A Two-Level Approach to Large Mixed-Integer Programs with Application to Cogeneration in Buildings

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Two-stage mixed-integer linear program (MILP)

$$
\begin{aligned}
\underset{y,x,v,w}{\text{minimize}} \quad & a^T y + b^T x + c^T v + d^T w \\
\text{subject to} \quad & y \in \mathbb{Z}_+^m, \quad x \in \{0,1\}^N \\
& v \in \mathbb{R}^N,\ Lx \le v \le Ux \\
& w \in \mathbb{R}^N,\ w \ge 0 \\
& Ay + Bx + Cv + Dw \le f
\end{aligned}
$$

- 1st-stage integer variables $y \in \mathbb{Z}_+^m$: $m \approx 10$
- 2nd-stage binary variables $x \in \{0,1\}^N$: $N \approx 10^6$
- Coupling constraints between 1st- and 2nd-stage: $\mathcal{O}(10^6)$
- No sparsity assumptions $\Rightarrow$ Not amenable to decomposition method

Cogeneration in commercial buildings

Renewable technologies: batteries, fuel cells, solar panels, ...

1. What new technologies to invest? How many units?
   - Current design problem: $\approx$ 10 integer variables
2. How to operate over a period of 10-20 years?
   - Long-term operation problem: $\mathcal{O}(10^6)$ binary variables

$$24 \text{ hours } \times 365 \text{ days } \times 10 \text{ years } \times 12 \text{ units } = 1,051,200$$

MILP model for cogeneration problem

$$
\begin{aligned}
\underset{y,p,g,u,u^{\max},w}{\text{minimize}} \quad & \sum_j C_j\, y_j && \text{capital cost (1st-stage)} \\
& + \sum_j \sum_{t \in \mathcal{T}} G_t\, g_{jt} && \text{gas cost} \\
& + \sum_j \sum_{t \in \mathcal{T}} M_j\, p_{jt} && \text{maintenance cost} \\
& + \sum_{t \in \mathcal{T}} P_t\, u_t && \text{purchased power cost} \\
& + \sum_{m \in \mathcal{M}} P_m^{\max}\, u_m^{\max} && \text{max power cost} \\
& + \sum_j \sum_{t \in \mathcal{T}} \sum_i W_j\, w_{ijt} && \text{switching cost}
\end{aligned}
$$

Demand constraints (coupling over technologies)

- Power demand

$$\sum_j \underbrace{p_{jt}}_{\text{power from techs}} + \underbrace{u_t}_{\text{purchased}} + \underbrace{b_t^{\text{out}}}_{\text{battery}} \geq D_t^P$$

- Heat demand

$$\sum_j \underbrace{q_{jt}}_{\text{heat from techs}} + \underbrace{q_t^{\text{out}}}_{\text{storage}} \geq D_t^Q$$

$t \in \text{Time} \qquad j \in \text{Techs (e.g., fuel cells)}$

Dynamical constraints (coupling over time)

- Power storage

$$b_{t+1} = (1 - L^P) b_t + b_t^{\text{in}} - b_t^{\text{out}}$$

- Heat storage

$$s_{t+1} = (1 - L^Q) s_t + q_t^{\text{in}} - q_t^{\text{out}}$$

efficiency loss coefficient $L^P, L^Q \in (0, 1)$

- Boundary condition

$$b_{\text{initial}} = b_{\text{final}}, \qquad s_{\text{initial}} = s_{\text{final}}$$

Operational constraints (binary variables)

- On-off constraints for technologies

$$x_{ijt} \;\in\; \{0,1\}$$

- Switching constraints (coupling over time for binary variables)

$$w_{ijt} \;\geq\; x_{ij(t+1)} \;-\; x_{ijt} \quad \text{switching-on decision}$$

$$w_{ijt} \;\geq\; x_{ijt} \;-\; x_{ij(t+1)} \quad \text{switching-off decision}$$

$$\Rightarrow \quad w_{ijt} \in \{0,1\} \text{ at optimal solution}$$

Other constraints

- Power rating of technology

$$R_j^{\min} x_{ijt} \le p_{ijt} \le R_j^{\max} x_{ijt}$$

- Gas consumption

$$\underbrace{E_j^P}_{\text{electric efficiency}} \times g_{jt} = p_{jt}$$

- Heat generation

$$\underbrace{q_t^{\text{in}}}_{\text{input to storage}} \le \sum_j \underbrace{E_j^Q}_{\text{thermal efficiency}} \times g_{jt}$$

