# Distributed Generation for Buildings:

## A Multi-Scale Optimization Approach

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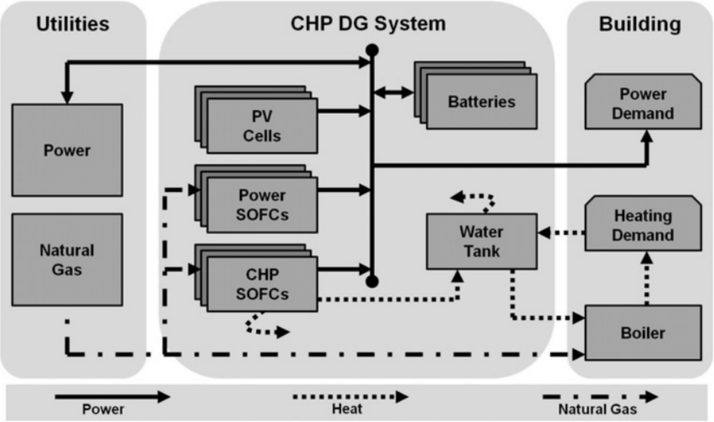
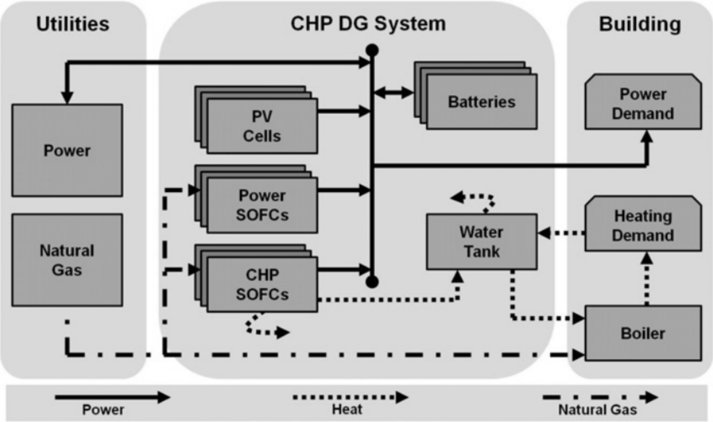
# Distributed generation for buildings




SOFC = Solid-Oxide Fuel Cell CHP = Combined Heat and Power

Pruitt, Braun, Newman 2013

# Distributed generation for buildings




SOFC = Solid-Oxide Fuel Cell    CHP = Combined Heat and Power

Pruitt, Braun, Newman 2013

- Current investment decisions: What technology? How many units?
- Long-term operation scheduling: When to turn on/off machines?

# Distributed generation for buildings

1. What new technologies to implement? How many units?
   - Current design problem: $\approx 10$ integer variables
2. How to operate over a period of 10-20 years?
   - Long-term operation problem: $\mathcal{O}(10^6)$ binary variables
3. How to cope with uncertainty in problem data?
   - Future demands and prices: $\mathcal{O}(10^4)$ scenarios

$\Rightarrow$ Large two-stage stochastic mix-integer program (MIP)

In this talk, we focus on the first two issues

# Two-stage mixed-integer program (MIP)

| | | | |
|---|---|---|---|
| minimize | current design (1st-stage cost) | + | long-term operation (2nd-stage cost) |

subject to demand constraints (e.g., heat, power)

coupling over technologies

dynamics constraints (e.g., storage, battery)

coupling over time periods

operation constraints (e.g., technology units on/off)

other constraints (e.g., max power output)

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- Large MIP ⇒ beyond the scope of current MIP solvers
- Coupling constraints ⇒ Not amenable to decomposition techniques (fixing 1st-stage decisions does not decouple 2nd-stage problem)

# Objective

$$
\begin{aligned}
\underset{n,p,w,u,u^{\max}}{\text{minimize}} \quad & \sum_j C_j\, n_j && \text{capital cost (1st-stage)} \\
& + \sum_{j,t\in\mathcal{T}} M_j\, p_{jt} && \text{operating \& maintenance cost} \\
& + \sum_{i,j,t\in\mathcal{T}} W_j\, w_{ijt} && \text{switching cost} \\
& + \sum_{t\in\mathcal{T}} P_t\, u_t && \text{purchased power cost}
\end{aligned}
$$

