A new origin for fermion masses

Shailesh Chandrasekharan

Duke University

Work done in collaboration with
Venkitesh Ayyar
(related work PRD 91 (2015) 065035, 93 (2016) 081701(R))

Other work: S. Catterall, JHEP 1601 (2016) 121, He et. al., arXiv:1603.08376

Lattice for Beyond the Standard Model Physics, Argonne National Laboratory, April 22-23, 2016

US Department of Energy,
Office of Science, Nuclear Physics Division

Outline

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- Conclusions

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This Talk: There is more to the story!

Review of the “well known results”

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Z_2 Higgs-Yukawa models

$$S = \int d^d x \left\{ \left(\partial_\mu \phi(x)\right)^2 + m^2 \left(\phi(x)\right)^2 + \lambda \left(\phi(x)\right)^4 \right.$$
$$\left. + \overline{\psi}(x) \gamma_\mu \partial_\mu \psi(x) + g \; \phi(x) \overline{\psi}(x) \psi(x) \right\}$$

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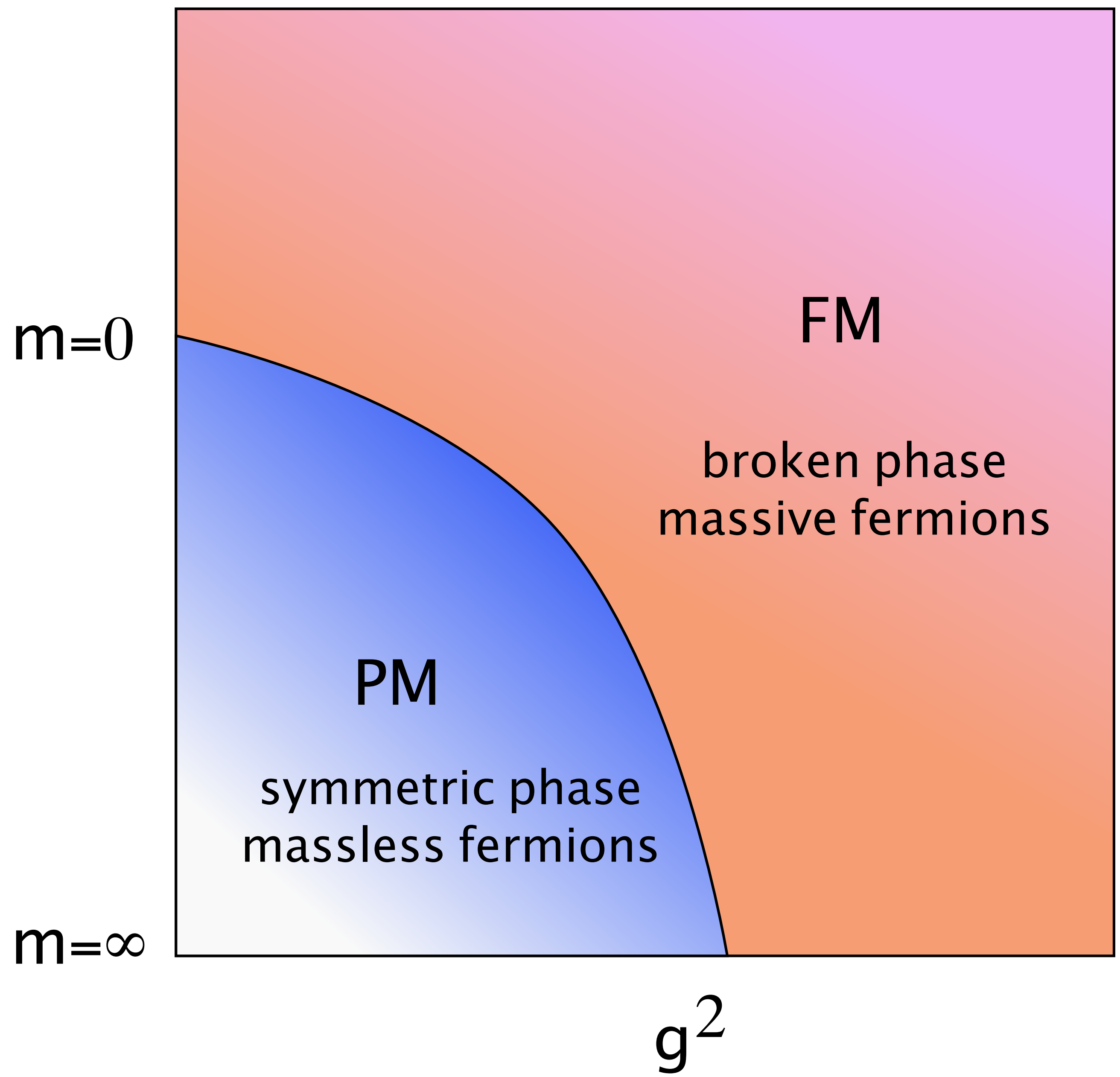
Symmetry that protects the mass term:

$$\psi(x) \to i\gamma_5 \psi(x), \quad \overline{\psi}(x) \to i\overline{\psi}(x)\gamma_5, \quad \phi(x) \to -\phi(x)$$

$$\gamma_5^2 = 1, \quad \gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$$

Phase diagram (fixed λ)



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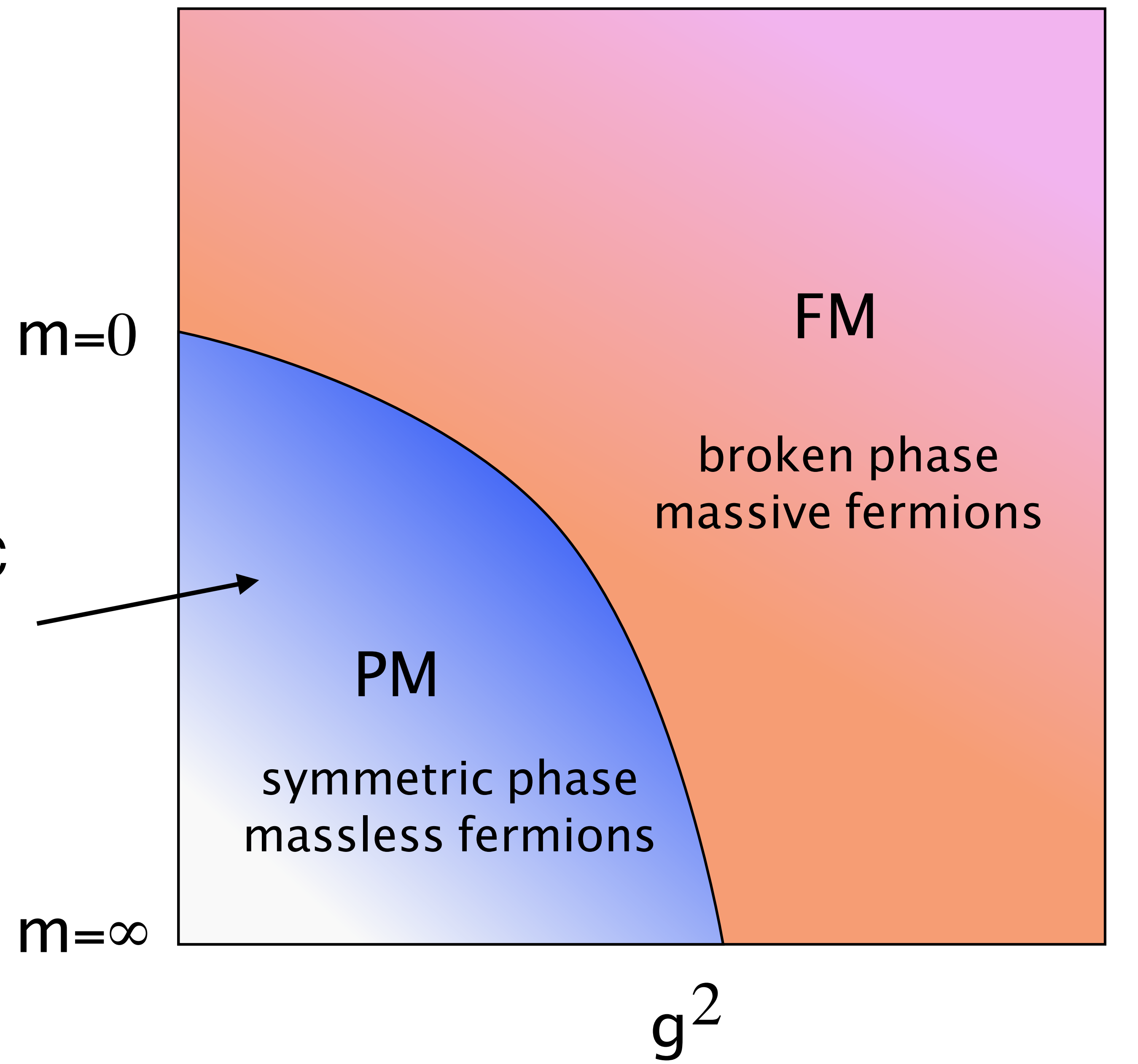
paramagnetic