An example using commercial MILP solver: CPLEX

Days	Binary	Continuous	Constraint
4	1,152	2,994	6,698
7	2,016	5,226	11,738
14	4,032	10,434	23,498
28	8,064	20,850	47,018
84	24,192	62,514	$1.41 \cdot 10^5$
364	$1.05 \cdot 10^5$	$2.71 \cdot 10^5$	$6.11 \cdot 10^5$

- Problem size increases linearly with horizon length
- A ten-year model:

  $\approx$ 10^6 binary 2.7×10^6 continuous 6.1×10^6 constraints

An example using commercial MILP solver: CPLEX

Days	Time(s)	Nodes	LP-iter	Bat	Boil	Chp	Pow	Stor
4	2	0	4,000	1	1	2	0	6
7	241	2,807	$4.49 \cdot 10^5$	2	1	0	1	0
14	1,210	10,352	$2.73 \cdot 10^6$	2	1	0	1	0
28	5,690	47,770	$6.02 \cdot 10^6$	2	1	0	1	0
84	18,000	11,179	$2.37 \cdot 10^6$	4	1	0	1	0

84-day reaches 5-hour limit, but relative gap still $\geq 5\%$

- Exponential increase in complexity

  (Can't solve a 1-year problem)

- First-stage decisions vary as we change the horizon length

  (Need to solve long horizon problems)

Two ideas

- Primal (variable) coarsening

  Daily profile representation to coarsen variables

- Dual (constraint) coarsening

  Aggregation of constraints to coarsen constraints

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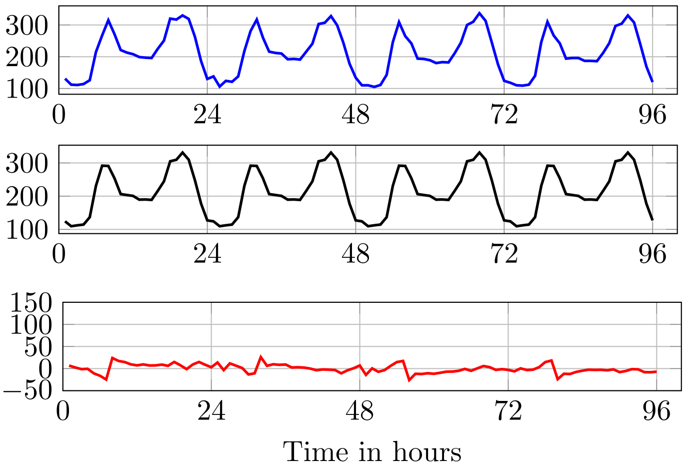
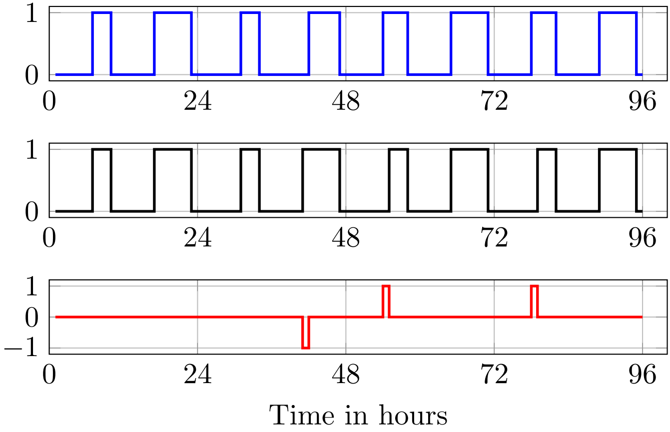
Demand profiles = periodic profile + perturbations



Similar cyclic structure in **weekly**, **monthly**, and **yearly** demand

On-off operational profiles = base profile + variations

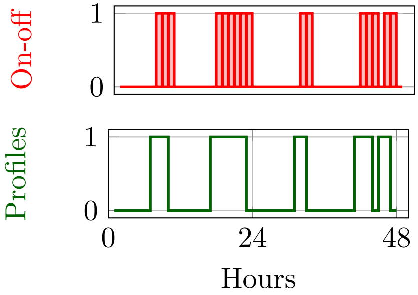


Coarsen **hourly** on/off operations to **daily** profiles

Coarsening hourly on/off decisions

Replace hourly $x_t \in \{0,1\}$ by daily profiles $X_k \in \{0,1\}^{24}$

$$\begin{pmatrix} x_{24(i-1)} \\ \vdots \\ x_{24i-1} \end{pmatrix} = \sum_{k \in \mathcal{K}} \bar{x}_{ik} X_k \quad \text{and} \quad \bar{x}_{ik} \in \{0,1\}, \quad \sum_{k \in \mathcal{K}} \bar{x}_{ik} = 1$$

