STOCHASTIC SIMULATION OF PREDICTIVE SPACE-TIME SCENARIOS OF WIND SPEED USING OBSERVATIONS AND PHYSICAL MODELS

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We propose a statistical space-time model for predicting atmospheric wind speed based on deterministic numerical weather predictions and historical measurements. We consider a Gaussian multivariate space-time framework that combines multiple sources of past physical model outputs and measurements along with model predictions in order to produce a probabilistic wind speed forecast within the prediction window. We illustrate this strategy on a ground wind speed forecast for several months in 2012 for a region near the Great Lakes in the United States. The results show that the prediction is improved in the mean-squared sense relative to the numerical forecasts as well as in probabilistic scores. Moreover, the samples are shown to produce realistic wind scenarios based on the sample spectrum.

1. Introduction. In this study we propose a statistical space-time model for predicting atmospheric wind speed based on numerical weather predictions and historical measurements. We focus on a region around Lake Michigan in the United States; however, the framework proposed here is not specific to that region. The wind speed predictions are based on deterministic numerical weather prediction (NWP) model outputs in a framework that integrates past dependence between observational measurements and the NWP model outputs. The aim of this work is to improve the wind speed forecasts provided by the NWP model based on the past relation, which is modeled linearly, between measurements and NWP forecasts.

Atmospheric surface wind prediction is important for the energy, agricultural, and security sectors, and it has received considerable attention in the past several years. Several components of the wind field can be predicted: the zonal and meridional components [16, 26], wind speed [6, 25], and wind direction [2]. Recent work on wind speed and wind power statistical prediction focuses on the generation of predictive scenarios that enable scientists to account for the prediction error [20, 19]. However, few criteria of quality assessment of scenarios have been proposed; multivariate (multiple time-step ahead and/or space or ensemble forecasts) criteria can be used but do not account for the nature of time trajectory of scenarios. In [19] an event-based criterion is proposed to assess the quality of scenarios to reproduce wind events and to compare scenarios from different models.

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In general, predicting wind speed based on measurements and physical model outputs results in multivariate space-time processes that are typically inhomogeneous because of the presence of different types of data. Multivariate space-time modeling has been an area of intense research in the past two decades; see [10] for a review of bivariate geostatistical modeling, and see [3], where hierarchical Bayesian modeling is discussed for multiple dependent datasets.

Combining multiple sources of data is an increasingly important field of research because of the large variety of sources of data available today. Data fusion is also part of multivariate modeling, and various statistical models have been proposed. For example, in [11] a Bayesian hierarchical model is presented that combines model outputs and observed measurements to provide spatial prediction for chemical species. A hidden process is used to represent the unobserved “true” concentration of sulfur dioxide, and the sources of data are affine transformations of this “true” process. A similar approach was used in a space-time context for multiple measurements of snow water equivalent data in [8]. In [17], a hierarchical model based on a spatial random effects model is presented that combines several outputs of regional climate models in a spatial framework. In [4], a space-time hierarchical Bayesian model is proposed to fuse measurements and model outputs of air-quality data, with an extension of a downscaling model introduced some years ago. A hierarchical approach to multivariate spatial modeling and prediction is developed in [22, 23], where the specification of the conditional and a marginal distribution is made instead of specifying the joint distribution, which involves cross-covariances. Indeed the modeling of multivariate covariance structure is challenging and is still an on-going research area; see [1, 12, 5].

Forecast uncertainty can be accounted for through ensemble forecasts; however, this strategy is known to be often uncalibrated and underdispersive. In the context of improving numerical forecasts, statistical methods have been proposed to provide probabilistic forecasts; such methods postprocess the single or ensemble forecasts and tend to address the issue of bias and dispersion. These methods, known as model output statistics (MOS) and ensemble model output statistics, are used to identify shortcomings of the raw ensemble from past measurement-forecast pairs. In [21], a finite mixture model, called Bayesian model averaging, is introduced for producing probabilistic forecasts based on ensemble forecasts. In [13], a regression model between the measurements and the members of the ensemble forecast is proposed as a postprocessing statistical tool. The assessment of multivariate predictive distribution has been discussed in [15], where tools to assess calibration and sharpness of the predictive distributions are investigated.

In this paper, we propose a bivariate space-time Gaussian process to improve forecasts from an NWP model. The forecasts of wind speed are combined with historical measurements data and provide scenarios of prediction. A particularly important aspect of our model is that it accounts for the space-time dependence between the two datasets. To the best of our knowledge, this dependence is not accounted for by the MOS methods proposed in the literature for wind speed. Moreover, in this work we consider a framework where future information from the NWP is used, whereas MOS methods commonly
work with contemporaneous information in space and time. The model is specified in a
hierarchical way in order to avoid the characterization of the full space-time bivariate
covariance. This specification, initially proposed in [22, 23] in a spatial context, is here
extended to a space-time modeling.

The paper is organized as follows. In Section 2 we introduce the modeling context
and the model. In Section 3 we describe the two sources of data that are used and
combined. In Section 4 the model is validated on different months of the year, and the
quality of space-time prediction at one out-of-sample station is assessed. We highlight the
improvements in terms of the forecasting accuracy of the proposed model with respect
to the NWP forecasts. We conclude in Section 5 by presenting general improvements
made by the model with respect to the NWP data, and we highlight some perspectives
to improve the shortcomings of the models.

2. Statistical model for NWP model outputs. In this section we introduce a
Gaussian modeling framework that embeds the space-time dependence between mea-
sured observations and NWP model forecasts. In [22, 23], a model is presented that
conveniently combines spatial data; here we extend at model to a space-time context.

