

# Exploiting Algebraic Dependences Between Local Partial Derivatives in Jacobian Accumulation \*

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We introduce a novel undirected graph model for the optimal Jacobian accumulation problem. This new graph addresses the need illustrated in [1] for accumulation techniques capable of exploiting algebraic dependences between the local partial derivatives.

Let  $\mathbf{G}$  be a linearized computational graph (LCG). The corresponding *path-clique graph* (CPG)  $\mathbf{G}^p$  has vertex set  $V^p = E \in \mathbf{G}$ ; two vertices  $i, j \in V^p$  are adjacent iff their corresponding edges in  $\mathbf{G}$  lie on a common path. We allow two vertices to be *merged* if their respective labels are identified as equivalent.

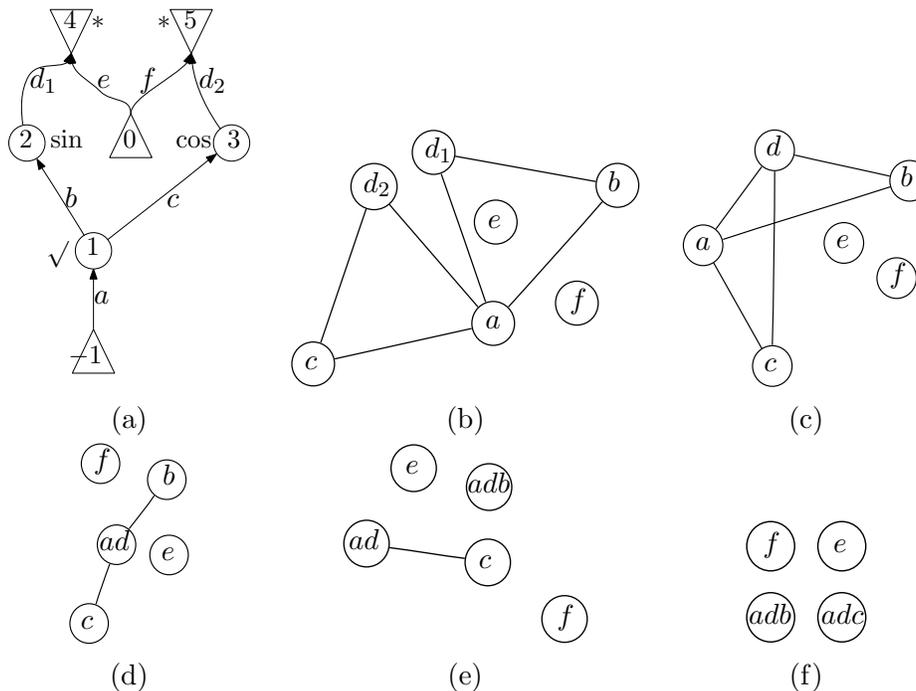


Figure 1: An example LCG  $\mathbf{G}$  (a), the corresponding CPG  $\mathbf{G}^p$  (b),  $\mathbf{G}^p$  after vertices  $d_1$  and  $d_2$  have been merged (c),  $\mathbf{G}^p$  after the contraction of edges  $\{a, d\}$  (d),  $\{ad, b\}$  (e), and  $\{ad, c\}$  (f).

A sequence of *edge contraction* operations reduces a CPG to an edgeless graph whose vertices yield the values of Jacobian entries. Each edge contraction implies a single *fused multiply-add* operation in the generated derivative code.

## References

- [1] Uwe Naumann. Optimal Jacobian accumulation is np-complete. *Math. Prog.* Online First.

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