MISE is the Jump Number Problem

Andrew Lyons `lyonsam@gmail.com`

Let $G = (V, E)$ be a finite directed acyclic graph without loops or multiple edges, and let $L = v_1 v_2 \dots v_k$ be a linear extension of G. A *jump* (resp. *immediate successor*) is a pair $v_i v_{i+1}$ of consecutive elements in L such that $(v_i, v_{i+1}) \notin E$ (resp. $(v_i, v_{i+1}) \in E$). The jump number of G is the minimum number of jumps in a linear extension, or, equivalently, the minimum number of edges whose addition results in a Hamiltonian path in G.

Proposition. *MISE is equivalent to the jump number problem.*

Proof. Observe that every consecutive pair of elements in L must either be a jump or an immediate successor. In particular, we have

$$|\{\text{jumps in } L\}| + |\{\text{immediate successors in } L\}| = n - 1$$

Thus, any linear extension that maximizes the number of immediate successors will also minimize the number of jumps. □

As explained in [1], any two dags with the same transitive closure are equivalent with respect to the jump number problem. It follows then that it may be considered a problem on posets. Pulleyblank [3] showed that determining the jump number of a poset is NP-complete.

Though the jump number problem is NP-complete even for chordal bipartite graphs [2], it is a well-studied problem and can be solved in polynomial time for a wide array of subclasses of dags and posets.

References

[1] M. Chein and M. Habib. The jump number of dags and posets: An introduction. In M. Deza and I. G. Rosenberg, editors, *Annals of Discrete Math.*, volume 9, pages 189–194. 1980.

[2] Haiko Müller. Alternating cycle-free matchings. *Order*, 7(1):11–21, 1990.

[3] William Pulleyblank. On minimizing setups in precedence constrained scheduling. Unpublished Manuscript, 1982.