2.1. Modeling objectives. The modeling context is the following. Let us assume that
both measured observations \( Y_{\text{Obs}} \) and NWP forecasts \( Y_{\text{NWP}} \) are available from time \( t_1 \)
to time \( t_k \). In the following, the term “observations” refers to the observational measure-
ments. Observations are available at \( J_0 \) locations \( S = \{s_1, ..., s_{J_0}\} \), and NWP forecasts
are available over a grid that covers these stations. The NWP model is run every day for
a period of \( h \) hours; time can be written in terms of blocks of length \( h \). Henceforth, we
consider a time window of \( h = 24 \) hours. We denote by \( b_i \) the \( i \)th time block of length
\( h \), \( b_i = \{t_{k_i}, ..., t_{k_i+h-1}\} \).

The objective here is to predict the measurements \( Y_{\text{Obs}} \) between time \( t_{kK} \) and \( t_{kK+h-1} \)
at stations \( S = \{s_1, ..., s_{J_0}\} \) and possibly at locations \( \{s_{J_0+1}, ..., s_J\} \) where no historical
measurements are recorded, from NWP forecasts that are available between \( t_{kK} \) and
This can be summarized by

\[
\begin{pmatrix}
    y_{\text{Obs}}^a(b_1; s_1, \ldots, s_J) \\
    y_{\text{Obs}}^a(b_2; s_1, \ldots, s_J) \\
    \vdots \\
    y_{\text{Obs}}^a(b_K; s_1, \ldots, s_J) \\
    y_{\text{Obs}}^u(t_{kK}; s_1, \ldots, s_{J_0}, s_{J_0+1}, \ldots, s_J) \\
    \vdots \\
    y_{\text{Obs}}^u(t_{kK+h-1}; s_1, \ldots, s_{J_0}, s_{J_0+1}, \ldots, s_J)
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
    y_{\text{NWP}}^a(b_1; s_1, \ldots, s_J) \\
    y_{\text{NWP}}^a(b_2; s_1, \ldots, s_J) \\
    \vdots \\
    y_{\text{NWP}}^a(b_K; s_1, \ldots, s_J) \\
    y_{\text{NWP}}^a(t_{kK}; s_1, \ldots, s_{J_0}, s_{J_0+1}, \ldots, s_J) \\
    \vdots \\
    y_{\text{NWP}}^a(t_{kK+h-1}; s_1, \ldots, s_{J_0}, s_{J_0+1}, \ldots, s_J)
\end{pmatrix},
\]

where the superscript “a” stands for available and “u” for unavailable quantities.

In this context the model is trained on the following available pairs:

\[
\left\{(y_{\text{Obs}}^a(b_1; S), y_{\text{NWP}}^a(b_1; S)), (y_{\text{Obs}}^a(b_2; S), y_{\text{NWP}}^a(b_2; S)), \ldots, (y_{\text{Obs}}^a(b_K; S), y_{\text{NWP}}^a(b_K; S))\right\},
\]

and the prediction is made from \(y_{\text{NWP}}^a(b_{K+1}; S, s_{J_0+1}, \ldots, s_J)\) to estimate \(y_{\text{Obs}}^a(b_{K+1}; S, s_{J_0+1}, \ldots, s_J)\), where \(b_{K+1} = \{t_{kK}, \ldots, t_{kK+h-1}\}\). Each day of \(h = 24\) hours, the Weather and Research (WRF) model is run independently from the previous day because WRF is initialized from a reanalysis or assimilated dataset. In a probabilistic sense, we aim to compute

\[
P(y_{\text{Obs}}^a(b_{K+1})|y_{\text{NWP}}^a(b_{K+1}), y_{\text{Obs}}^a(b_{1:K}), y_{\text{NWP}}^a(b_{1:K})) = \\
\int P(y_{\text{Obs}}^a(b_{K+1}), \theta|y_{\text{NWP}}^a(b_{K+1}), y_{\text{Obs}}^a(b_{1}), \ldots, y_{\text{Obs}}^a(b_{K}), y_{\text{NWP}}^a(b_{1}), \ldots, y_{\text{NWP}}^a(b_{K+1})) d\theta,
\]

where \(\theta\) is a random set of model parameters, blocks \(b_{1:K}\) are available, and \(b_{K+1}\) is a predicted block; and the spatial components are suppressed for brevity. Note that \(b_{K+1}\) is not necessarily a block coming right after \(b_K\), but rather a day that is not observed. To simplify the computation of (2.2), we now make several assumptions. First, we assume (i) that we have approximate independence of \(y_{\text{Obs}}^a(b_{K+1})\) on \(y_{\text{Obs}}^a(b_{1:K}), y_{\text{NWP}}^a(b_{1:K})\) \textit{conditional on} \(y_{\text{NWP}}^a(b_{K+1})\). In hierarchical models such as ours, which has NWP predictions as its first layer and the observation sites as the second layer, one commonly
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assumes that random variables on the second layer are independent conditional on the realizations of the ones in the first layer; see [9]. This assumption is correct if the additional randomness occurs from the noise of different, unrelated sensors. In our case, since we are considering the error of NWP models, the difference between prediction and observations most likely is due to features not modeled by NWP. They may be the use of lower resolution or models that have been obtained by some level of space-time homogenization of the physics of the model considered. In this case, the difference is the modeling of subscale noise, which can be assumed to have short temporal correlation scales; see [18]. Moreover, our use of 24-hour temporal blocks as opposed to every time index would strengthen the validity of approximate conditional independence on NWP simulations of wind realizations at observation sites. The independence of \( y_{\text{Obs}}^b(b_{K+1}) \) on \( y_{\text{NWP}}^b(b_{1,K}) \) conditional on \( y_{\text{NWP}}^b(b_{K+1}) \) may also be a good approximation given the short temporal correlation scales of subscale noise discussed above.

