New characterizations of social influence in social networks

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joint work with:

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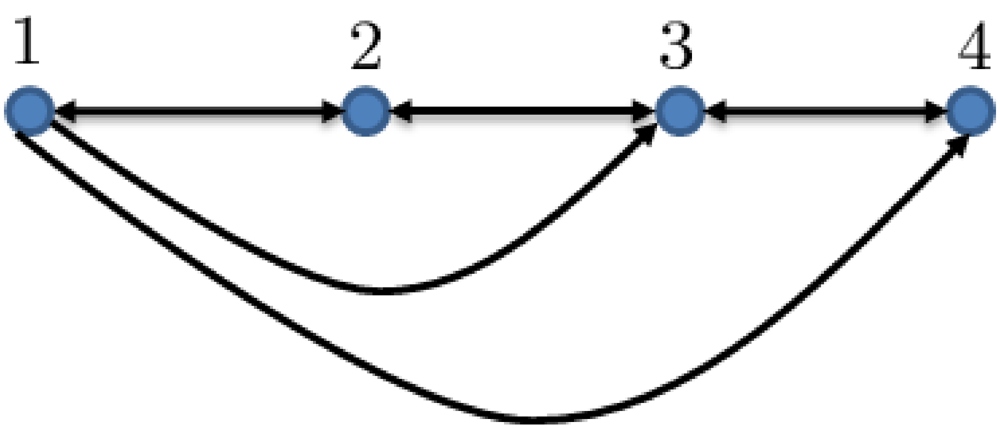
DeGroot model

- Agent's belief

$$x_i \in [0,1]$$

- Belief update

$$\begin{bmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} & W & \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix}$$



$$W = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/3 & 0 & 1/3 & 1/3 \end{bmatrix}$$

⋆ Row stochastic matrix

$$W\mathbb{1} = \mathbb{1}$$

⋆ Strongly connected graphs $\Rightarrow$ consensus

⋆ Spectrum

$$\lambda_1(W) = 1, \qquad |\lambda_i(W)| < 1, \qquad i = 2, \ldots, n$$

DeGroot '74, DeMarzo *et al.* '03, Golub and Jackson '10, ...

OTHER MODELS

Blondel *et al.* '09, Acemoglu and Ozdaglar '10, Yildiz *et al.* '11, Jadbabaie *et al.* '12
Mohajer and Touri '13, Etesami *et al.* '13, Ghaderi and Srikant '13