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& + \sum_{t\in\mathcal{T}} P_t \, u_t && \text{purchased power cost} \\
& + \sum_{m\in\mathcal{M}} P_m^{\max} \, u_m^{\max} && \text{maximum purchased power cost}
\end{aligned}$$

hours $\mathcal{T} = \{1, 2, \ldots\}$, months $\mathcal{M} = \{\text{January, February, } \ldots\}$

$$u_m^{\max} \geq u_t, \quad \forall t \in \{\text{hours in month } m\}$$

INTER-TEMPORAL COUPLING

# Coupling constraints

- Maximum purchased power constraint

$$u_m^{\max} \geq u_t, \quad \forall t \in \{\text{hours in month } m\}$$

coupling over hours in months

- Storage with lossy coefficient $Q \in (0, 1)$

$$s_{t+1} = (1 - Q)s_t + s_t^{\text{in}} - s_t^{\text{out}}, \quad \forall t \in \mathcal{T}$$

coupling over immediate hours

- Power demand in hour $t$

$$\sum_j p_{jt} + u_t + s_t^{\text{out}} \geq D_t, \quad \forall t \in \mathcal{T}$$

coupling over technologies (e.g., Power SOFCs, CHP SOFCs)

# Binary variables

- On-off decisions

$$y_{ijt} \in \{0,1\} \qquad \text{for unit } i \text{ of tech } j \text{ at time } t$$

- Switching decisions

$$w_{ijt} \geq y_{ij(t+1)} - y_{ijt} \quad \text{switching-on decision}$$

$$w_{ijt} \geq y_{ijt} - y_{ij(t+1)} \quad \text{switching-off decision}$$

$$\Rightarrow \quad w_{ijt} \in \{0,1\}$$

# Challenges of the two-stage MIP

- A small number ($\approx 10$) of integral variables at 1st-stage
- A large number $\mathcal{O}(10^6)$ of binary variables at 2nd-stage

| | |
|---|---|
| Hourly operation for 10 years | 87,600 hours $\times$ |
| Multiple units over several technologies | 12 units |
| | $> 10^6$ binary variables |

- Coupling constraints over time and technologies

Not obvious how to decompose over time periods

# Outline

- Exploit multi-period structure

  (daily, weekly, and yearly demand cycles for buildings)

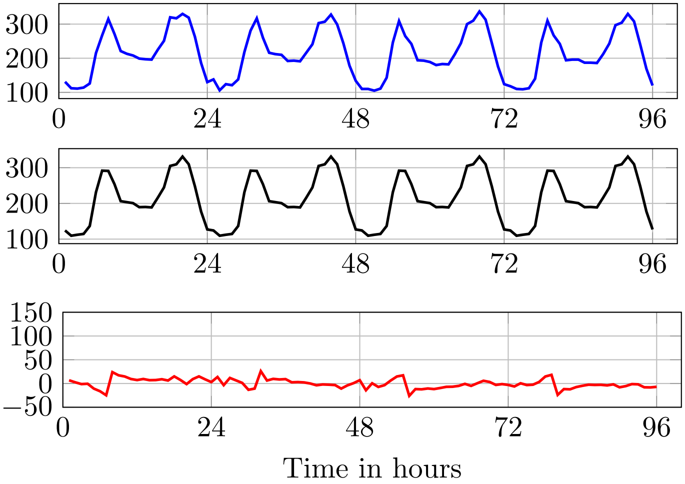
- Column generation for dimension reduction

  (parameterization in daily operation profiles)

- Computational results

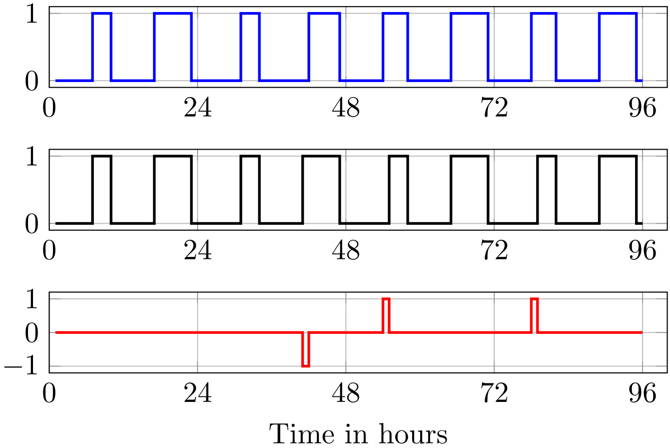
  (tractable problems with performance bounds)