As a result, assumption (ii) implies that the integrand in (2.2) can be approximated as

\[
p (y_{\text{Obs}}^b(b_{K+1}), \theta | y_{\text{NWP}}^b(b_{K+1}), y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})) \\
= p (y_{\text{Obs}}^a(b_{K+1}) | \theta, y_{\text{NWP}}^a(b_{K+1}), y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})) \\
p (\theta | y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})) \\
\approx p (y_{\text{Obs}}^a(b_{K+1}) | \theta, y_{\text{NWP}}^a(b_{K+1})) p (\theta | y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})).
\]

(2.3)

In this study, we assume (ii) that \( \theta^* \) can be obtained by maximizing the likelihood

\[
\theta^* = \argmax_{\theta} \mathcal{L} (\theta; y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})) \\
= \argmax_{\theta} p (\theta | y_{\text{Obs}}^a(b_{1,K}), y_{\text{NWP}}^a(b_{1,K})).
\]

With assumptions (i-ii) and thus using (2.3) in (2.2) we obtain that

\[
\int p (y_{\text{Obs}}^a(b_{K+1}), \theta | y_{\text{NWP}}^a(b_{K+1}), y_{\text{Obs}}^a(b_{1}), ..., y_{\text{Obs}}^a(b_{K})), y_{\text{NWP}}^a(b_{1}), ..., y_{\text{NWP}}^a(b_{K})) d\theta \\
\approx p (y_{\text{Obs}}^a(b_{K+1}) | \theta^*, y_{\text{NWP}}^a(b_{K+1})).
\]

(2.5)

In what follows we consider multivariate normal distributions for (2.5). In our approach we have found a productive approach is to model statistically the output of NWP itself. In other words, one can consider NWP as a noisy realization of a latent underlying process \( NWP^V \) (which models the evolution of spatially averaged quantities). With NWP conditional on this \( NWP^V \) we then assume it to be independent for two different temporal blocks, that is all temporal correlation between successive blocks is due to \( NWP^V \) itself. The same reasoning now applies by replacing \( NWP^V \) in our earlier discussion. Note that we never forecast the NWP output using the statistical model we develop; we forecast only its relationship to the observations. Thus, we do not need to model explicitly the temporal correlation between different blocks of NWP as long as a sample is produced by the WRF model by a (for the purpose of this paper) black-box mechanism that emulates the correct interblock correlation by its relationship
to NWP. Moreover, if such an assumption does not hold completely, it can lead only to more conservative forecasts. Another way our approach can be thought of is as a regression approach with noisy NWP predictors and the observational unit being one temporal block over the entire geographical area. As a result, the likelihood in (2.4) factorizes in product form for different temporal blocks.

To summarize our approach, for a given statistical model, we first estimate \( \theta^* \) from the available data (model and observations) using (2.4). Then, using (2.5), we obtain a predictive distribution by conditioning only on the NWP predictions for the same temporal block and plugging-in the maximum likelihood estimate \( \theta^* \).

\[
p(y_{\text{Obs}}^u(b_{K+1})|y_{\text{NWP}}^a(b_{K+1}), y_{\text{Obs}}^o(b_{1:K}), y_{\text{NWP}}^a(b_{1:K})) \\
\approx p(y_{\text{Obs}}^u(b_{K+1})|\theta^*, y_{\text{NWP}}^a(b_{K+1}))
\]

(2.6)

These choices are motivated by computational tractability, by the fact that we assume that the information missed by NWP is subscale-type information, which, as mentioned above, is assumed to have short time correlations conditional on NWP realizations, and by the fact that we do not forecast NWP itself, but rather the relationship between NWP and observations. In Section 2.2 we review a hierarchical approach for Gaussian processes, and in Section 2.3 we present the model used for the mean and covariance functions that introduce the parametrization \( \theta \).

2.2. Hierarchical bivariate model. Gaussian processes are chosen for their convenience in expressing conditional distributions. As in other studies, we use the wind speed data directly without any transformation [14, 11, 23]. Moreover, we have not observed a significant departure from normality within the data sets used for this study. Square-root or Box-Cox transformations can be used to preprocess the data, but we do not expect that will influence the modeling choices. We write the joint distribution of the process \((Y_{\text{Obs}}, Y_{\text{NWP}})\) as

\[
(Y_{\text{Obs}}, Y_{\text{NWP}}) \sim \mathcal{N}
\left(
\begin{pmatrix}
\mu_{\text{Obs}} \\
\mu_{\text{NWP}}
\end{pmatrix},
\begin{pmatrix}
\Sigma_{\text{Obs}} & \Sigma_{\text{Obs},\text{NWP}} \\
\Sigma_{\text{Obs},\text{NWP}}^T & \Sigma_{\text{NWP}}
\end{pmatrix}
\right).
\]

(2.7)

The positive-definiteness of block matrices is generally difficult to ensure when specifying the three blocks in (2.7) independently. Therefore, to avoid the specification of the full covariance in (2.7), we follow the hierarchical conditional modeling proposed by [22, 23], and we model \((Y_{\text{Obs}}|Y_{\text{NWP}})\) and \((Y_{\text{NWP}})\), where \((Y_{\text{Obs}}|Y_{\text{NWP}})\) stands for the conditional distribution of \(Y_{\text{Obs}}\) given \(Y_{\text{NWP}}\). When \((Y_{\text{Obs}}, Y_{\text{NWP}})\) is a Gaussian process, \((Y_{\text{Obs}}|Y_{\text{NWP}})\) and \((Y_{\text{NWP}})\) follow a Gaussian distribution; then only first- and second-order structures are to be specified. Consequently the model is described by the following distributions:

\[
(Y_{\text{Obs}}|Y_{\text{NWP}}) \sim \mathcal{N}
\left(
\mu_{\text{Obs}|\text{NWP}}, \Sigma_{\text{Obs}|\text{NWP}}
\right).
\]

(2.8)

A linear dependence between \(Y_{\text{Obs}}\) and \(Y_{\text{NWP}}\) agrees reasonably with the data analysis, so we choose the following dependence:

\[
\mu_{\text{Obs}|\text{NWP}} = \mathbb{E}(Y_{\text{Obs}}|Y_{\text{NWP}}) = \mu + \Lambda Y_{\text{NWP}}
\]