$\langle \overline{\psi}\psi \rangle \;=\; 0$



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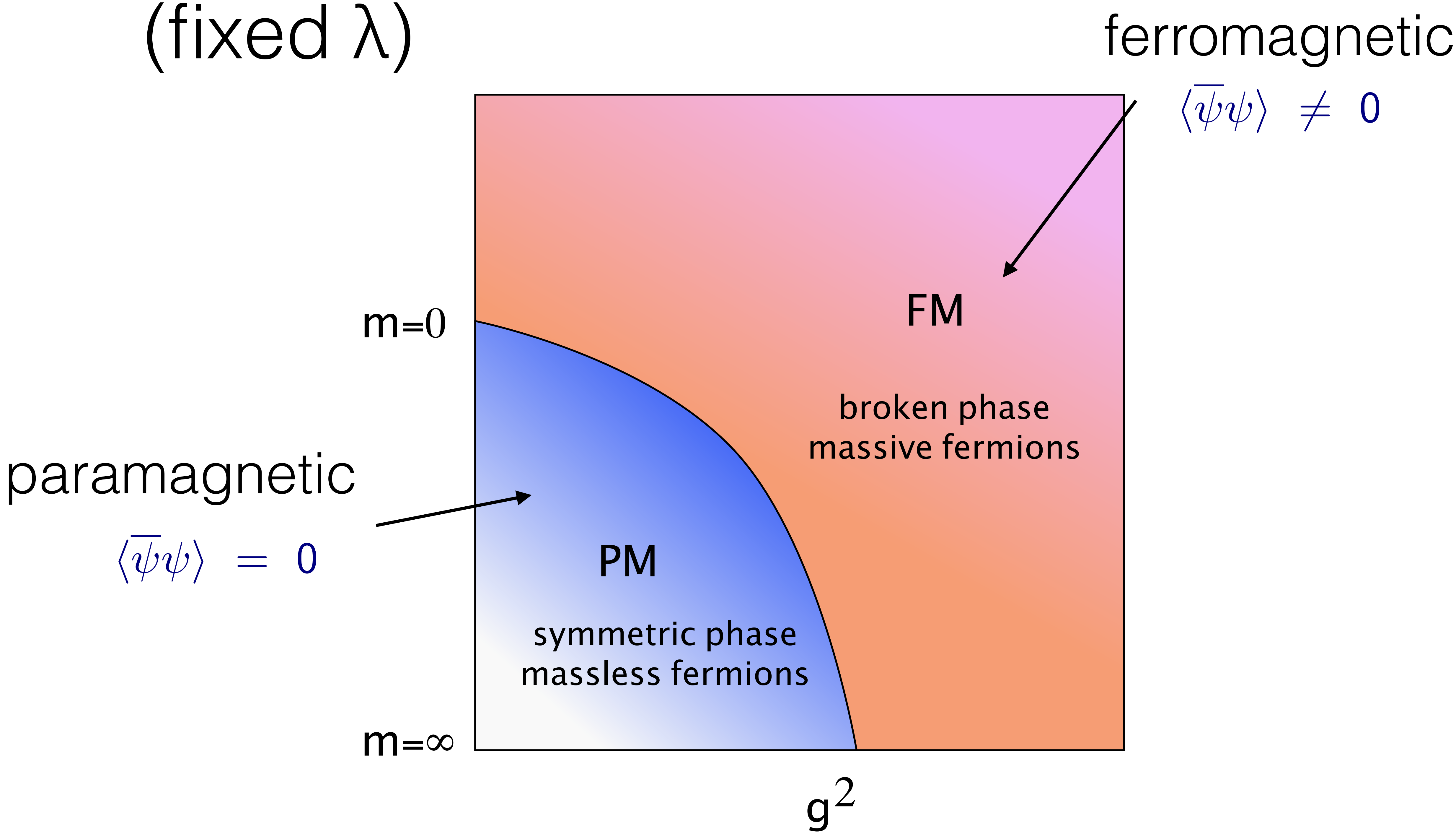

