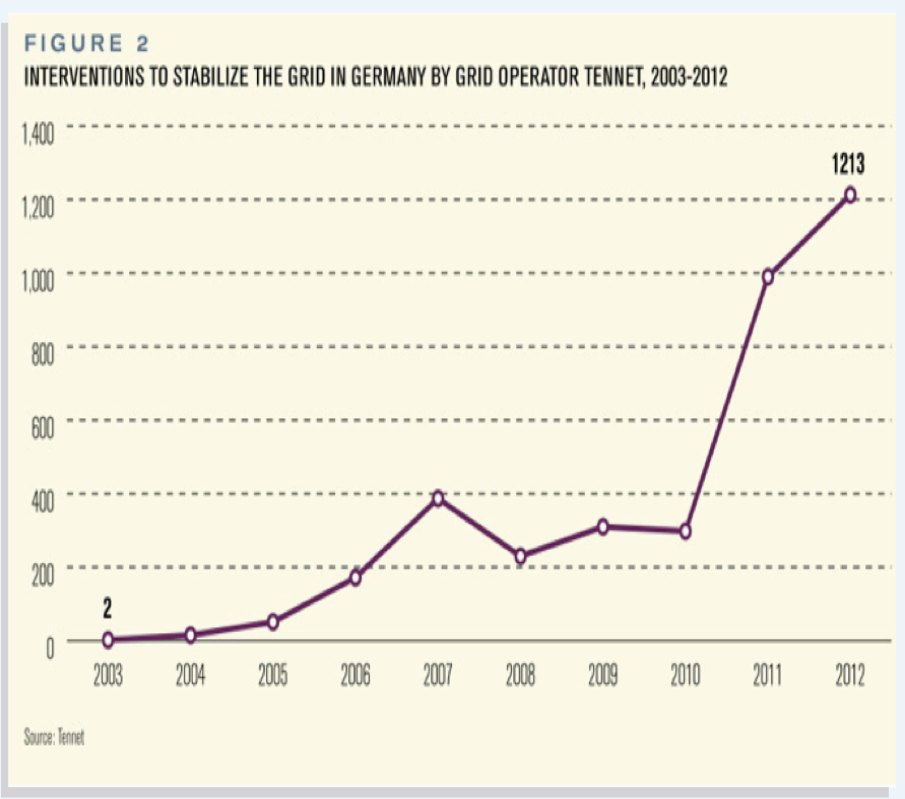
Solar Radiation winter 2008 in the COMED AREA



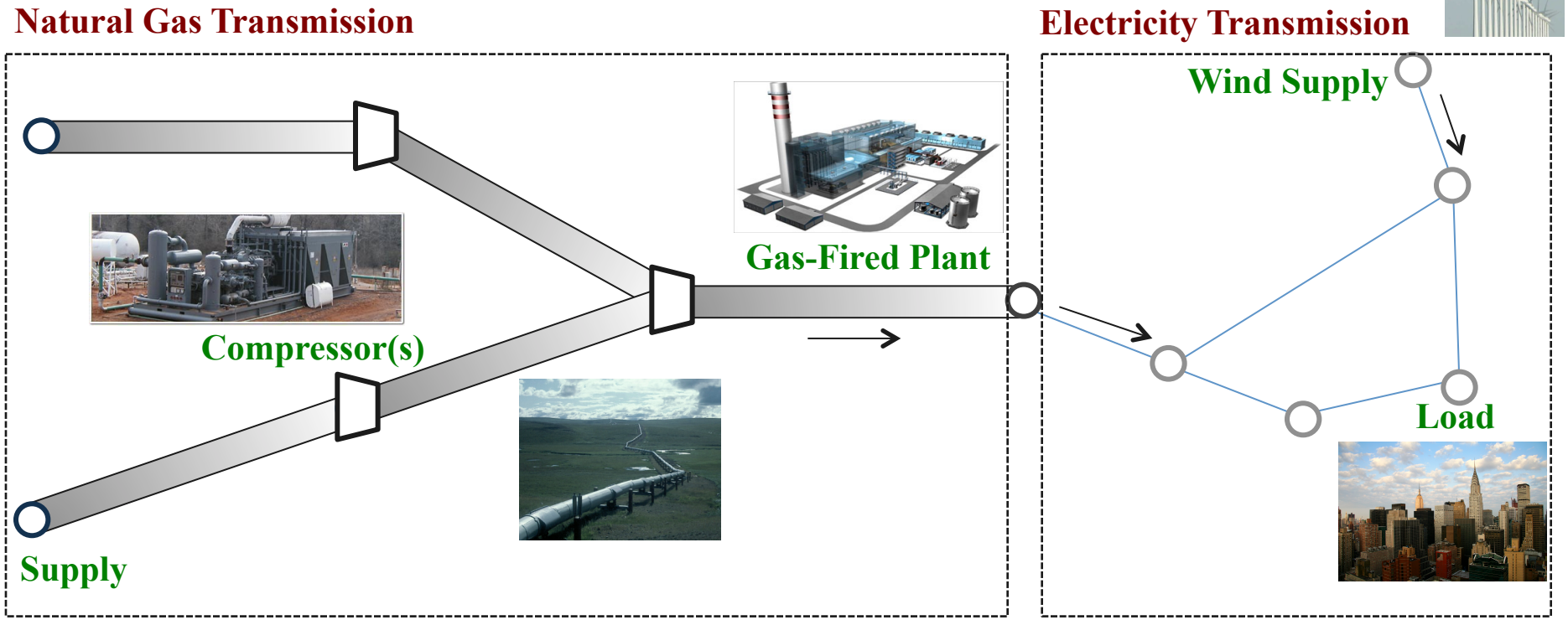
F. Gilbert and M.A.

Maybe things will turn out all right anyways? Doubtful -- Germany Energy Landscape





Increase Natural Gas Usage ➔ Very different issues



- Gas Pipelines Provide Storage Capacity to Mitigate Fluctuations (Ramps & Capacity)
- But, Gas Travels at 30-50 mph (Storage Has to be Built up Well in Advance)
- Aggressive Gas Withdrawals Cascade Upstream the Pipeline (Compromises Stability)
- Shortages of Natural Gas for Power Plants Not Uncommon

Complexity Drivers over next 15 years

- More pro-active consumer ("prosumer")
- Significant increase of uncertainty in supply, demand, and inertia due to renewables expansion and prosumers.
- Increased Reliance on Distributed Generation and Natural Gas.
- Changes in Power Flow patterns
- Increased Dynamic Ranges, shorter analyses time scales and real times.
- Rapidly Increasing Data Streams.
- New Devices. (PHEV, PMUs, HVDC, mass and local storage)