Steady-state social influence

- Consensus belief

$$\lim_{k \to \infty} x(k) = \lim_{k \to \infty} W^k x(0)$$

$$= \mathbb{1}\, \mu^T x(0)$$

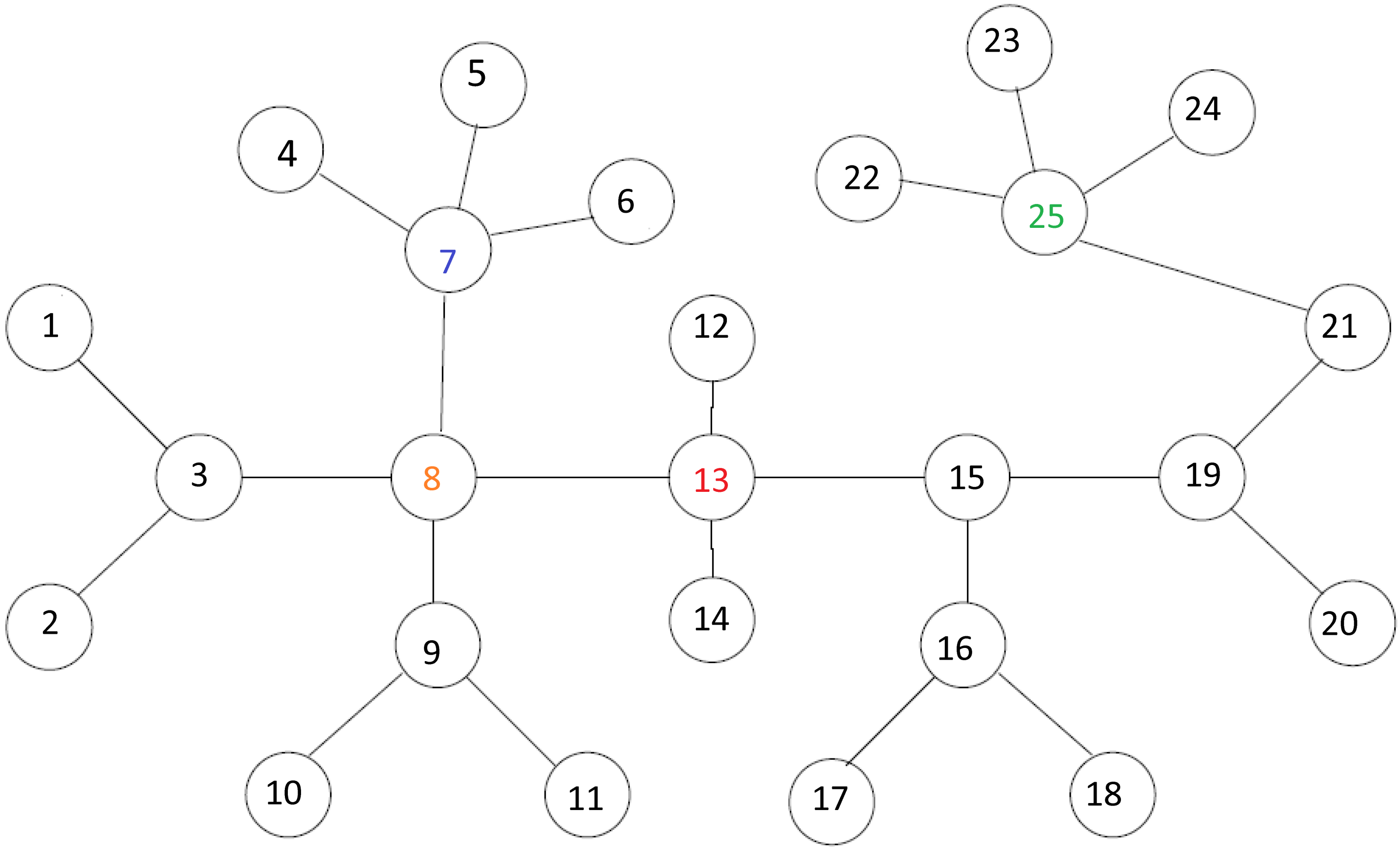
left eigenvector: $\mu^T W = \mu^T$

$\mu_i \rightarrow$ social influence of agent i

DeMarzo *et al.* '03

SHORTCOMING: does not capture transient behavior

A motivating example



IDENTICAL influence: $13, 8, 7, 25$

Alternative characterization

order of influence: $13 > 8 > 7 > 25$

Outline

- Social influence that accounts for transient behavior

- Identification of influential agents in social networks

- Elementwise convexity and coordinate descent

Alternative characterization of social influence

Forceful agents: do not update beliefs $x_{\mathrm{FA}}(k) \equiv \alpha$

Regular agents: consensus-type belief update

$$x_{\mathrm{RA}}(k) \to \alpha \quad \text{as} \quad k \to \infty$$

- INFLUENCE OF FORCEFUL AGENTS
  - ⋆ Decay rate
  - ⋆ Cumulative effect

Simplifying assumptions: $\alpha = 0$ $x_{\mathrm{RA}}(0) = \mathbb{1}$

Transient behavior

$$\begin{bmatrix} x_{\mathrm{FA}}(k+1) \\ x_{\mathrm{RA}}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ W_0 & W_{\mathrm{RA}} \end{bmatrix} \begin{bmatrix} x_{\mathrm{FA}}(k) \\ x_{\mathrm{RA}}(k) \end{bmatrix}$$

$$x_{\mathrm{RA}}(k+1) = W_{\mathrm{RA}}\, x_{\mathrm{RA}}(k)$$

- Decay rate

$$\max_i \{ |\lambda_i(W_{\mathrm{RA}})| \} < 1$$

- Cumulative effect (ℓ_1 norm)

$$\sum_{k=0}^{\infty} x_{\mathrm{RA}}(k) = (I + W_{\mathrm{RA}} + W_{\mathrm{RA}}^2 + \cdots)\, \mathbb{1} = (I - W_{\mathrm{RA}})^{-1}\, \mathbb{1}$$