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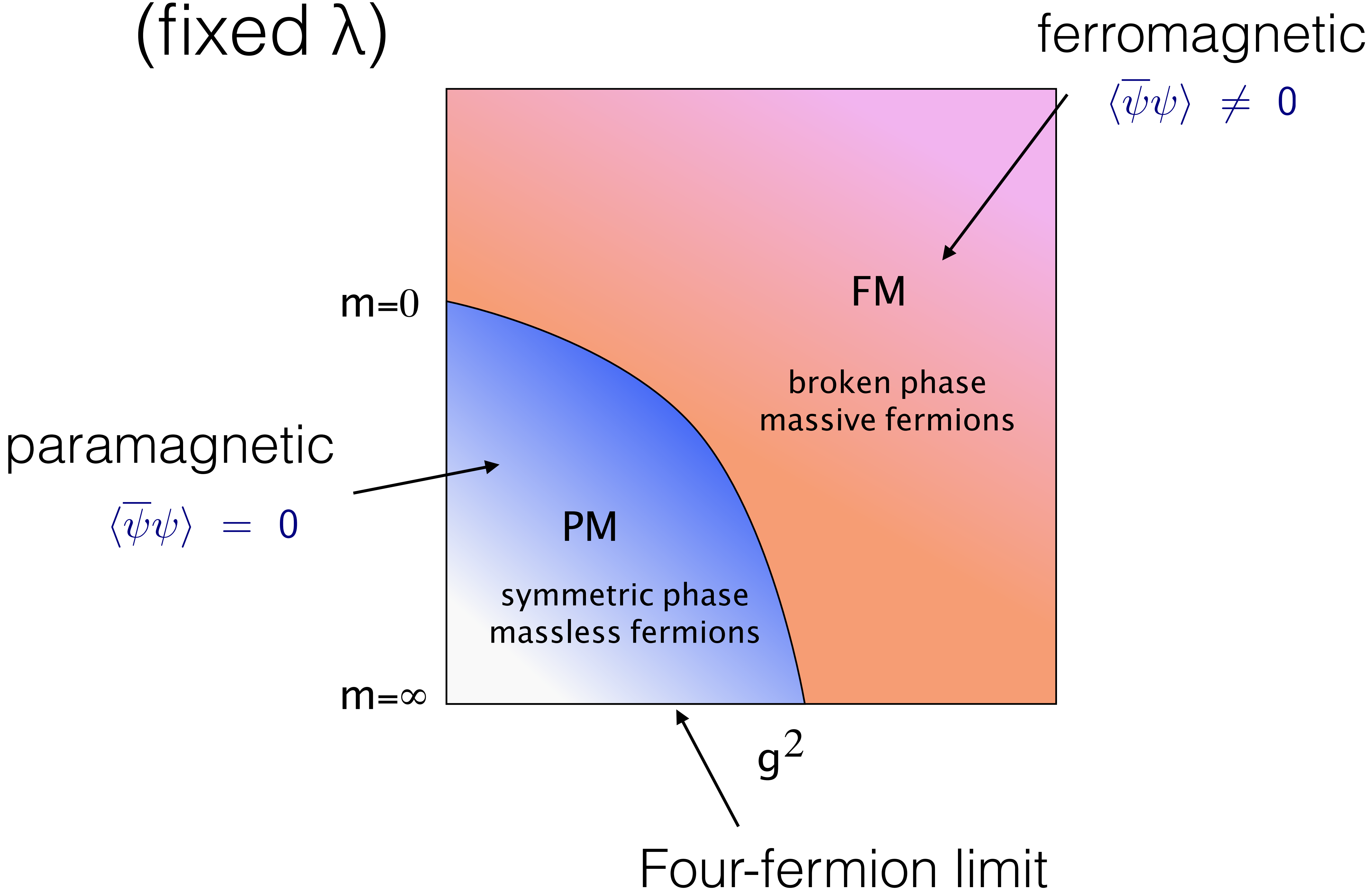