(2.9)
and

$$Y_{\text{NWP}} \sim \mathcal{N}(\mu_{\text{NWP}}, \Sigma_{\text{NWP}}).$$

From these equations, we express the full joint distribution given by (2.7) as

$$
\begin{pmatrix}
Y_{\text{Obs}} \\
Y_{\text{NWP}}
\end{pmatrix}
\sim \mathcal{N}
\left(
\begin{pmatrix}
\mu + \Lambda \mu_{\text{NWP}} \\
\mu_{\text{NWP}}
\end{pmatrix},
\begin{pmatrix}
\Sigma_{\text{Obs|NWP}} + \Lambda \Sigma_{\text{NWP}} \Lambda^T & \Lambda \Sigma_{\text{NWP}} \\
\Lambda \Sigma_{\text{NWP}}^T & \Sigma_{\text{NWP}}
\end{pmatrix}
\right).
$$

2.3. Statistical model. To provide time prediction and to ensure model parsimony, we propose a parameterization in space and time of the involved quantities such that the first- and second-order structures of the conditional and the marginal distributions defined by (2.8) and (2.10) are specified following an exploratory analysis of the datasets.

2.3.1. Marginal mean structure of $Y_{\text{NWP}}$. The empirical mean function of $Y_{\text{NWP}}$ exhibits spatial patterns associated with the geographical coordinates but also with several parameters of the NWP model because of the large water mass of the lake (especially the land use, which is a categorical variable that represents the type of land used in the parameterization of the NWP model). Time-periodic effects are present in the first-order structure of $Y_{\text{NWP}}$ and are accounted for through harmonics of different frequencies. In Figure 7, these spatial and temporal patterns are plotted. We write

$$E(Y_{\text{NWP}}(t, s)) = \beta_0 + \beta_1 \cos \left(\frac{2\pi t}{24}\right) + \beta_2 \sin \left(\frac{2\pi t}{24}\right) + \beta_3 \cos \left(\frac{2\pi t}{12}\right) +$$

$$\beta_4 \sin \left(\frac{2\pi t}{12}\right) + \beta_5 \cos \left(\frac{2\pi t}{8}\right) + \beta_6 \sin \left(\frac{2\pi t}{8}\right) +$$

$$\left(\alpha_0^{(\text{LU}(s))} + \alpha_1(s)\right),$$

where $t$ is measured in hours, $\text{LU}(s)$ is an integer that represents the land use associated with station $s$ used in the model; $(\alpha_0^d)_{d=1,\ldots,n}$, with $n$ the number of possible land uses and $(\alpha_1(j))_{j=1,\ldots,J_0}$ and $(\beta_k)_{k=0,\ldots,6}$ are real numbers.

2.3.2. Marginal covariance structure of $Y_{\text{NWP}}$. The block structure of the space-time covariance of the data suggests expressing wind speed at each station as a linear transformation of an unobserved common signal with added noise; see the top panels of Figure 4. Intuitively we can think of this common signal as an average flow over the studied region. The wind speed at each site is a linear transformation of this average flow. The temporal dynamics of the unobserved signal is modeled with a squared exponential covariance. The following structure is used:

$$Y(b_i, s_j) = L_{nj} Y_0(b_i) + \epsilon_{s_j}(b_i),$$

where $b_i$ is a temporal window of $h = 24$ lags, $s_j$ the spatial location and $L_{nj}$ is an $h \times h$-matrix. The various $\epsilon_{s_j}$ are assumed independent from each other and from $Y_0$. 

This model is inspired in part by an earlier study [7] where the $L$ operators were used to represent a known functional relation. In our case, $L_{s_j}$ is a parameterized matrix that is inferred from the data.

The overall covariance of $Y$ has the following structure:

$$\text{cov}(Y(.,s_i), Y(.,s_j)) = (L_{s_j}K_0L_{s_i}^T) + \delta_{i-j}K_{s_i},$$

where $\delta$ stands for the Kronecker symbol and for $j \in \{1, \ldots, J_0\}$. The $h \times h$-matrices $K_{s_j}$ are written as

$$K_{s_j}[l,k] = a_{s_j} \exp(-b_{s_j}(|t_k - t_l|^2)) + \delta_{k-l}c_{s_j}$$

and

$$K_0[l,k] = a_0 \exp(-b_0(|t_k - t_l|^2)) + \delta_{k-l}c_0.$$

Following the data analysis, the $h \times h$-matrices $L_{s_j}$ are parametrized as tridiagonal matrices. Given the study of the variance in Figure 1, the diagonal and off-diagonal quantities are modeled with a quadratic dependence in time and spatially dependent coefficients. The diagonal, subdiagonal and superdiagonal of the matrix $L_{s_j}$ are written respectively as

$$L_{s_j}[i,i] = (1 + a_{1,Lat}(s_j) + a_{2,Long}(s_j)) + (1 + a_{3,Lat}(s_j) + a_{4,Long}(s_j)) \times i + (1 + a_{5,Lat}(s_j) + a_{6,Long}(s_j)) \times i^2,$$

$$L_{s_j}[i,i-1] = (1 + a_{7,Lat}(s_j) + a_{8,Long}(s_j)) + (1 + a_{9,Lat}(s_j) + a_{10,Long}(s_j)) \times i + (1 + a_{11,Lat}(s_j) + a_{12,Long}(s_j)) \times i^2,$$

$$L_{s_j}[i,i+1] = (1 + a_{13,Lat}(s_j) + a_{14,Long}(s_j)) + (1 + a_{15,Lat}(s_j) + a_{16,Long}(s_j)) \times i + (1 + a_{17,Lat}(s_j) + a_{18,Long}(s_j)) \times i^2,$$

for $i \in \{1, \ldots, h\}$. We work in relatively small areas and use distances in latitude and longitude here and for the rest of this work.