- Larger state spaces, smaller time scales, more physics, more scenarios,

Total US R&D Investment

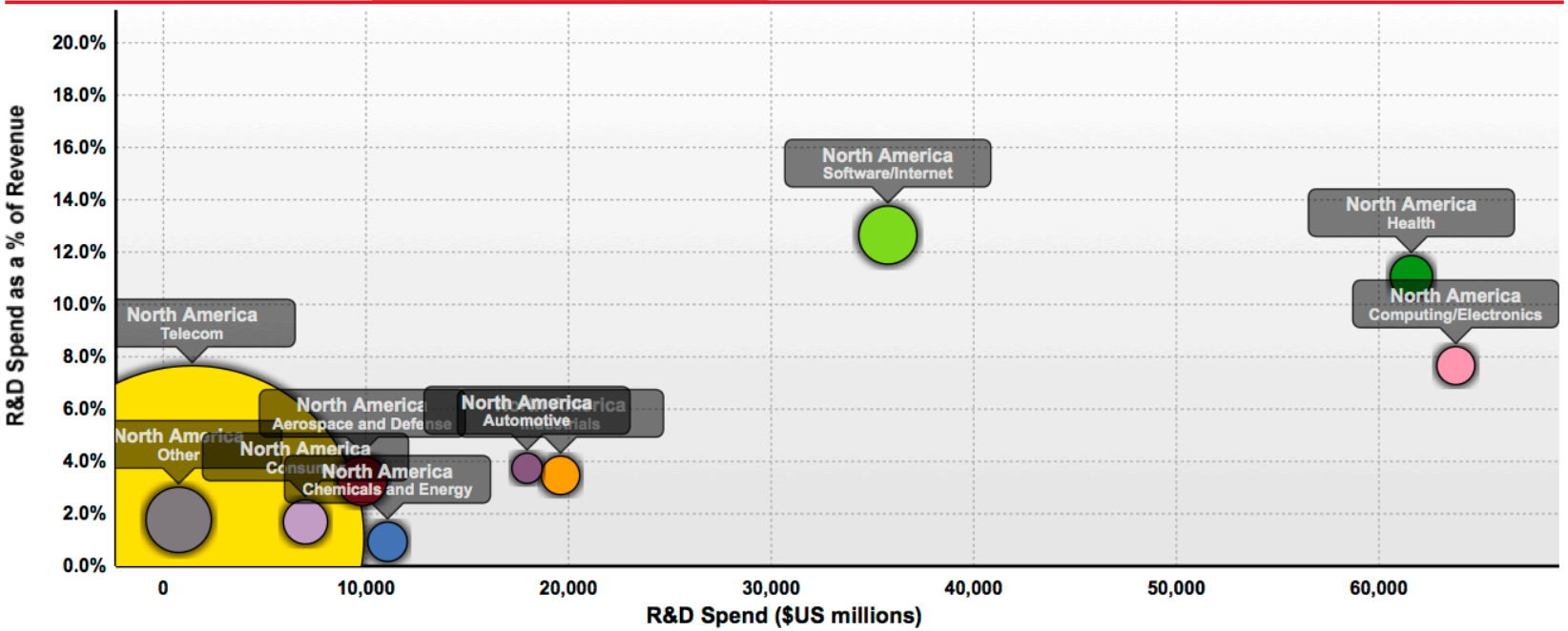
Formerly Booz & Company

The Global Innovation 1000: **Comparison of R&D Spending by Regions and Industries**

HELP

This graph allows you to compare R&D as a percentage of revenue (R&D "intensity") and total R&D spend by regions and industries as it changes from 2004-2011.

Analyze Region | Analyze Industry | Legend



2. Unusual/unexpected mathematics

Classical OPF Problem

Cost

$$\min \sum_{k\in\mathcal{G}} f_k\left(P_{Gk}\right) = \sum_{k\in\mathcal{G}} c_{2k}P_{Gk}^2 + c_{1k}P_{Gk} + c_{0k}$$

Engineering Constraints

subject to

$$P_{Gk}^{\min} \le P_{Gk} \le P_{Gk}^{\max}$$

$$Q_{Gk}^{\min} \le Q_{Gk} \le Q_{Gk}^{\max}$$

$$\left(V_k^{\min}\right)^2 \le V_{dk}^2 + V_{qk}^2 \le \left(V_k^{\max}\right)^2$$

$$|S_{lm}| \le S_{lm}^{\max}$$

Physical Laws

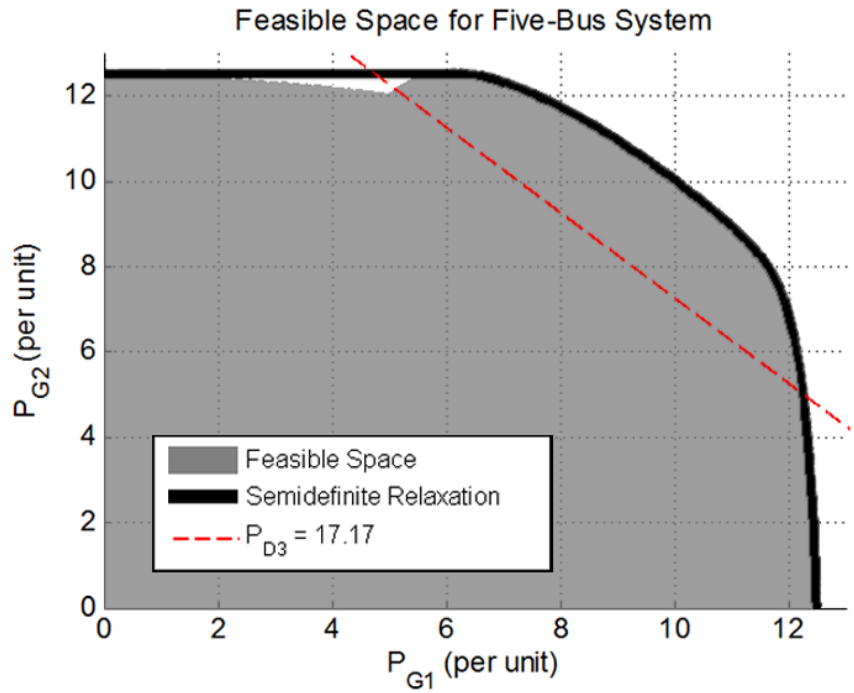
$$P_{Gk} - P_{Dk} = V_{dk}\sum_{i=1}^{n}\left(\mathrm{G}_{ik}V_{di} - \mathrm{B}_{ik}V_{qi}\right) + V_{qk}\sum_{i=1}^{n}\left(\mathrm{B}_{ik}V_{di} + \mathrm{G}_{ik}V_{qi}\right)$$

$$Q_{Gk} - Q_{Dk} = V_{dk}\sum_{i=1}^{n}\left(-\mathrm{B}_{ik}V_{di} - \mathrm{G}_{ik}V_{qi}\right) + V_{qk}\sum_{i=1}^{n}\left(\mathrm{G}_{ik}V_{di} - \mathrm{B}_{ik}V_{qi}\right)$$

Rectangular voltage coordinates: $\vec{V}_i = V_{di} + jV_{qi}$

SDP Relaxations

- SDP relaxations were shown to be exact in many cases (Javaei and Low)
- Higher order moment relaxations of Laserre have been shown to significantly fill the gap (Molzahn, Lesieutre, Hiskens).
- However, practical estimates of the order of relaxations are lacking.
- We do not have sufficiently fast SDP implementations.



Source: D. Molzahn (ANL)

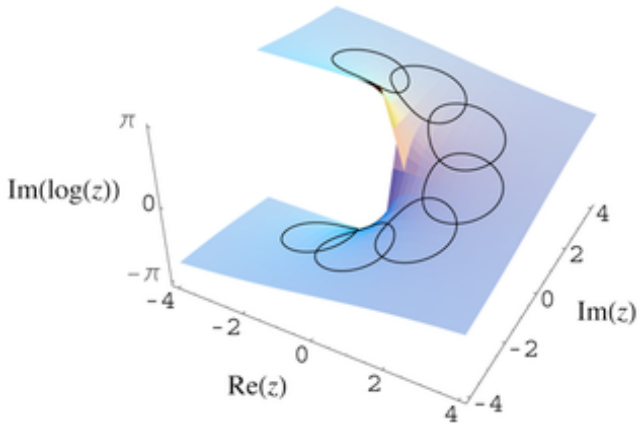
Holomorphic Embedding Analytic Continuation for global solution of power flow equations

- Embed the Quadratic Equations of ACPF in a holomorphic function.
- At s=0, trivial solution, at s=1 the solution you want.
- Approximate at s=0 efficiently with a series (polynomial, or RF approx)
- Do *analytic* continuation.
- Patent-protected (one PES meeting paper)
- 2012: (Battelle) "Battelle Acquires Equity Position in Gridquant"

Wikipedia (HELM)

$$\Sigma_k Y_{ik} V_k + Y_i^{sh} V_i = \frac{s_i^*}{v_i^*}$$

$$\sum_k Y_{ik} V_k(s) + Y_i^{sh} V_i(s) = s \frac{s_i^*}{v_i^*(s^*)}$$



A. Trias (GRIDQUANT)

Random Graphs (RG)

- Essential if aim towards expert free planning (10s yrs horizons)
- Current RG models do not reproduce power networks well

Graph type	Vertices, edges	Clust coeff	ASPL	Diameter
Western US	4941, 6594	0.0166	21.75	
Random (of WECC)	4941, 6594	**0.00049**	**10**	
Nordic	4789, 5571	0.0801	18.99	
Random (of nordic)	4789, 5571	**0.00054**	**8.7**	
Eastern inter	49597, 62985	0.071	35.8	96
Pref. attach (eastern)	49597, 62966	**0.0006**	**7.2**	18
Small world p=0.0882 (eastern)	49597, 62906	**0.27**	36.6	96

M Halappanavar, PNNL. (ASPL== average shortest path length)

Statistical Mechanics of Cascade Failures

- Cascades are very rare and very costly events.
- 2003 EIC outage: 50million people w/o electricity.
- We do not have enough data for a data-driven model.
- There is a lot of work in descriptive cascade models.
- Is there a probabilistic, first-principles cascade model?

- Is there an energy function?
- This would allow chemistry-like statistical mechanics.
- It would require good noise models.