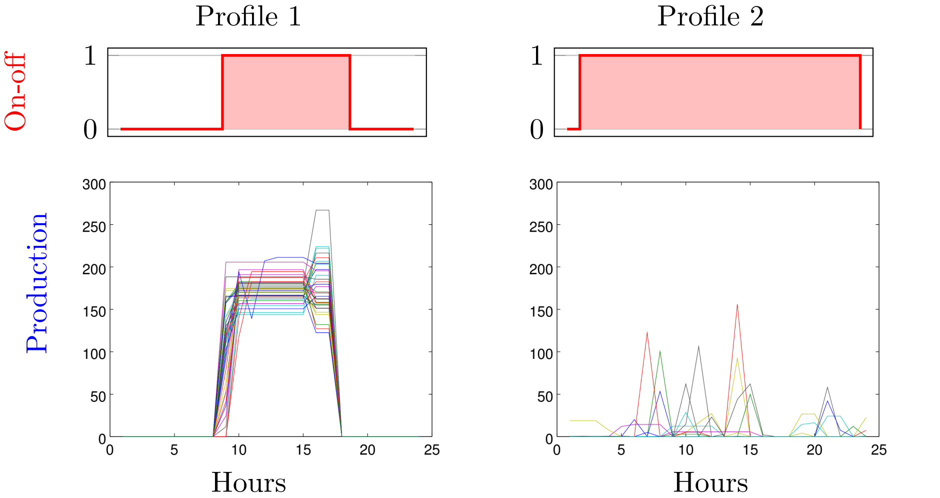


Coarsening

Choose different daily operational profile to meet demand

... reduce problem size of MILP by order of magnitude

Coarsening hourly continuous variables



$$v_i = \sum_{k=1}^{K}\sum_{j=1}^{I_k} \bar{v}_{ijk} V_{jk}, \quad 0 \le \bar{v}_{ijk} \le 1, \quad \sum_{k=1}^{K}\sum_{j=1}^{I_k} \bar{v}_{ijk} = 1, \quad \sum_{j=1}^{I_k} \bar{v}_{ijk} = \bar{x}_{ik}$$

Primal variable coarsening (Semi-coarse model)

$$\underset{y,\bar{x},\bar{v},\bar{w}}{\text{minimize}} \quad a^T y + \bar{b}^T \bar{x} + \bar{c}^T \bar{v} + \bar{d}^T \bar{w}$$

$$\text{subject to} \quad \sum_{k=1}^{K} \bar{x}_{ik} = 1, \quad \bar{x}_{ik} \in \{0,1\} \quad \text{Coarsened binary vars.}$$

$$\sum_{k=1}^{K} \sum_{j=1}^{I_k} \bar{v}_{ijk} = 1, \quad \sum_{j=1}^{I_k} \bar{v}_{ijk} = \bar{x}_{ik}, \quad 0 \le \bar{v}_{ijk} \le 1$$

$$\sum_{j=1}^{J} \bar{w}_{ij} = 1, \quad 0 \le \bar{w}_{ij} \le 1 \quad \text{Coarsened continuous vars.}$$

$$Ay + \bar{B}\bar{x} + \bar{C}\bar{v} + \bar{D}\bar{w} \le f \quad \text{Profiles embedded in } \bar{B}, \bar{C}, \bar{D}$$

Feasible solution and upper bound

Fact: Let $(\bar{x}, \bar{v}, \bar{w})$ be a feasible point of the semi-coarse model. Then it follows that the corresponding fine-scale variables (x, v, w) are feasible in the original MILP.

Fact: The semi-coarse model is a tightening of the original MILP, and its solution provides an upper bound. The two problems are equivalent if the optimal profiles from the solution of are included in the semi-coarse model.