2.3.3. Conditional mean structure of $(Y_{\text{Obs}}|Y_{\text{NWP}})$. Scatterplots of observations and model outputs suggest that a linear dependence between the variables is reasonable. In [22], several configurations of the transition matrix $\Lambda$ are proposed depending on its use. For instance, a transition matrix from atmospheric pressure to wind speed is derived from geostrophic equations in [23]. The observations exhibit daily and half-daily periodicity (with various intensities depending on the month of the year) and spatial patterns; see Figure 3. However, the relation between the two datasets does not exhibit
significant time-dependence that requires a time-varying dependence. We use spatial and temporal neighbors to explain the observed wind speed. The land use (LU) is included in the transition matrix, since it defines different behaviors in the NWP model data. We choose the following transition between the two datasets:

\[
E(Y_{\text{Obs}}(t, s) | Y_{\text{NWP}}(t, s)) = \mu(t, s) + (\Lambda Y_{\text{NWP}})(t, s), \quad \text{with}
\]

\[
\mu(t, s) = \left( \beta_0 + \beta_1 \cos \left( \frac{2\pi t}{24} \right) + \beta_2 \sin \left( \frac{2\pi t}{24} \right) + \beta_3 \cos \left( \frac{2\pi t}{12} \right) + \beta_4 \sin \left( \frac{2\pi t}{12} \right) \right) (1 + \beta_5 \text{Lat}(s) + \beta_6 \text{Long}(s)),
\]

\[
(\Lambda Y_{\text{NWP}})(t, s) = \sum_{i=1}^{h} \alpha^{(\text{LU}(s))}(|t - t_i|) \sum_{k=1}^{3} f_k(\Delta \text{Lat}, \Delta \text{Long})(s, s_k) Y_{\text{NWP}}(t_i, s_k),
\]

where

- \( \alpha^{(l)}(.) \) are temporal weights, parameterized according to \( \alpha^{(l)}(\Delta t) = \theta_0^{(l)} \exp(-\theta_1^{(l)} |\Delta t|) + \theta_2^{(l)} \), for the time difference \( \Delta t \) in \( \{0, ..., h - 1\} \); the integer \( l \in \{1, ..., n\} \) is the land-use value of the closest grid point of \( s \); \( \alpha^{(l)}(0) = 1 \) for identifiability purposes;

- \( f \) are linear functions of the differences in latitude and in longitude \( \Delta \text{Lat}(s_i, s_j) = |\text{Lat}(s_i) - \text{Lat}(s_j)| \) and \( \Delta \text{Long}(s_i, s_j) = |\text{Long}(s_i) - \text{Long}(s_j)| \); and

- \( s_1, s_2, s_3 \) are nearest spatial neighbor grid points of \( s \) selected according to the radial distance, but other distances are possible. Moreover, for simplicity we consider here nearest neighbors, but other choices of predictors can be made, such as upwind stations.

2.3.4. Conditional covariance structure of \( (Y_{\text{Obs}}|Y_{\text{NWP}}) \). Analysis of the empirical conditional covariance suggests the use of the parametric shape proposed in (2.13), with a different set of parameters.

2.4. Estimation of the parameters. Maximum likelihood is chosen for estimating the parameters. The likelihood of the model for the observed dataset \( y_{\text{Obs}}(t_1, ..., t_T) \);
Similarly the conditional distribution is written as

$$\mathcal{L}(\theta; y_{\text{Obs}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0}), y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0}))$$

$$= p_0(y_{\text{Obs}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0}), y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0}))$$

$$= p_0(y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0}))$$

$$= p_0(y_{\text{Obs}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})|y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})).$$

This is the particular instantiation of (2.4).

Each day, the WRF model is run independently from the previous day therefore we consider statistical independence between each day, which leads to the following product:

$$p_0(y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})) = \prod_{i=1}^{K} p_0(y_{\text{NWP}}(t_{k_i}; S), \ldots, y_{\text{NWP}}(t_{k_i+23}; S))$$

$$= \prod_{i=1}^{K} p_0(y_{\text{NWP}}(b_i; S)),$$

where \( S = \{s_1, \ldots, s_{J_0}\} \) and \( \{b_1, \ldots, b_K\} = \{t_1, \ldots, t_{24}, t_{25}, \ldots, t_T\} \) with \( b_i = \{t_{k_i}, \ldots, t_{k_i+23}\} \).

For each \( i \in \{1, \ldots, K\} \) we have

$$p_0(y_{\text{NWP}}(b_i; S)) = \frac{1}{\sqrt{(2\pi)^{J_0} \det(\Sigma_{\text{NWP}})}} \exp \left( -\frac{1}{2}(y_{\text{NWP}}(b_i; S) - \mu_{\text{NWP}})^T \Sigma_{\text{NWP}}^{-1}(y_{\text{NWP}}(b_i; S) - \mu_{\text{NWP}}) \right),$$

where \( \mu_{\text{NWP}} \) and \( \Sigma_{\text{NWP}} \) are the parametric mean and covariance, respectively expressed in (2.12) and (2.13). The log-likelihood associated with the marginal of \( Y_{\text{NWP}} \) is then expressed as

$$\log(p_0(y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})))$$

$$= -\frac{1}{2} \sum_{i=1}^{K} \left( \log((2\pi)^{J_0}) + \log(\det(\Sigma_{\text{NWP}})) \right)$$

$$\quad + (y_{\text{NWP}}(b_i; S) - \mu_{\text{NWP}})^T \Sigma_{\text{NWP}}^{-1}(y_{\text{NWP}}(b_i; S) - \mu_{\text{NWP}}).$$