# Demand profiles = periodic profile + perturbations




Similar cyclic structure in **weekly**, **monthly**, and **yearly** demand

# On-off operational profiles = base profile + variations



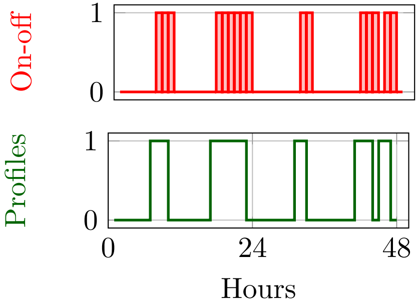

Coarsen **hourly** on/off operations to **daily** profiles

Hope: a small number of daily profiles suffice

# Coarsening hourly on/off decisions

Replace hourly $y_t \in \{0,1\}$ by daily profiles $Z_k$

$$\begin{pmatrix} y_{24(d-1)} \\ \vdots \\ y_{24d-1} \end{pmatrix} = \sum_{k\in\mathcal{K}} \lambda_{dk} Z_k \quad \text{and} \quad \lambda_{dk} \in \{0,1\}, \sum_{k\in\mathcal{K}} \lambda_{dk} = 1$$




## Coarsening

Choose different daily operational profile to meet demand.

... reduce problem size of two-stage MIP by order of magnitude

# Column generation algorithm

1. Generate a database of columns (daily profiles)
2. Initialize active set of columns
3. Solve two-stage MIP parameterized with active columns
4. Compute negative reduced cost for inactive columns
5. Add inactive columns with most negative reduced costs
6. Go to Step 3 until stopping criteria are satisfied

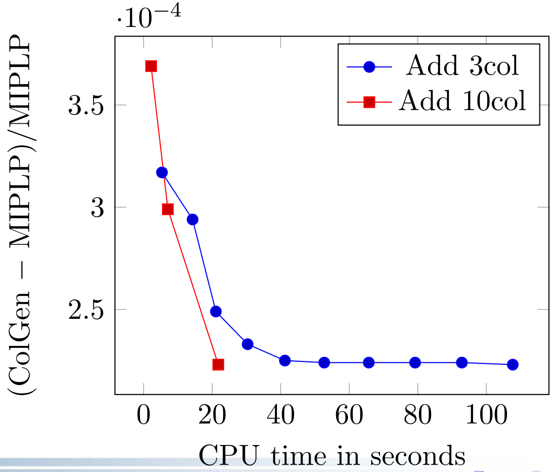
Provides a feasible solution $\Rightarrow$ an upper bound

Relaxes second-stage integrality (MIPLP) $\Rightarrow$ a lower bound

# One-month example

| # of integer | binary | linear vars. | constraints | GAMS/Cplex |
|---|---|---|---|---|
| 4 | 5, 208 | 13, 423 | 25, 823 | 1,020 seconds |

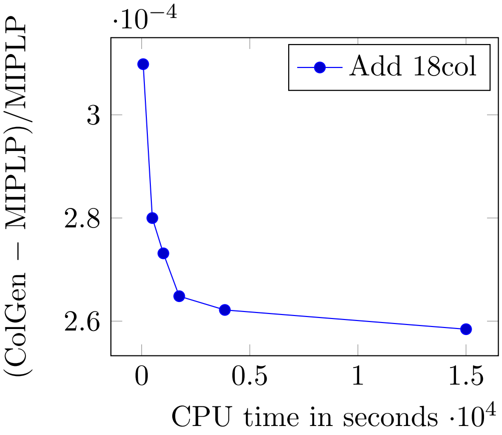
Column generation



# One-year example

| # of integer | binary | linear vars. | constraints | GAMS/Cplex |
|---|---|---|---|---|
| 4 | 63,875 | 158,045 | 322,295 | Out of memory (3340Mb) |

Column generation



# Conclusions

- Formulated two-stage MIP for distributed generation
  - Two-stage MIP with $\mathcal{O}(10^6)$ binary variables
  - Coupling constraints over time periods and technologies
- Column generation framework for dimension reduction
- Numerical results are promising

  Solve one-year model with relative gap $\mathcal{O}(10^{-4})$ in 4 hours

  Ongoing work:
  - Coarsening dual variables (constraints)
  - Analysis of solution quality from ColGen
  - Test on data from different building types

# Coarsening dual variables (constraints)

- Parameterize using only daily profiles (Semi-Coarse Model)
- Reduce number of constraints (Coarse Model)