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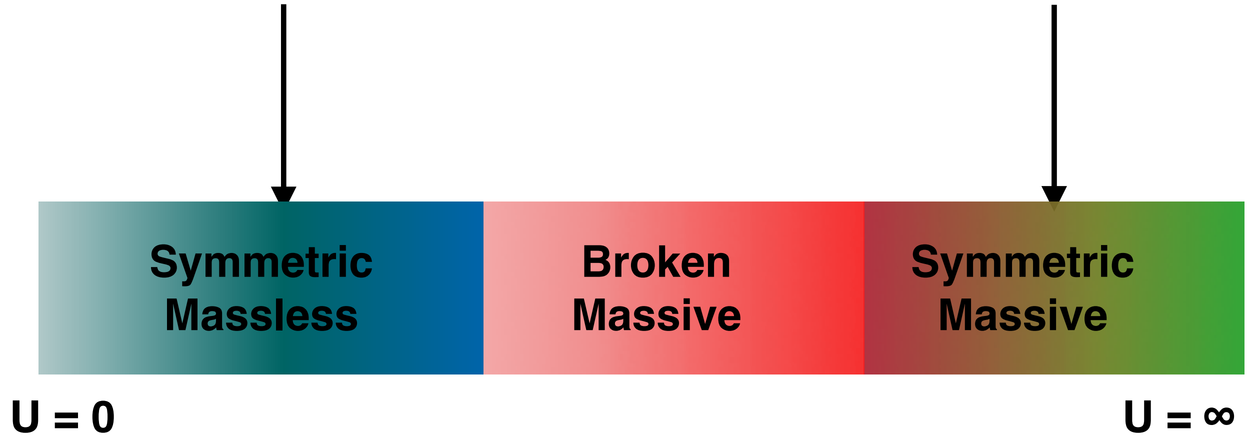

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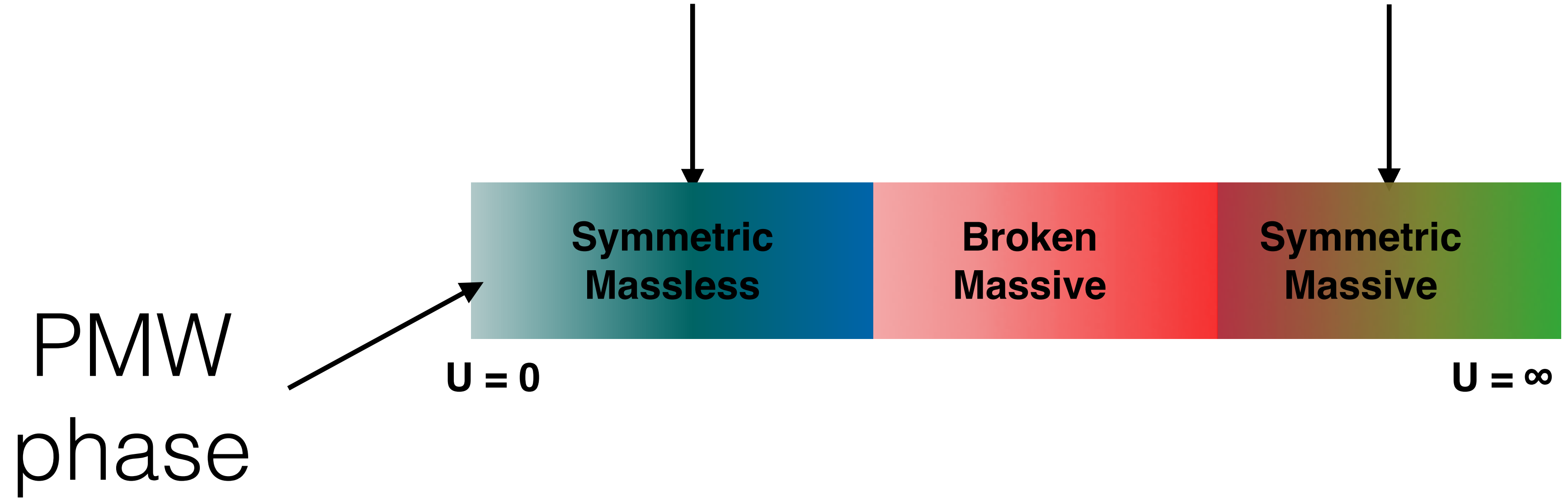