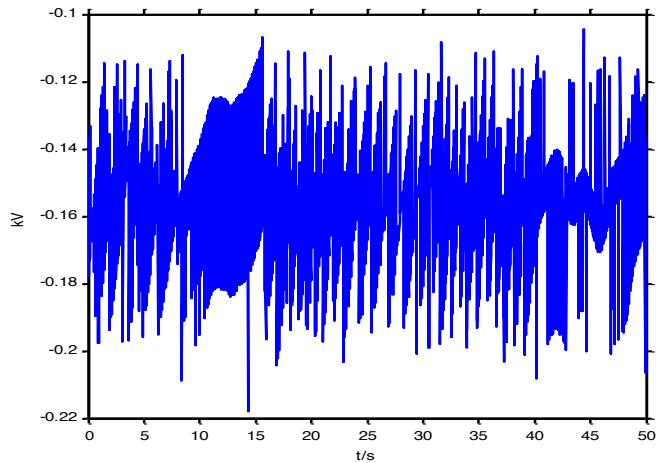
$$\dot{\mathbf{x}} = \mathbf{A}\nabla_{\mathbf{x}}\Phi(\mathbf{x}) + \zeta$$

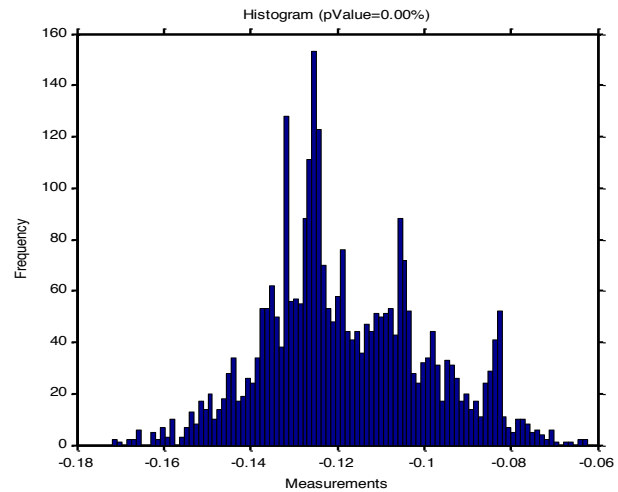
- Extension of Hamiltonian dynamics.
- A negative sdf, invertible,
- Does not model controls.

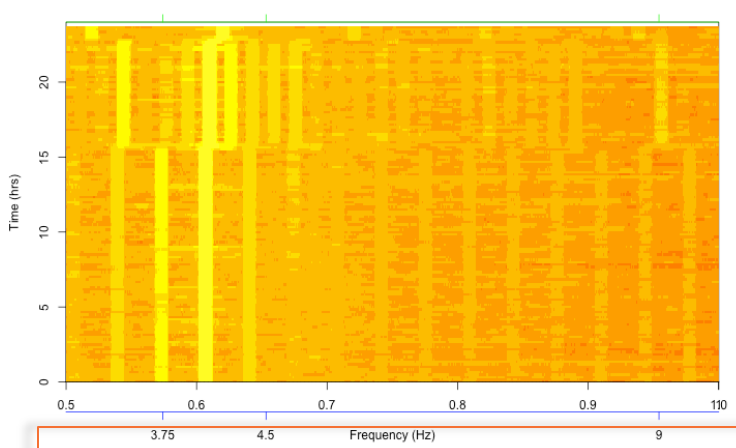
Courtesy: Chris de Marco, Wisconsin

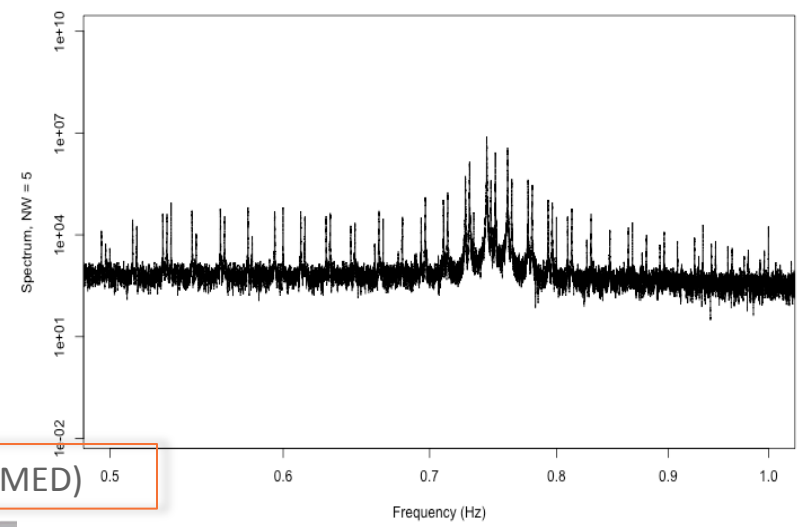
Statistical Models of Sub-Second Noise

- Evidence of non-Gaussian, heavy-tailed, complex spectrum.
- Too Complex for AR models to cut it.









H. Huang,(PNNL) C. Haley, M.A., D. Schooley (COMED)

Summary: Unexpected/Unusual Mathematics

- SDP relaxation very promising.
- Analytic continuation is intriguing
- Good random graph models for grid are a challenge
- Hamiltonian-based cascade models
- Sub-second noise is more complex than one might think.

3. Stochasticity/Probabilistic Modeling

Sources of Uncertainty

- Reformulations: there appears to be a big push for probabilistic modeling and decision making in light of increased renewable penetration.
- See the FERC Technical Conferences.
- However, such techniques do not appear to be used in a major way operationally (at all?)
- Upfront cost increases, but so does reliability.
- To model:
  - Weather
  - Demand
  - Sub-second noise.
  - Fuel Cost
  - Reliability
  - ….

Stochastic unit commitment/economic dispatch w. wind

$$\min \quad \mathrm{COST} = \frac{1}{N_s}\sum_{s\in S}\left(\sum_{j\in N}\sum_{k\in T} c^p_{sjk} + c^u_{jk} + c^d_{jk}\right)$$

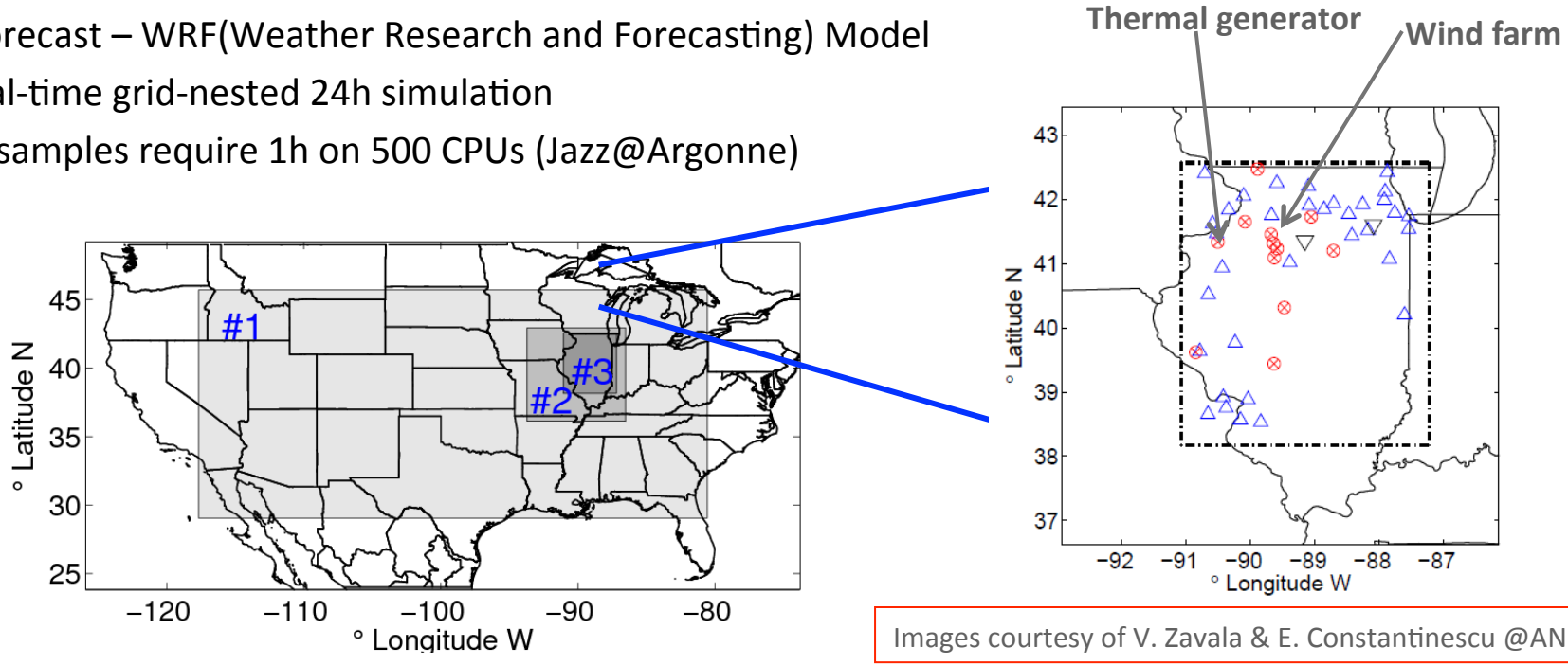
$$\text{s.t.} \quad \sum_{j\in N} p_{sjk} + \sum_{j\in N_{wind}} p^{wind}_{sjk} = D_k, s\in S, k\in T$$

$$\sum_{j\in N} \overline{p}_{sjk} + \sum_{j\in N_{wind}} p^{wind}_{sjk} \geq D_k + R_k, s\in S, k\in T$$

ramping constr., min. up/down constr.