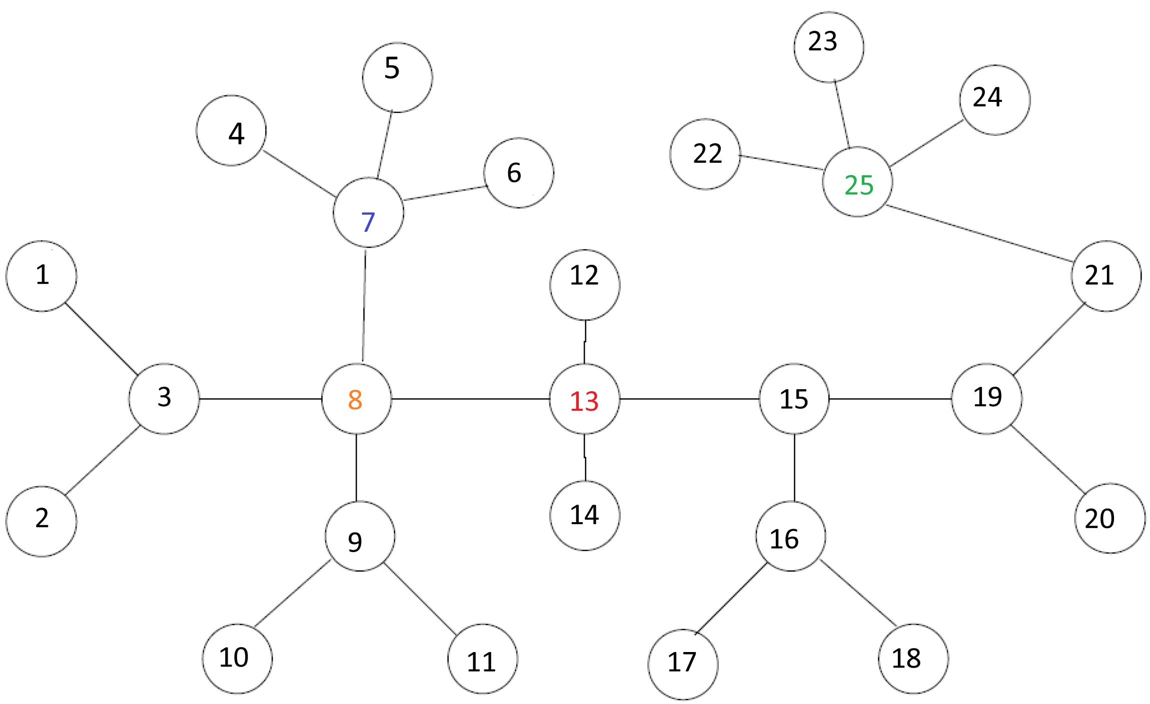
Markov chain interpretation: Expected number of steps before absorption

Total cumulative effect

$$J = \sum_{k=0}^{\infty} \mathbb{1}^T x_{\mathrm{RA}}(k)$$

$$= \mathbb{1}^T (I - W_{\mathrm{RA}})^{-1} \mathbb{1}$$

smaller total cumulative effect $\Rightarrow$ bigger influence of FAs



Order of influence:

$13 > 8 > 7 > 25$

Design of social networks

- Optimal selection of forceful agents

- Creation of optimal social links (in the paper)

- Elementwise convexity

- Coordinate descent method

Optimal selection of forceful agents

$$\begin{array}{ll} \underset{\phi}{\text{minimize}} & f(\phi) \;=\; \mathbb{1}^T (I \,-\, (I \,-\, \text{diag}(\phi))\, W\, (I \,-\, \text{diag}(\phi)))^{-1} \mathbb{1} \\ \text{subject to} & \phi_i \;\in\; \{0,1\}, \qquad i \;=\; 1,\ldots,n \\ & \mathbb{1}^T \phi \;=\; N_{\text{FA}} \end{array}$$