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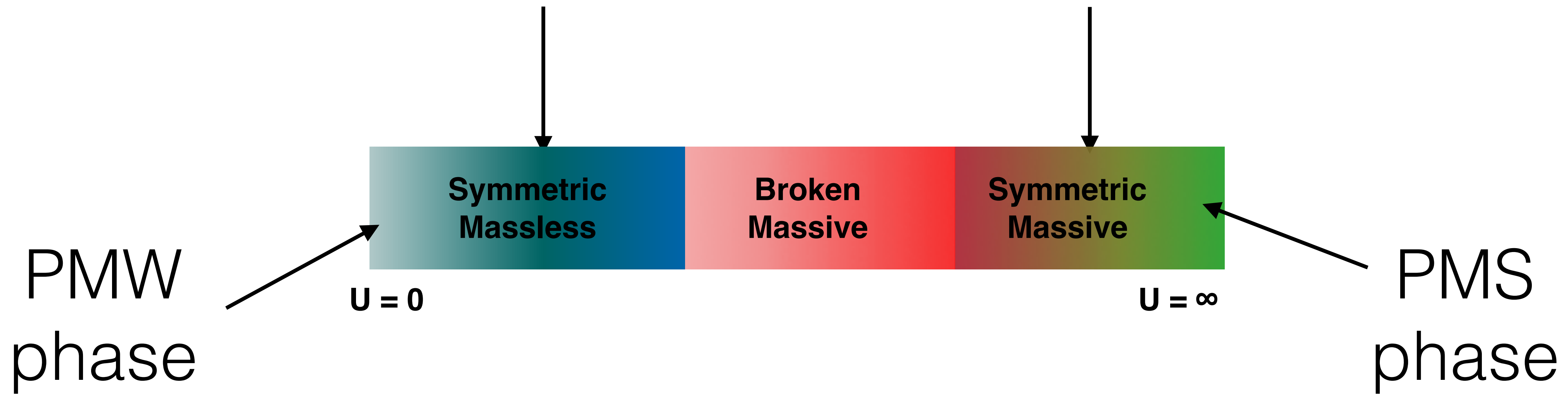

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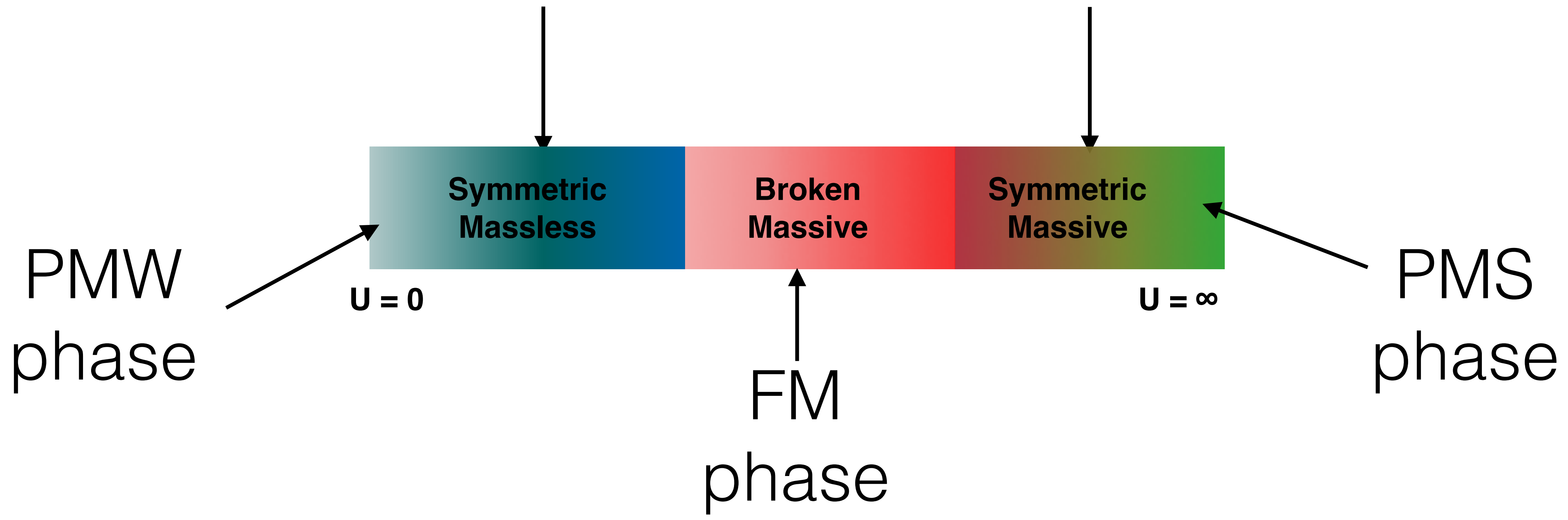