Dual constraint coarsening (Coarse model)

- Still have many hourly constraints

$$Ay + \bar{B}\bar{x} + \bar{C}\bar{v} + \bar{D}\bar{w} \le f$$

- Sum m consecutive rows over one period (e.g., $m = 24$)

$$\cdots + \begin{pmatrix} C_{11} \\ \vdots \\ C_{m1} \end{pmatrix} \bar{v}_1 + \cdots + \begin{pmatrix} C_{1n} \\ \vdots \\ C_{mn} \end{pmatrix} \bar{v}_n + \cdots \le \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix}$$

$$\Rightarrow \cdots + \left(\sum_{t=1}^{m} C_{t1} \right) \bar{v}_1 + \cdots + \left(\sum_{t=1}^{m} C_{tn} \right) \bar{v}_n + \cdots \le \sum_{t=1}^{m} f_t$$

Reduce the number of constraints by a factor of m

Related work on aggregation techniques

- Zipkin 1980, 1981, ...
  Bounds on row/column-aggregation in linear programs
- Birge 1985, Clay and Grossmann 1997, ...
  Constraint aggregation in stochastic programs
- Balas 1965, Glover 1968, 1977, Geoffrion 1969, ...
  Surrogate constraints in pure integer programs
- Rogers et al. '91, ...
  Aggregation and disaggregation in optimization
- Elhallaoui et al. '05, ...
  Dynamic aggregation of constraints in column generation

Dual constraint coarsening (Coarse model)

$$
\begin{aligned}
\underset{y,\bar{x},\bar{v},\bar{w}}{\text{minimize}} \quad & a^T y + \bar{b}^T \bar{x} + \bar{c}^T \bar{v} + \bar{d}^T \bar{w} \\
\text{subject to} \quad & \sum_{k=1}^{K} \bar{x}_{ik} = 1, \quad \bar{x}_{ik} \in \{0,1\} \quad \text{Coarsened binary vars.} \\
& \sum_{k=1}^{K} \sum_{j=1}^{I_k} \bar{v}_{ijk} = 1, \quad \sum_{j=1}^{I_k} \bar{v}_{ijk} = \bar{x}_{ik}, \quad 0 \le \bar{v}_{ijk} \le 1 \\
& \sum_{j=1}^{J} \bar{w}_{ij} = 1, \quad 0 \le \bar{w}_{ij} \le 1 \quad \text{Coarsened continuous vars.} \\
& \hat{A} y + \hat{\hat{B}} \bar{x} + \hat{\hat{C}} \bar{v} + \hat{\hat{D}} \bar{w} \le \hat{f} \quad \text{Aggregated daily constraints}
\end{aligned}
$$

- Add violated constraints and resolve until satisfying all constraints in the semi-coarse model

MILPs do NOT warm start (Use LP-relaxation warm start)

Algorithm for solving the semi-coarse model

Phase I: **LP warm-start**

Solve LP-relaxation of coarse model

while *Not feasible in the LP-relaxation of the semi-coarse model* **do**

  Add violated constraints

  Solve the LP-relaxation of the coarse model

end

Phase II: **MILP iterations**

Solve coarse MILP with constraints identified in Phase I

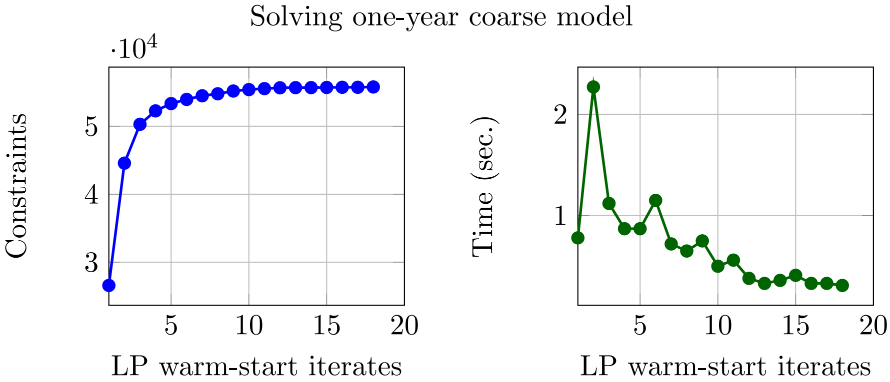
while *Not feasible in the semi-coarse MILP model* **do**