Two sources of nonconvexity

⋆ Boolean constraints

⋆ Objective function

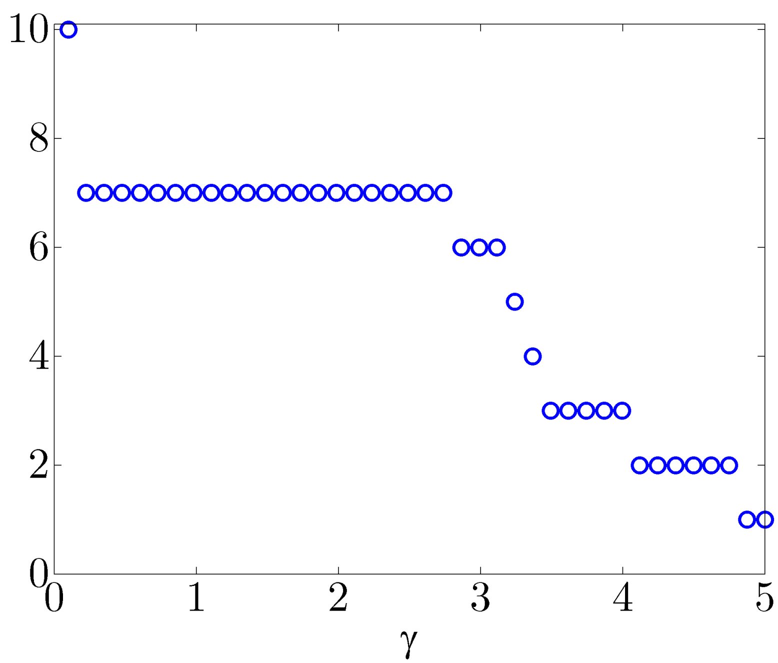
Related leader selection problem

Patterson and Bamieh ’10, Clark and Poovendran ’11, Fardad *et al.* ’11,
Lin *et al.* ’11, Clark *et al.* ’12, Kawashima and Egerstedt ’12

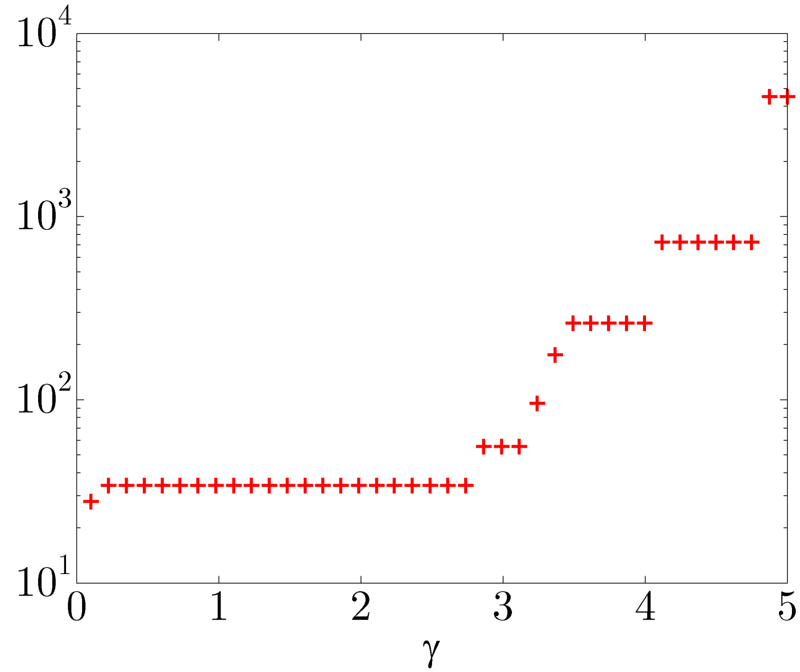
Soft constraint ℓ_1-regularization

$$\begin{array}{ll} \underset{\phi}{\text{minimize}} & f(\phi) \, + \, \gamma \, \mathbb{1}^T \phi \\ \text{subject to} & \phi_i \, \in \, [0,1], \quad i \, = \, 1, \ldots, n \end{array}$$