- Most data approaches lose coherence after 2-3 hrs.
- Numerical weather prediction (NWP) expensive
- NWP was never meant to simulate < 1 km resolution.

- Wind Forecast – WRF(Weather Research and Forecasting) Model
  - Real-time grid-nested 24h simulation
  - 30 samples require 1h on 500 CPUs (Jazz@Argonne)



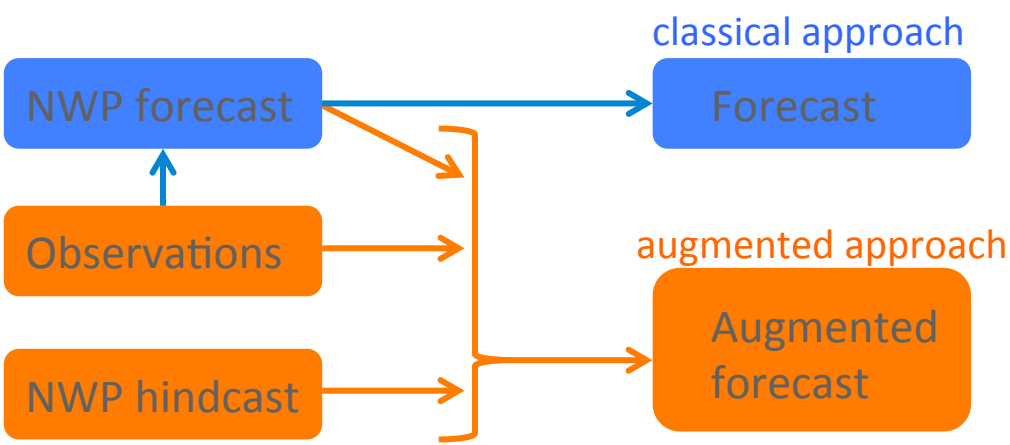
Images courtesy of V. Zavala & E. Constantinescu @ANL

Hybrid NWP/Data Models

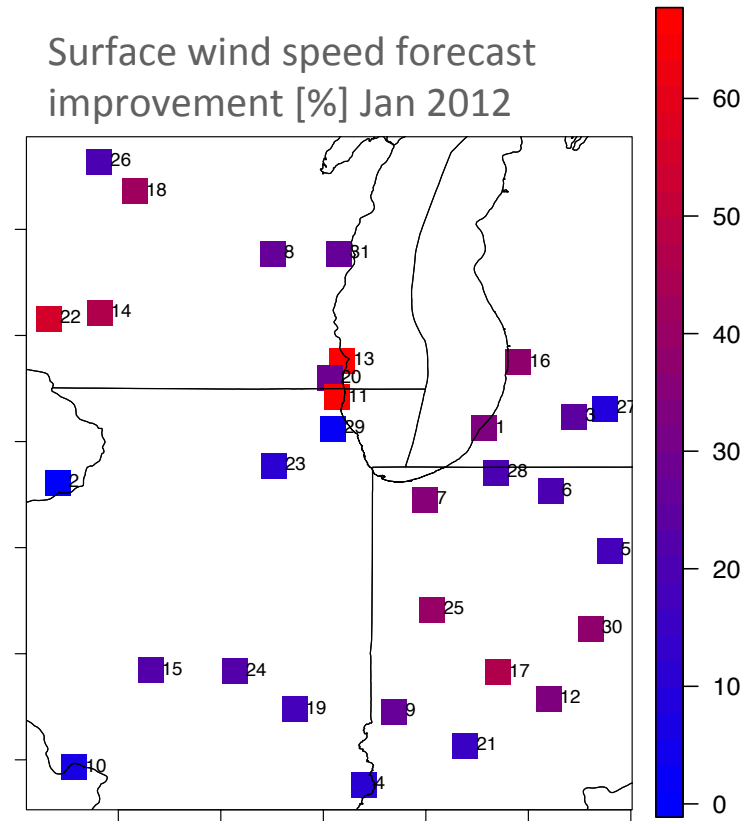
- joint distribution of weather simulations (NWP) and observations (Obs)

$$\begin{pmatrix} Y_{\text{Obs}} \\ Y_{\text{NWP}} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{\text{Obs}} \\ \mu_{\text{NWP}} \end{pmatrix}, \begin{pmatrix} \Sigma_{\text{Obs}} & \Sigma_{\text{Obs,NWP}} \\ \Sigma_{\text{Obs,NWP}}^T & \Sigma_{\text{NWP}} \end{pmatrix} \right)$$

- wind speed prediction fusing NWP and Data

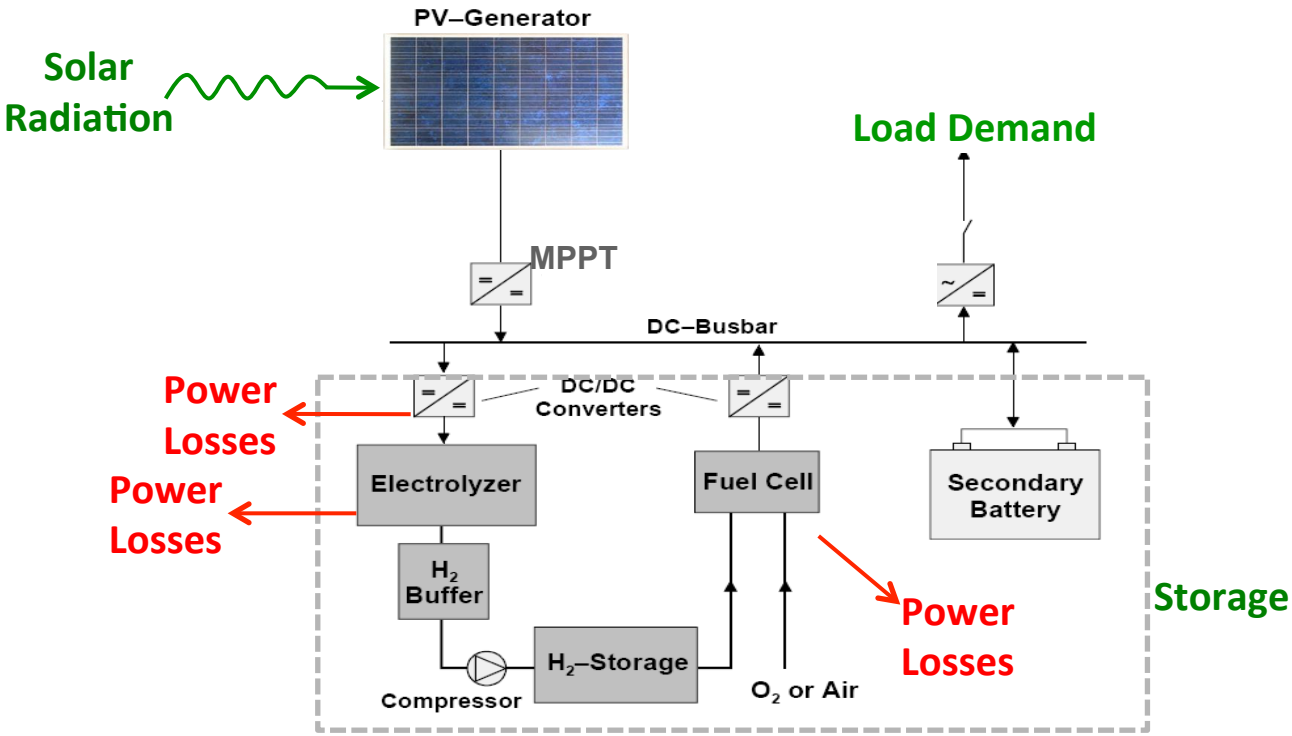


Surface wind speed forecast improvement [%] Jan 2012



Julie Bessac, Emil Constantinescu, and Mihai Anitescu; *Stochastic simulation of predictive space-time scenarios of wind speed using observations and physical models*, 2015.