  Add violated constraints

  Solve the coarse MILP model

end

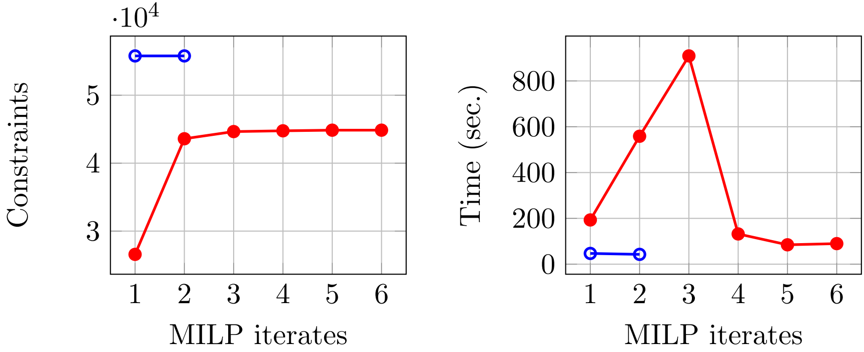
LP warm-start phase



- Fewer constraints added as LP iterates progress
- Almost negligible cost compared with MILP

Effect of LP warm-start

Solving one-year coarse model with or without LP warm-start



With LP warm-start:

- Fewer MILP iterates
- Orders of magnitude faster in each MILP iterate

observed in different types of building examples

So far ...

A large two-stage MILP with coupling constraints

$\Rightarrow$ beyond state-of-the-art MILP solvers

- Primal variable coarsening

  Daily profile representation $\Rightarrow$ semi-coarse MILP model

- Dual constraint coarsening

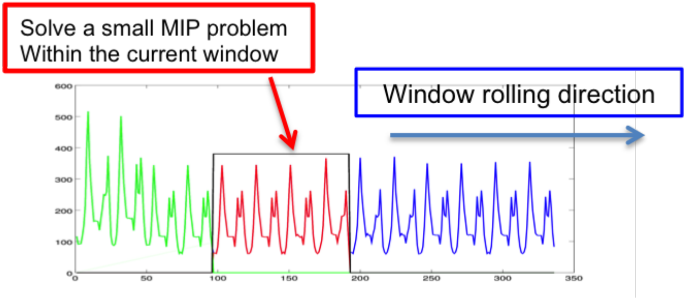
  Constraint aggregation $\Rightarrow$ coarse MILP model

- Iterative resolve coarse model with added constraints

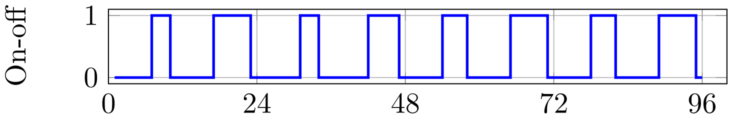
Next: Profile generation and selection

Profile generation: Moving horizon method

- Solve a large number of **small full model** MILPs



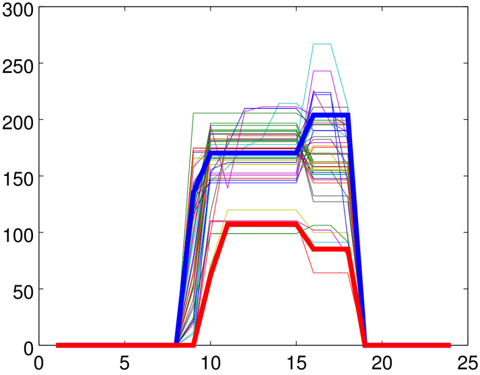
- Extract profiles from solutions of small MILPs



$\Rightarrow$ 4 daily on-off profiles

Profile selection

- Sampling techniques (uniform or frequency-based)
- Envelope methods (profiles with max/min ℓ_1 or ℓ_∞ norms)
- Clustering algorithms (k-means with ℓ_2-distance)



k-means with 2-3 cluster centers (bold) works well in practice

Solving coarse MILPs for a ten-year model

Iter	Binary	Continuous	Constraint	Objective Value
1	131,040	254,935	589,321	3,797,389.08
2	131,040	254,935	589,617	3,797,565.30
3	131,040	254,935	589,629	3,797,568.94