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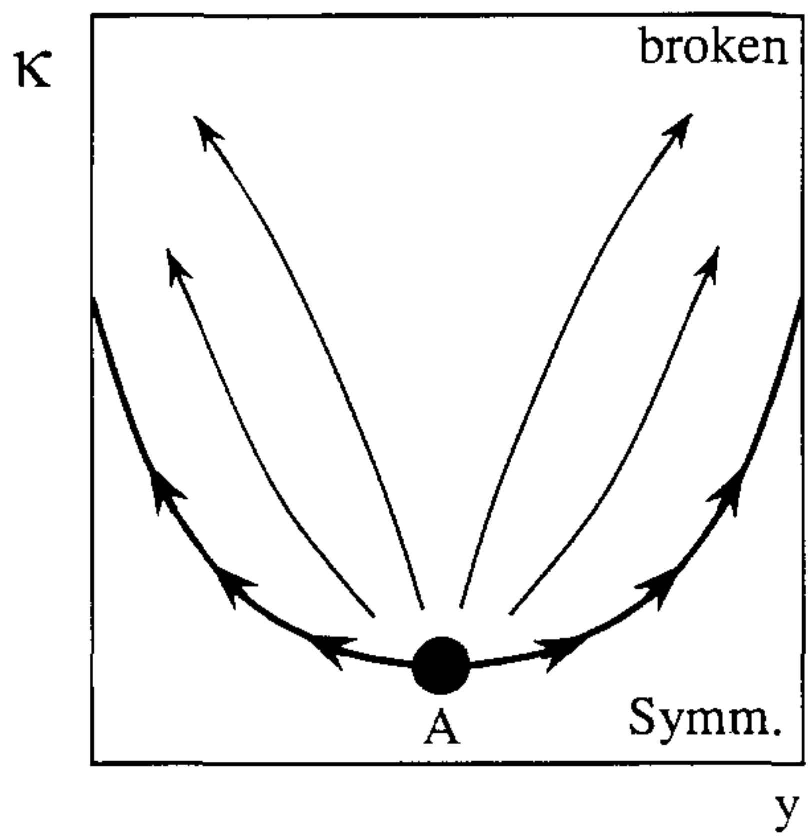
PLB 220, (1989) 435