Similarly the conditional distribution is written as

$$\log(p_0(y_{\text{Obs}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})|y_{\text{NWP}}(t_1, \ldots, t_T; s_1, \ldots, s_{J_0})))$$

$$= -\frac{1}{2} \sum_{i=1}^{K} \left( \log((2\pi)^{J_0}) + \log(\det(\Sigma_{\text{Obs|NWP}})) \right)$$

$$\quad + (y_{\text{Obs}}(b_i; S) - \mu - \Lambda y_{\text{NWP}}(b_i; S))^T \Sigma_{\text{Obs|NWP}}^{-1}(y_{\text{Obs}}(b_i; S) - \mu - \Lambda y_{\text{NWP}}(b_i; S)).$$

In practice, a preliminary least-squares estimation of the parameters is realized between the empirical and parametric first- and second-order structures of \( Y_{\text{Obs}} \) and \( Y_{\text{NWP}} \). These estimates are used as initial conditions of the maximum likelihood procedure.
2.5. **Kriging.** Predictions of $Y_{\text{Obs}}$ from $Y_{\text{NWP}}$ are obtained from the kriging equations [28], with the mean and covariance defined by (2.11). For $t_0$ in $\{t_{k+1}, ..., t_{k+h}\}$ and $s_0$ in $\{1, ..., J_0, J_0 + 1, ..., J\}$ defined in (2.1), we have

\begin{equation}
(Y_{\text{Obs}}(t_0; s_0)|Y_{\text{NWP}}(t_{k+1}, ..., t_{k+h}; 1, ..., J_0, J_0 + 1, ..., J)) \sim \mathcal{N}\left(\hat{\mu}_{\text{Obs}}(t_0; s_0), \hat{\Sigma}_{\text{Obs}}(t_0; s_0)\right)
\end{equation}

with

\begin{equation}
\hat{\mu}_{\text{Obs}}(t_0; s_0) = (\mu + \Lambda \mu_{\text{NWP}})(t_0; s_0) + c_0^T \Sigma_{\text{NWP}}^{-1}(\langle b_K; 1, ..., J \rangle; \langle b_K; 1, ..., J \rangle)((Y_{\text{NWP}} - \mu_{\text{NWP}})(b_K; 1, ..., J)),
\end{equation}

\begin{equation}
\hat{\Sigma}_{\text{Obs}}(t_0; s_0) = \Sigma_{\text{Obs}}((t_0; s_0); (t_0; s_0)) + c_0^T \Sigma_{\text{NWP}}^{-1}(\langle b_K; 1, ..., J \rangle; \langle b_K; 1, ..., J \rangle)c_0,
\end{equation}

\begin{equation}
c_0 = \Sigma_{\text{Obs},\text{NWP}}((t_0; s_0); (b_K; 1, ..., J)).
\end{equation}

The distribution (2.14) is used to generate the scenarios of prediction of wind speed in Section 4. This is in fact the predictive distribution presented in (2.6).

3. **Wind data.** In order to improve forecasts from the considered numerical model, two sources of data are combined: ground measurements and WRF model outputs. The measurement data are recorded across an irregular network, and at each observational station, we pick the closest gridded point of NWP outputs. As a result, the two datasets have the same number of spatial locations; however, the proposed model is not restricted to this spatial layout and can handle datasets with different numbers of stations. In the following, the time series of the two datasets are filtered in time by a moving average process over a window of one hour to remove small-scale effects and focus on a larger temporal scale; they are picked every hour.

3.1. **Direct observations.** Observational data are extracted from the Automated Surface Observing System (ASOS) network, available at ftp://ftp.ncdc.noaa.gov/pub/data/asos-onemin. The network of collecting stations covers the U.S. territory. The studied data are 1-minute data selected from the states of Wisconsin, Illinois, Indiana, and Michigan; see Figure 2. The measured wind speed is discretized in integer knots (one knot is about 0.5 m/s). We do not apply any additional treatment to account for this discretization since the data are filtered over a window of 1 hour, see [25] for a discussion of the discretization of wind speed. The orography of this region is simple and flat; however, the presence of Lake Michigan has a strong impact on the wind conditions. Several months are investigated and reveal different behaviors; in particular, periodicities differ from winter to spring and summer months. In the following, for homogeneity purposes the dataset is subdivided in 3 spatial clusters, depicted in Figure 2. A spatial clustering is performed on wind speed in order to distinguish among different average regional weather conditions. This is a proxy for different NWP forecast behaviors. These three clusters are treated independently hereafter.
3.2. **Numerical weather prediction data.** State-of-the-art NWP forecasts are generated by using WRF v3.6 [24] which is a state-of-the-art numerical weather prediction system designed to serve both operational forecasting and atmospheric research needs. WRF has a comprehensive description of the atmospheric physics that includes cloud parameterization, land-surface models, atmosphere-ocean coupling, and broad radiation models. The terrain resolution can go up to 30 seconds of a degree (less than 1 km²). The NWP forecasts are initialized by using the North American Regional Reanalysis fields dataset that covers the North American continent (160W-20W; 10N-80N) with a resolution of 10 minutes of a degree, 29 pressure levels (1000-100 hPa, excluding the surface), every three hours from the year 1979 until the present. Simulations are started every day during January, May and August 2012 and cover the continental United States on a grid of 25x25 km with a time resolution of 10 minutes.

4. **Results.** In this section, we first analyze the estimated parameters and then explore qualitatively and quantitatively the ability of the model to provide accurate forecasts. Three months of the year (January, May, and August) are considered and are studied independently in order to investigate the model performance under different conditions. For each month, the model is trained on two-thirds of the month and validated on the remaining third. The training periods are rolled over all the possible permutations.