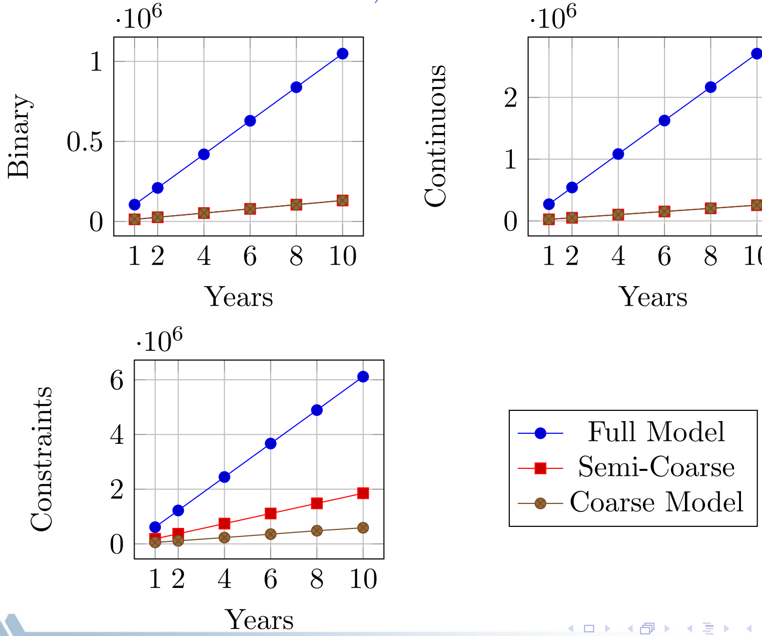
Iter	Time	Nodes	LP-iter	Bat	Boil	Chp	Pow	Stor
1	7,821.83	3,620	$1.06 \cdot 10^6$	0	1	1	1	1
2	1,341.28	472	$2.38 \cdot 10^5$	0	1	1	1	1
3	1,125.19	938	$1.47 \cdot 10^5$	0	1	1	1	1

Solution of the first iterate is remarkably good!

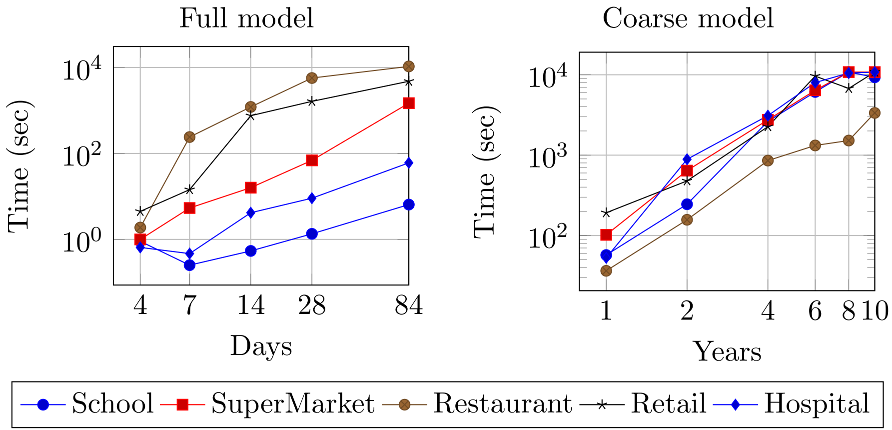
- Close objective value (up to 4 digits)
- Consistent first-stage solutions

observed in different types of building examples

Problem size for the full, semi-coarse and coarse models



Solution time for different types of building examples



- Much more scalable with time horizon
- Less variability with different buildings

Conclusions

- A multi-level approach for large two-stage MILP

  with $O(10^6)$ binary, continuous vars, and coupling constraints

  - Profile representation for binary and continuous variables
  - Aggregation of constraints with LP warm-start

  LP warm-start works very well for MILP resolves

More details: Preprint ANL/MCS-P5332-0415

Fu Lin, Sven Leyffer, and Todd Munson

A Two-Level Approach to Large Mixed-Integer Programs with Application to Cogeneration in Energy-Efficient Buildings

AMPL codes publicly available at

http://www.mcs.anl.gov/~fulin/codes/DistrGen.zip.

Wish list and extensions

- Wish list:
  1. How to analyze upper bounds?
  2. How to obtain lower bounds?
  3. How to dynamically add/remove profiles?
- Extensions:
  1. Transmission expansion problem
     (Francisco Munoz & Jean-Paul Watson at Sandia)
  2. Multi-level graph analysis for MILP
     (Mahantesh Halappanavar at PNNL)
  3. Connections to multigrid and receding-horizon control
     (Victor Zavala at ANL)