Hybrid Photovoltaic-H2 System



- **Operating Costs Driven by <u>Uncertain</u> Radiation** *Ulleberg, 2004*
  - **Performance Deteriorates by Multiple <u>Power Losses</u>**

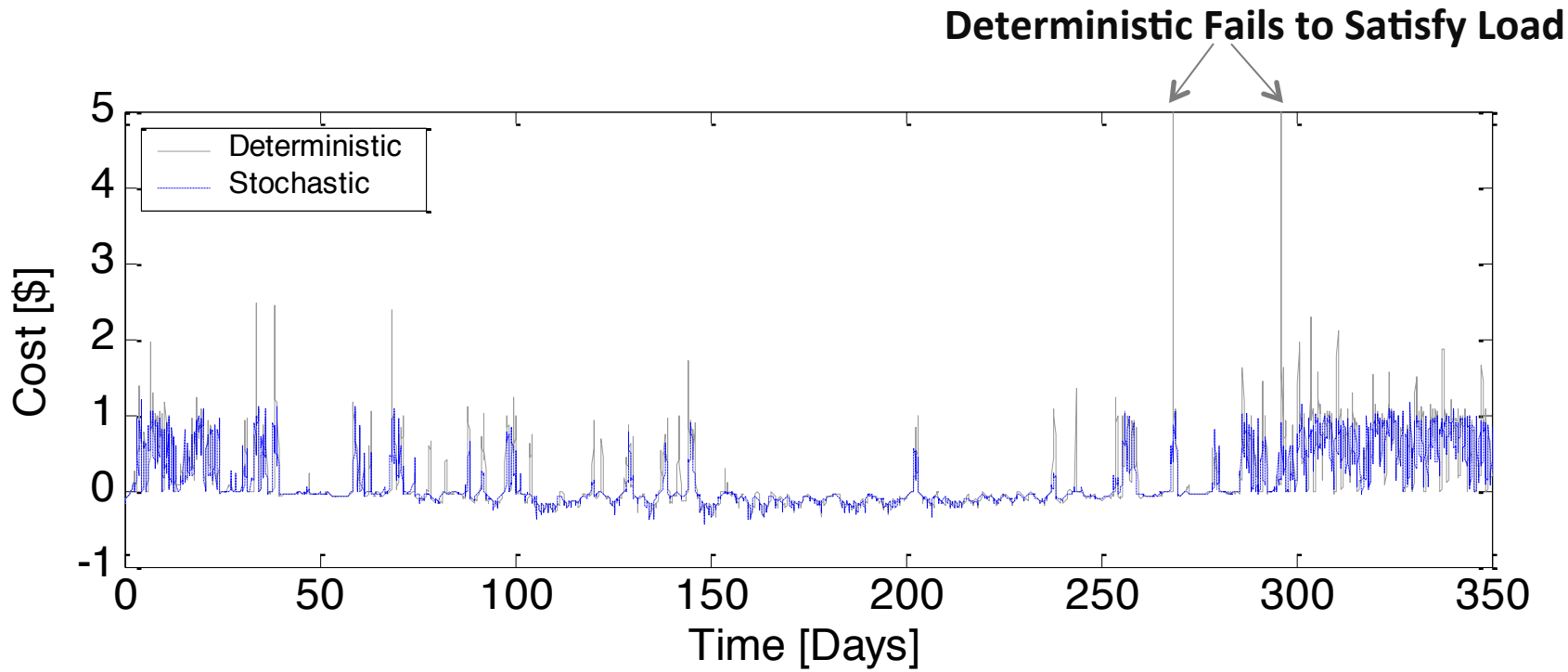
Model:

Problem: Operate to Minimize Operating Costs + Maximize H_2 Production

<u>Subject to:</u> Energy Balances; State-of-Charge, Fuel Cell and Electrolyzer Limits

Hybrid Photovoltaic-H2 System

Load Satisfaction Deterministic ("Optimization on Mean") vs. Stochastic



Therefore, the alternative to stochastic programming can turn out infeasible !!

Handling Stochastic Effects Particularly Critical in Grid-Independent Systems

V. Zavala, M.A, T. Krause

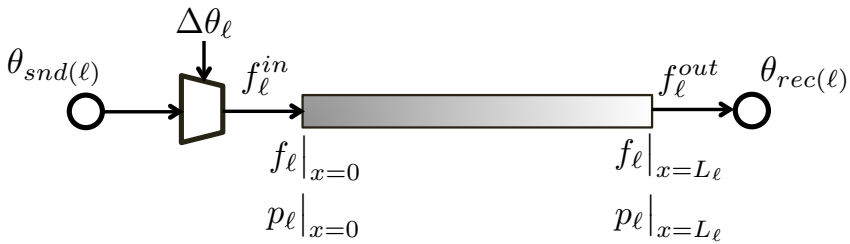
Gas Networks

Transport Equations for link $\ell \in \mathcal{L} := \mathcal{L}_p \cup \mathcal{L}_a$

$$\frac{\partial p_\ell}{\partial t} + \frac{1}{A_\ell}\frac{p_\ell}{\rho_\ell}\frac{\partial f_\ell}{\partial x} = 0$$

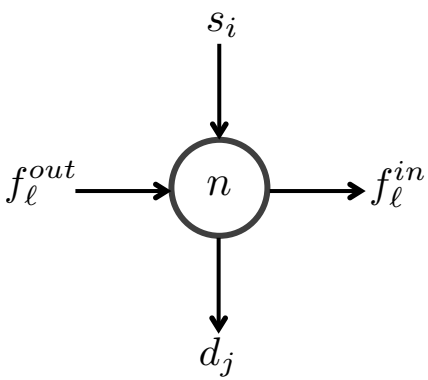
$$\frac{1}{A_\ell}\frac{\partial f_\ell}{\partial t} + \frac{\partial p_\ell}{\partial x} + \frac{8\lambda_\ell}{\pi^2 D_\ell^5}\frac{f_\ell|f_\ell|}{\rho_\ell} = 0$$

$$\begin{aligned} f_\ell\big|_{x=0} &= f_\ell^{in} \\ f_\ell\big|_{x=L_\ell} &= f_\ell^{out} \\ p_\ell\big|_{x=L_\ell} &= \theta_{rec(\ell)} \\ p_\ell\big|_{x=0} &= \theta_{snd(\ell)},\ \ell \in \mathcal{L}_p \\ p_\ell\big|_{x=0} &= \theta_{snd(\ell)} + \Delta\theta_\ell,\ \ell \in \mathcal{L}_a \end{aligned}$$



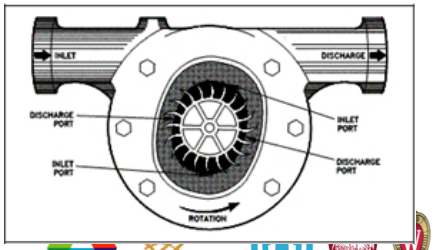
Conservation at node $n \in \mathcal{N}$

$$\sum_{\ell:rec(\ell)=n} f_\ell^{out} + \sum_{i:sup(i)=n} s_i - \sum_{\ell:snd(\ell)=n} f_\ell^{in} - \sum_{j:dem(j)=n} d_j = 0$$