4.1. **Analysis of the estimated parameters.** In this section, we investigate the maximum likelihood estimation of the mean and covariance of the process. First, the empirical
mean and covariance are compared with the fitted parametric ones proposed in Section 2. The mean of the process \((Y_{\text{Obs}}, Y_{\text{NWP}})\) is depicted in Figure 3; for each station, the mean at each hour of the day is plotted. The structure of the estimated mean of the two processes is accurately reproduced in terms of temporal and spatial patterns. In Figure 4, the empirical and fitted space-time correlation are plotted. A great part of the structure is captured by the proposed parametric shapes; however, the global shapes tend to be smoothed by the parametric models. The nonseparability between space and time that is visible on the empirical off-diagonal blocks is not entirely captured by the parametric model on the top panels. The analysis of the matrices \(L_s\) that are involved in the covariance model (2.13) reveals different configurations given the subregion and the period of the year. These can be expected since these operators can be interpreted as a linear projector of a process that is common to all the stations. Average air flows differ according to the season and the location; the dependence from a common process that would contain this information is likely to differ in space and in time across the year.

The matrix \(\Lambda\), which appears in both the mean and covariance components, is important since it links the NWP forecasts to the objective predictive quantities. The analysis of \(\Lambda\) reveals that the intensity of temporal dependence varies with the land use; however, the temporal persistence is curtailed to a few hours across the different land-uses.
Fig 4: Empirical (left column) and fitted parametric (right column) space-time correlation estimated in January 2012 in the subregion C2. Top panels: NWP data and NWP parametric model; bottom panels: empirical conditional correlation and associated fitted ones.
The table below shows the parameter values for the model:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NWP mean</td>
<td></td>
</tr>
<tr>
<td>NWP covariance</td>
<td></td>
</tr>
<tr>
<td>Conditional mean</td>
<td></td>
</tr>
<tr>
<td>Conditional covariance</td>
<td></td>
</tr>
</tbody>
</table>

Fig 5: Maximum likelihood estimation of the parameters of the model and associated Gaussian samples. Vertical lines separate the different sets of parameters. From left to right are the parameters of $\mu_{NWP}$, $\Sigma_{NWP}$, $\mu_{Obs|NWP}$, and $\Sigma_{Obs|NWP}$.

In the second step, the uncertainty associated with the estimation of the parameters is accounted for. Following [27], samples from a normal distribution are generated, with the mean given by the maximum likelihood estimated parameters and the covariance given by the inverse of the hessian of the log-likelihood. In Figure 5, the maximum likelihood estimation of the parameters and the associated samples are plotted. The parameters that present the most estimation variance are several parameters $a_i$ that appear in the matrices $L$ in Section 2.3.2. In the parameters of $\mu_{Obs|NWP}$, parameters with a high estimation variance are the ones associated with $\mu$ defined in Section 2.3.3. In these cases, a lack of data in the estimation of these specific parameters may cause this high estimation variance. Notice that the improvement in the predictive variance is less than 5% when the uncertainty is accounted for in the generation of the predictive scenarios in comparison with the predictive variance when the uncertainty on the parameters is not accounted for.

4.2. Assessment of the quality of the predictive model. In this part, samples are generated from the predictive distribution defined by Equation (2.14); they are called scenarios or samples in the following. The mean of these samples can be used as a pointwise prediction, but the objective here is to embed the uncertainty associated with the prediction by working with samples from the predictive distribution.

4.2.1. Qualitative exploration of the predictions. First, as a visual assessment of the prediction, we investigate observed time series and generated predictive scenarios for a part of the months of January and August; see Figure 6. Measured wind speed, which is to be predicted, is plotted as a reference in order to evaluate the accuracy of the prediction. NWP wind forecasts are also plotted because they are predictors and a target to be improved with respect to the measurements. For both months under display, the global trend of the measured time series is well captured by the predictive mean and by
Fig 6: Time series of wind speed at the station with the median RMSE. January 2012 (top) and August 2012 (bottom) for six days. Left panels: 50 predictive samples are plotted; right panel: 3 samples are plotted.

the scenarios. The predictive samples cover the measurements that are to be predicted (see left panels); and the predictive mean realizes, most of the time, an improvement with respect to the NWP forecasts. Moreover, each sample has a temporal dynamics consistent with the observed temporal behavior (right panels). The improvement of the proposed prediction is more visible in August (bottom panels), this is likely because of the periodic components that are stronger in this period of the year and that are well captured by the model; see also Figure 3. Furthermore, the spread of the scenarios is more important in January than in August, likely because of the fact that wind speed has more variability in winter as illustrated in the observed variances in Table 1, which makes it less predictable. We note that the scenarios are not spreading at the end of each prediction window as observed in the literature. The reason is that the NWP predictors are available over the entire prediction window and such spread increase is not obvious in the model–measurement discrepancy.

In Figure 7, mean wind speed at each hour of the day is depicted for the measurements, NWP forecasts, and forecasts from the model at a station that has the median RMSE in subregion C2. The temporal evolution of the mean differs from the measurements to the NWP data; however, the proposed model is able to compensate for this discrepancy well, which is also visible in time series of Figure 6. In Figure 8 we show the mean wind speed at each station in August. The mean is estimated for the measurements, the NWP forecasts, and the predictions from our model. The NWP forecasts show a higher mean than the measurements, especially around the lake, likely due to the parameterization
of the NWP model. The proposed predictive model is able to correct this overestimation and provide a mean consistent with the measured one. Moreover, we note that the spatial structure of measurement is well captured by our forecasts.

The variance of the processes is shown in Figure 9. The proposed model also corrects in space and time the variances that are not well captured by the NWP model. Stations identified as 7 and 9 are near the lake, and the variance present in the NWP forecast is consistent with the mean overprediction. In general, the space-time correlation of the NWP forecasts and the model predictions are relatively consistent with the space-time correlations of measurements, which is reflected in part in the performance metrics considered below.