number of forceful agents



total cumulative effect



Elementwise convexity and coordinate descent

$f(\phi)$ convex w.r.t. each element $\phi_i \in [0,1]$

Coordinate descent

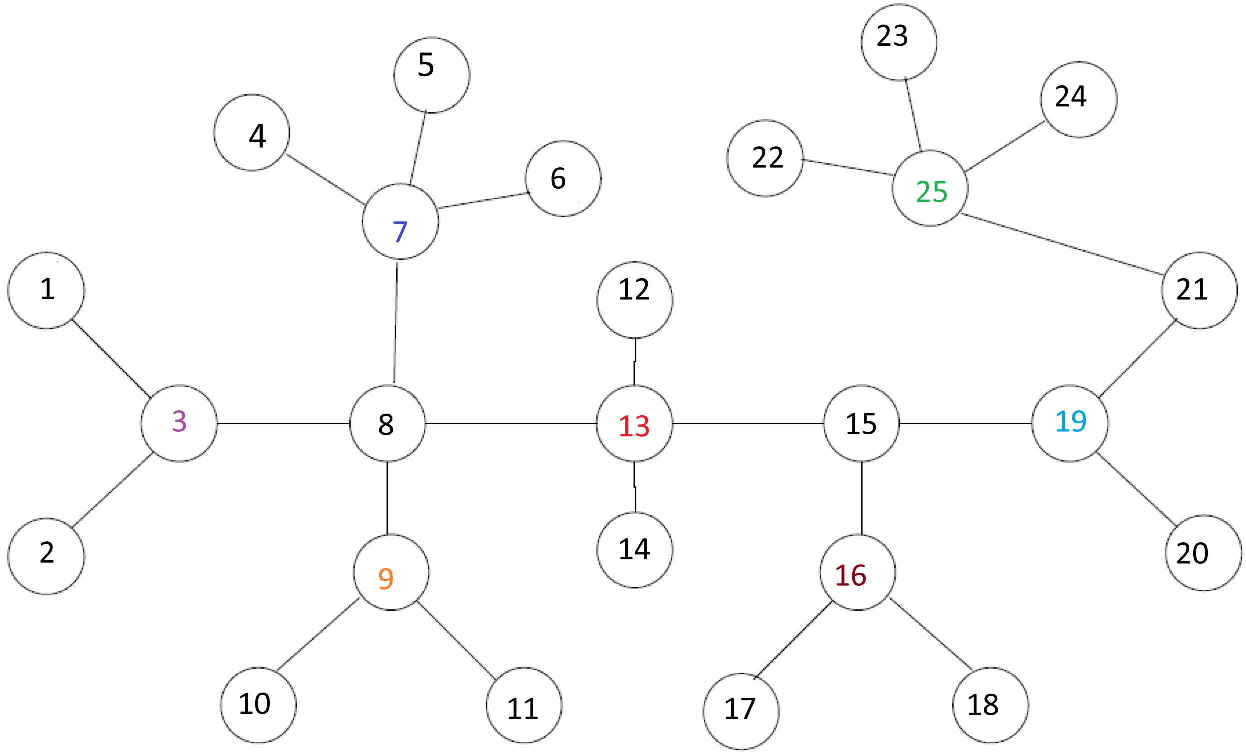
$$\begin{array}{ll} \underset{\phi_i}{\text{minimize}} & f(\phi_i) + \gamma\,\phi_i \\ \text{subject to} & \phi_i \in [0,1] \end{array}$$

smooth convex problem with a scalar variable

- Convergence to a stationary point of nonconvex problems

Bertsekas '99, Tseng '01

An example



	coordinate descent		exhaustive search	
N_{FA}	J	forceful agents	J	forceful agents
1	4508.0	25	1104.0	13
2	724.0	7, 25	334.0	8, 19
3	261.8	7, 13, 25	173.5	8, 15, 25
4	175.8	7, 13, 16, 25	129.5	7, 8, 15, 25
5	95.6	3, 7, 13, 16, 25	88.3	3, 7, 9, 15, 25
6	55.5	3, 7, 9, 13, 16, 25	55.5	3, 7, 9, 13, 16, 25
7	34.1	3, 7, 9, 13, 16, 19, 25	34.1	3, 7, 9, 13, 16, 19, 25

Concluding remarks

- Social influence that accounts for transition behavior
- Optimal selection of forceful agents

In the paper

- ⋆ Characterizations based on ℓ_2 and $\mathcal{H}_2$ norms
- ⋆ Optimal creation of social links

Future work

- ⋆ FA selection using other models
- ⋆ Lower bound via convex relaxations

www.umn.edu/∼mihailo/software/leaders/