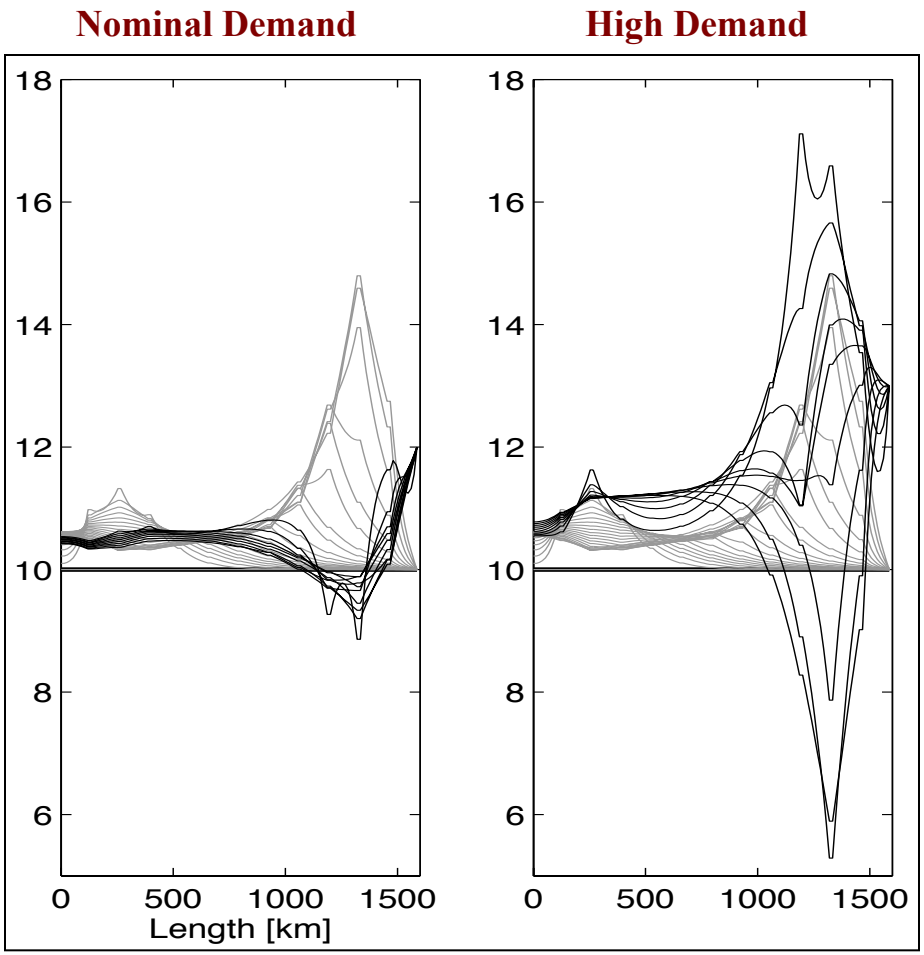


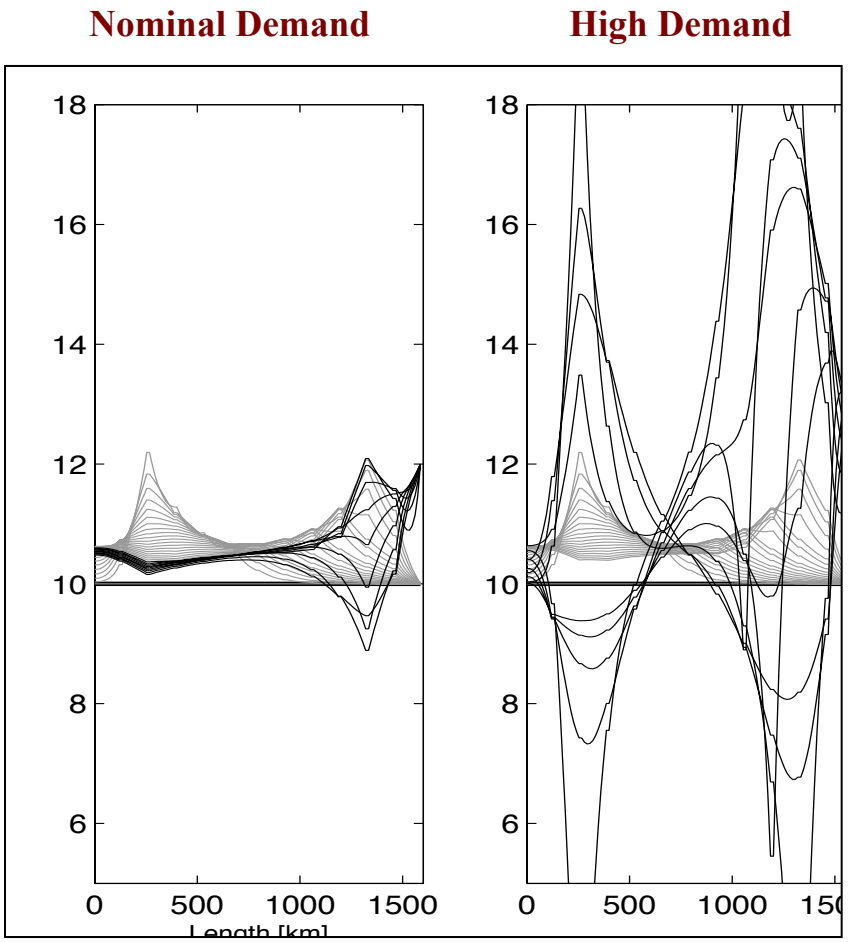
Compression Power for link $\ell \in \mathcal{L}_A$

$$P_\ell = f_\ell^{in} c_p T \left(\left(\frac{\theta_{snd(\ell)} + \Delta\theta_\ell}{\theta_{snd(\ell)}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$$



Stochastic Versus Deterministic Gas Pipeline Operations for Electricity

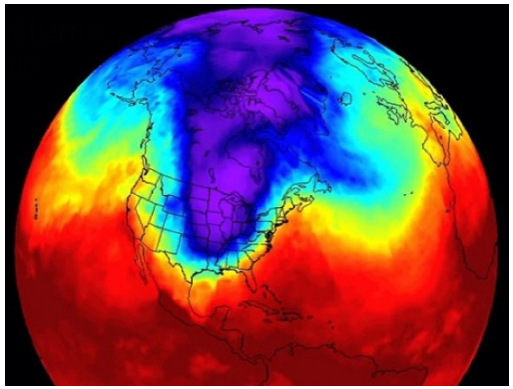




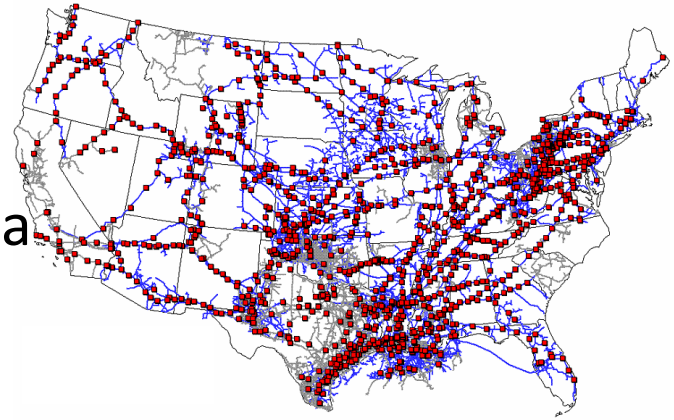
V. Zavala

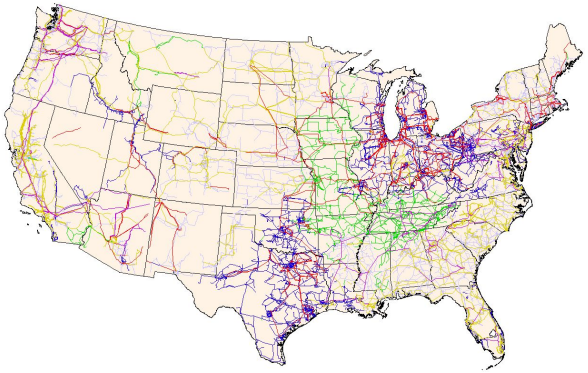
Converging Infrastructure leads to increased weather sensitivity

- (Forbes article on Feb 2014) events.
- **California**'s electric grid operator asked generators to reduce their gas usage.
- – In **Texas**, which **produces more natural gas than any other state**, the Electric Reliability Council of Texas (ERCOT) called a gas-scarcity **state of emergency.**
- – In **New England the price of natural gas to soar 20-fold** to more than $100 per thousand cubic feet. Coal, Jet Fuel, Oil plants were emergently started.
- This may have been foreseeable with probabilistic analysis tools.



U.S. Gas Transmission System





U.S. Electrical Transmission System

Picture and Challenges for Stochastic Modeling

- Space-time, non-stationary, models of uncertainty.
- High quality PDFs of weather.
- New physics: PDEs. DAEs.
- (addition cascades, sub-second noise)

- The benefits are increased reliability and more transparent pricing, but not yet operationally widespread.
- Challenge: Elicit Risk Functionals out of Decision Makers.