The spectral content of the scenarios and of the observations is estimated and depicted in Figure 10; the average spectrum of the estimated spectrum on each sample
Fig 9: Variance of the three processes (measurements, NWP outputs and prediction from the proposed model) at each station and each hour of the day in August in subregion C2 is also plotted. The estimated spectra of the scenarios cover most of the spectrum of the observations. The overall shape of the estimated spectrum and of the average spectrum indicates a robust agreement, especially in August where small frequencies are accurately captured. In this and other spectral estimates, the spectral content at high frequency is sometimes slightly overpredicted, we believe this because the forecasts do not attempt to correct for discontinuities at the boundaries between temporal blocks. Nevertheless, the features of the spectrum of the measurements appear well captured by our model. Therefore, our model appears to be appropriate as a realistic wind scenario generator.

4.2.2. **Quantitative assessment of the quality of the predictions.** As our second step, we assess quantitatively the overall improvement of the model in comparison with the WRF model outputs; see Table 1. We study general metrics since there are no specific user-applications here; however, we expect similar performances when using specific metrics. The root mean square error (RMSE) is computed for the predictive mean of the proposed distribution and for the NWP forecasts. We consider also the energy score (ES), which represents a multivariate generalization of the continuous ranked probability score (CRPS) (see [15, 19]). This metric is an omnibus metric that enables comparison of ensemble forecasts and scenarios with pointwise prediction; it is computed on predictive samples and on NWP forecasts. The energy score is a proper scoring rule, the lower the energy score, the better the proposed forecast.

In subregion $C_2$, the model shows the greatest improvement in terms of RMSE and energy score, likely because of the presence of Lake Michigan. Indeed, the NWP embeds this presence through the lake mask and land use, but this may be overestimated in comparison with the behaviors of the observations. The improvement of RMSE is more significant in May and August, likely because of the periodic components that are well captured by the model. The energy score clearly favors the proposed model in comparison
to the WRF outputs. The mean of the observations is well captured by the prediction made with the model. The variance is sometimes overestimated depending the subregion and the period of the year, but most of the variances are well reproduced.

5. Conclusions. We have introduced a statistical space-time modeling framework for the prediction of atmospheric wind speed based on deterministic numerical weather predictions and historical measurements. We have used a Gaussian multivariate space-time process that combines multiple sources of past physical model outputs and measurements along with model predictions to forecast wind speed at observation sites. We applied this strategy on ground-wind-speed forecasts for a region near the U.S. Great Lakes. The results show that the prediction is improved in the mean-squared sense as well as in probabilistic scores. Moreover, the samples are shown to produce realistic wind scenarios based on the sample spectrum. Using the proposed model, one can enable to correct the first- and second-order space-time structure of the numerical forecasts in order to match the structure of the measurements.

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Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>ES M/S</th>
<th>Mean(Y_{obs})</th>
<th>Var(Y_{obs})</th>
</tr>
</thead>
<tbody>
<tr>
<td>NWP (Jan. 2012, C_1)</td>
<td>1.85</td>
<td>M 4.6</td>
<td>5.31</td>
<td></td>
</tr>
<tr>
<td>Model (Jan. 2012, C_1)</td>
<td>1.65 (10.9%)</td>
<td>S 4.58</td>
<td>5.48</td>
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<tr>
<td>NWP (May 2012, C_1)</td>
<td>2.97</td>
<td>M 2.87</td>
<td>2.78</td>
<td></td>
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<tr>
<td>Model (May 2012, C_1)</td>
<td>1.9 (36%)</td>
<td>S 3.14</td>
<td>5.8</td>
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<tr>
<td>NWP (Aug. 2012, C_1)</td>
<td>1.73</td>
<td>M 2.49</td>
<td>2</td>
<td></td>
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<tr>
<td>Model (Aug. 2012, C_1)</td>
<td>1.13 (34.9%)</td>
<td>S 2.55</td>
<td>2.39</td>
<td></td>
</tr>
<tr>
<td>NWP (Jan. 2012, C_2)</td>
<td>2.55</td>
<td>M 4.32</td>
<td>5.05</td>
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<tr>
<td>Model (Jan. 2012, C_2)</td>
<td>1.81 (29%)</td>
<td>S 4.55</td>
<td>6.37</td>
<td></td>
</tr>
<tr>
<td>NWP (May 2012, C_2)</td>
<td>3.29</td>
<td>M 3.56</td>
<td>4.47</td>
<td></td>
</tr>
<tr>
<td>Model (May 2012, C_2)</td>
<td>1.86 (43.4%)</td>
<td>S 3.71</td>
<td>3.83</td>
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</tr>
<tr>
<td>NWP (Aug. 2012, C_2)</td>
<td>1.9</td>
<td>M 2.29</td>
<td>2.48</td>
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<tr>
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<td>1.31 (40.4%)</td>
<td>S 2.39</td>
<td>2.68</td>
<td></td>
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<tr>
<td>NWP (Jan. 2012, C_3)</td>
<td>2.05</td>
<td>M 4.29</td>
<td>5.04</td>
<td></td>
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<tr>
<td>Model (Jan. 2012, C_3)</td>
<td>1.82 (20%)</td>
<td>S 4.32</td>
<td>9.48</td>
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<tr>
<td>NWP (May 2012, C_3)</td>
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<td>3.36</td>
<td></td>
</tr>
<tr>
<td>Model (May 2012, C_3)</td>
<td>1.85 (21%)</td>
<td>S 3.39</td>
<td>6.44</td>
<td></td>
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<tr>
<td>NWP (Aug. 2012, C_3)</td>
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<td>2.22</td>
<td></td>
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<tr>
<td>Model (Aug. 2012, C_3)</td>
<td>1.22 (28.9%)</td>
<td>S 2.31</td>
<td>2.15</td>
<td></td>
</tr>
</tbody>
</table>

Statistics and metrics for the station, representing the median RMSE in each cluster denoted as C_i, for i = 1, 2, 3. ES is the energy score and M/S is measurements or samples. They are evaluated on the concerned month for time prediction. Associated with the model RMSE is the percentage of improvement of the model with respect to the NWP data.

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