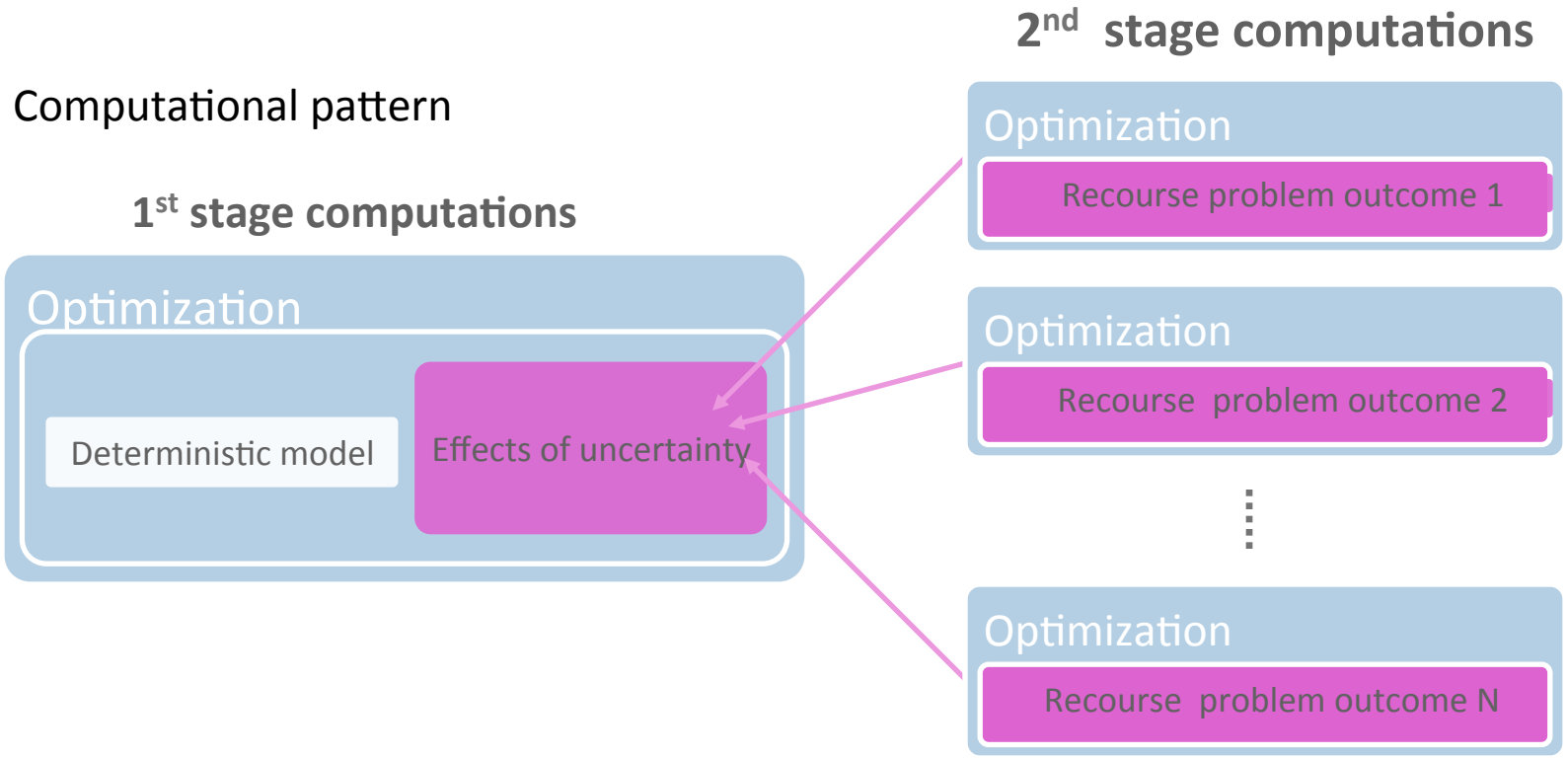
4 Computing/Supercomputing

State of Computing

- Computing is central on all these analyses.
- Parallel computing is starting to make inroads, but the use of supercomputing is rare.
- However, it is quite probable it will cost less than using humans for scenario screening and be much more accurate.
- E.g N-1 scenario dispatch over an RTO area, with stochasticity in demand and supply.
  - 10000 x 100 scenarios x 10000 degrees of freedom.
- It is certainly worth asking to what extent our approaches for certain problems scale to that level.

Stochastic programming - computational patterns

- Computational pattern



- Security-Constrained Problems.
- Two Stage Stochastic Programming
- Model Calibration.

Incomplete augmented factorization

- Revisiting sparse linear algebra for shared-memory, multi-core acceleration

$$\begin{bmatrix} K_i & B_i^T \\ B_i & 0 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} \qquad L_{22}U_{22} = -B_i^T K_i^{-1} B_i$$

- Halt the sparse factorization before factorizing the bottom-right block
- Sparsity of B_i is naturally accounted for (symbolic factorization)
- Dense matrix block operations ensure best multi-threaded performance
- Difficulty: symmetric indefinite matrices - pivot perturbations needed to maintain numerical stability
- Perturbation absorbed using preconditioned BiCGStab (all-at-once)
- 10x speed-up of the SC term computations

Test Problem	Solver	Threads/MPI	$B_i^T K_i^{-1} B$ Time (Sec.)
UC24	MA57	1	>3600
	PARDISO-SC	1	308.193
	PARDISO-SC	2	157.63
	PARDISO-SC	4	81.11

C. Petra, O. Schenk, M. Lubin, K. Gaertner, "*An augmented incomplete factorization approach for computing the Schur complement in stochastic optimization*", *SIAM Sci. Comp., 2014*

Full HPC evaluation of PIPS-IPM on power grid models

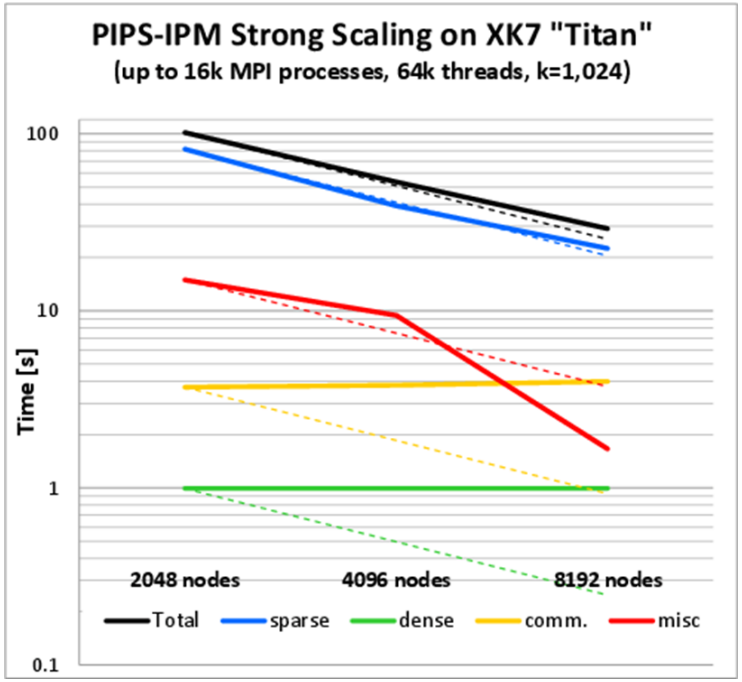
Relaxation of 24-hour unit commitment

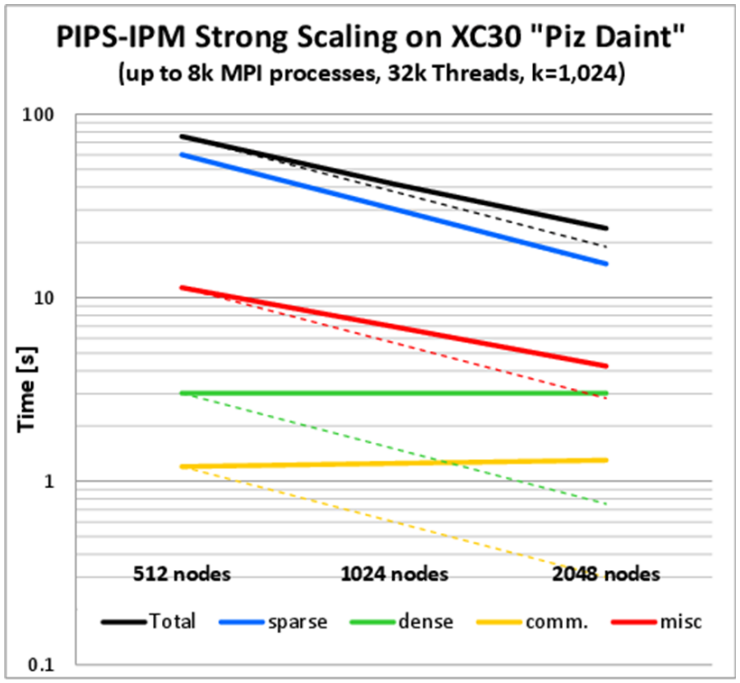
4 billion decisions variables
4.1 billion constraints
(32,768 scenarios)

Solution time: < 40 mins

FLOPS peak performance 12%

Parallel efficiency > 90%





C. Petra, O. Schenk, M. Anitescu, "*Real-time Stochastic Optimization of Complex Energy Systems on High Performance Computers*", accepted to IEEE Computing in Science & Engineering (CiSE), 2014.

Some Challenges

- The usual suspects: nonconvexity and integrality.
- Scenario Clustering. (Contingency x Load x Demand x Weather)
- Convergence of Planning and Operations ➔ Convergence of optimization and Dynamics (including MINLP + Dynamics)
- Very Long Time Horizons in Planning.
- Very large scale nonlinear programming, -- using decomposition.
- Rolling horizon: Warm/hot starting.

5. Decision-Oriented Algebraic Modeling for Complex Systems

State of Modeling in Optimization

- Optimization shines in the simplicity and power of its abstraction.
- Compare with partial differential equations.
- Optimization is one of the few areas where algebraic modeling is common (e.g AMPL, GAMS, PyOMO etc).
- On the other hand, for power grids we need to incorporate new patterns fast, and support parallelism, dynamics, PDEs.
- We may also be a very small market for AML providers.
- For huge problems, the pricing model per core does not suit us.

DIY Algebraic Modeling using Julia (Lubin et al)

- Julia: a high-level, high-performance, open-source dynamic language for technical computing
- keeps productivity of dynamic languages without giving up speed (2x of C/C++/Fortran)
- Macros instead of operator overloading (known to have poor performance).
- **Ideal for creating and extending AMLs without giving up speed JuMP (Lubin et al.)**

```
m = Model(:Max)
@defVar(m, 0 <= x[j=1:N] <= 1)
@setObjective(m, sum{profit[j] * x[j], j=1:N})
@addConstraint(m, sum{weight[j] * x[j], j = 1:N} <= C)
```

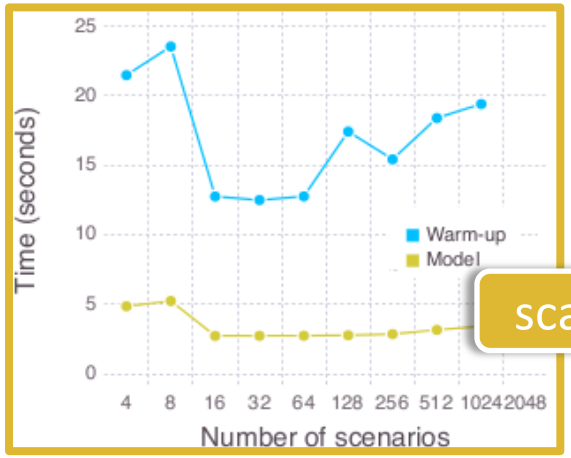
Table: Linear-quadratic control benchmark results. N=M is the grid size. Total time (in seconds) to process the model definition and produce the output file in LP and MPS formats (as available).

	JuMP/Julia		AMPL	Gurobi/C++		Pulp/PyPy		Pyomo
N	LP	MPS	MPS	LP	MPS	LP	MPS	LP
250	0.5	0.9	0.8	1.2	1.1	8.3	7.2	13.3
500	2.0	3.6	3.0	4.5	4.4	27.6	24.4	53.4
750	5.0	8.4	6.7	10.2	10.1	61.0	54.5	121.0
1,000	9.2	15.5	11.6	17.6	17.3	108.2	97.5	214.7

- Allows for different levels of user expertise within same language.
- We started using it last year, 3 of us spend > 50%, one 100%.

Example DIY: Parallel SP extension StochJuMP

- Extension of Julia JuMP for stochastic LP/QP/MILP/
- Interfaced with PIPS, runs efficiently on "Blues" LCRC cluster (MPI)
- **AMPL took 3 days to instantiate and 1 hour to run on BG.**



scalable

compact

StochJuMP

efficient

N	Load Module	Generate Model
4	3.528	1.409
8	3.694	1.415
16	3.334	1.169
32	2.541	0.917
64	2.863	0.996
128	3.462	0.768
256	2.620	0.723
512	2.871	0.689
1024	1.470	0.384
2048	-	0.500

RATIO OF STOCHJUMP TIMINGS OVER SOLVE TIME ($\times 100$).

```
m = StochasticModel(NS)

# Stage 0
@defVar(m, 0 <= Pgen_f[i=GENTHE] <= np_capThe[i])
@defVar(m, 0 <= PgenWin_f[i=GENWIN] <= np_capWin[i])
@defVar(m, -lineCutoff*Pmax[i] <= P_f[i=LIN] <= lineCutoff*Pmax[i])

# (forward) power flow equations
@addConstraint(m, pfeq_f[j=BUS],
              +sum{P_f[i], i=LIN; j==rec_bus[i]}
              -sum{P_f[i], i=LIN; j==snd_bus[i]}
              +sum{Pgen_f[i], i=GENTHE; j==bus_genThe[i]}
              +sum{PgenWin_f[i], i=GENWIN; j==bus_genWin[i]}
              -sum{loads[i], i=LOAD; j==bus_load[i]} >= 0)

@second_stage m node begin
    bl = StochasticBlock(m)
    # variables
    @defVar(bl, 0 <= Pgen[i=GENTHE] <= np_capThe[i])
    @defVar(bl, 0 <= PgenWin[i=GENWIN] <= windPower[node,i])
    @defVar(bl, -lineCutoff*Pmax[i] <= P[i=LIN] <= lineCutoff*Pmax[i])
    @addConstraint(bl, rampUpDown[g=GENTHE],
                  -0.1np_capThe[g] <= Pgen[g] - Pgen_f[g] <= 0.1np_capThe[g])
    # (spot) power flow equations
    @addConstraint(bl, pfeq[j=BUS],
                  +sum{P[i]-P_f[i], i=LIN; j==rec_bus[i]}
                  -sum{P[i]-P_f[i], i=LIN; j==snd_bus[i]}
                  +sum{Pgen[i]-Pgen_f[i], i=GENTHE; j==bus_genThe[i]}
                  +sum{PgenWin[i]-PgenWin_f[i], i=GENWIN; j==bus_genWin[i]} >= 0)
    @defVar(bl, t[GENTHE] >= 0)
    @addConstraint(bl, t_con1[g=GENTHE],
                   t[g] >= gen_cost_the[g]*Pgen_f[g] +
                   1.2gen_cost_the[g]*(Pgen[g]-Pgen_f[g]))
    @addConstraint(bl, t_con2[g=GENTHE],
                   t[g] >= gen_cost_the[g]*Pgen_f[g])
    @defVar(bl, tw[GENWIN] >= 0)
    @addConstraint(bl, t_w_con1[g=GENWIN],
                   tw[g] >= gen_cost_win[g]*PgenWin_f[g] +
                   1.2gen_cost_win[g]*(PgenWin[g]-PgenWin_f[g]))
    @addConstraint(bl, t_w_con2[g=GENWIN],
                   tw[g] >= gen_cost_win[g]*PgenWin_f[g])

    @setObjective(bl, Min, sum{t[g], g=GENTHE} + sum{tw[g], g=GENWIN})
end
```

J. Huchette, M. Lubin, **C. Petra**, *Parallel algebraic modeling for stochastic optimization.* SC'14 Workshop for High Performance Technical Computing in Dynamic Languages.

Some Challenges in Efficient Modeling

- If we are to break the barriers between analyses, a good algebraic modeling environment is necessary. Some challenges:
- PDEs -- Natural Gas is Hyperbolic.
- Dynamical/differential algebraic systems.
- Determining well posedness of composite models (easier in dynamics than in optimization – see SIMULINK or Modelica)
- Some abstraction of hierarchical modeling
  - interconnect-distribution-building
  - yearly-daily-hourly-second

Summary

- We manage the grid very well today, but it is not a solved problem.
- The US grid will change dramatically over the next 15 years.
- This will challenge our analytical abilities at multiple levels.
- Mathematical Foundations.
- Probabilistic Modeling
- New Algorithms
- Problem Representations

Unsolved and unapproached math conceptual challenges

- Global Optimization with nonconvex constraints.
- Equilibrium formulations with integer variables
- Probability density for cascade events.
- Random graphs
- Multiscale Math for Networks
- Spatio-temporal, nonstationary, data modeling.
- Rare event simulations
- Stochastic numerical simulation
- Multistage stochastic programming
- Error models for engineered physical processes
- Non-Gaussian noise
- Hybrid, multisource data analysis.

Appendix: links used (verified Nov 3, 2015)

Slide 7: Sources:
http://washington.cbslocal.com/2015/04/07/white-house-state-department-lose-power-due-to-scattered-outages/

http://www.cnn.com/2013/02/08/us/superdome-power-outage/

https://emp.lbl.gov/publications/final-report-august-14-2003-blackout-united-states-and-canada-causes-and-recommendation (sec 6, page 73)

Slide 13:
http://instituteforenergyresearch.org/analysis/germanys-electricity-market-balance-must-pay-flexible-back-power/

- Slide 16:
  http://www.strategyand.pwc.com/global/home/what-we-think/global-innovation-1000/rd-intensity-vs-spend-2013-v2stage
- Slide 23
- http://www.forbes.com/sites/christopherhelman/2014/02/08/how-can-a-nation-awash-in-natural-gas-have-shortages-and-what-to-do-about-it/