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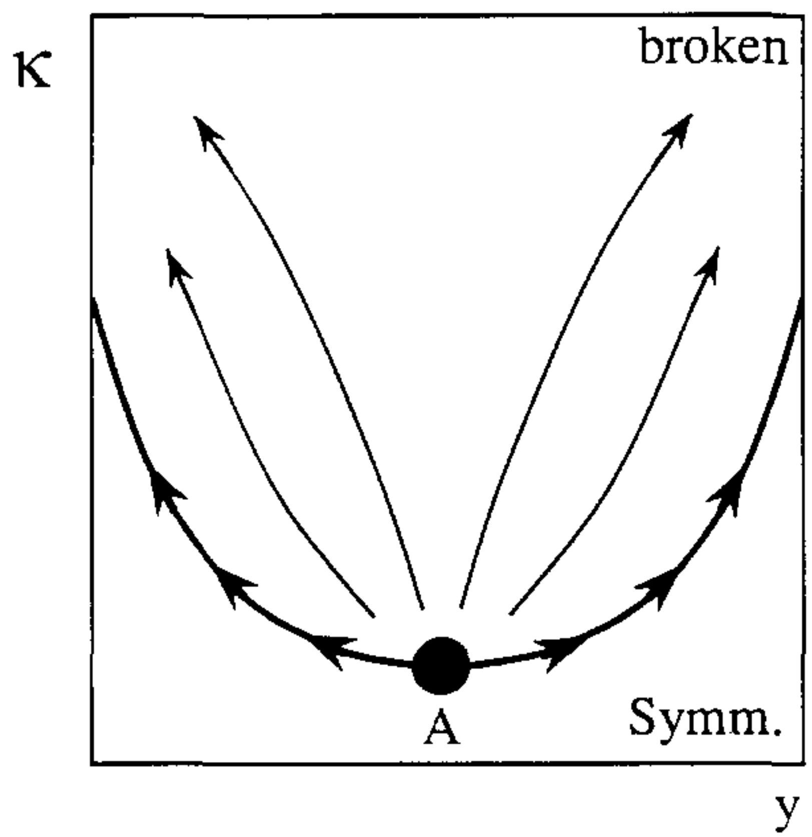

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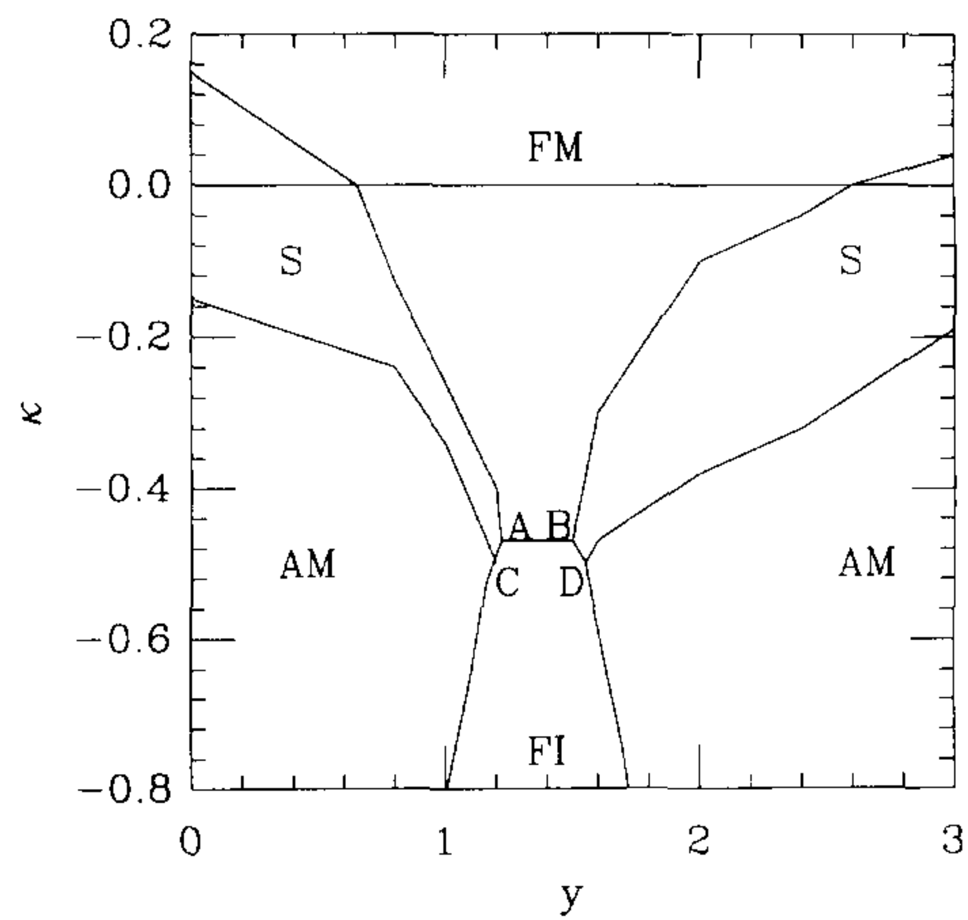

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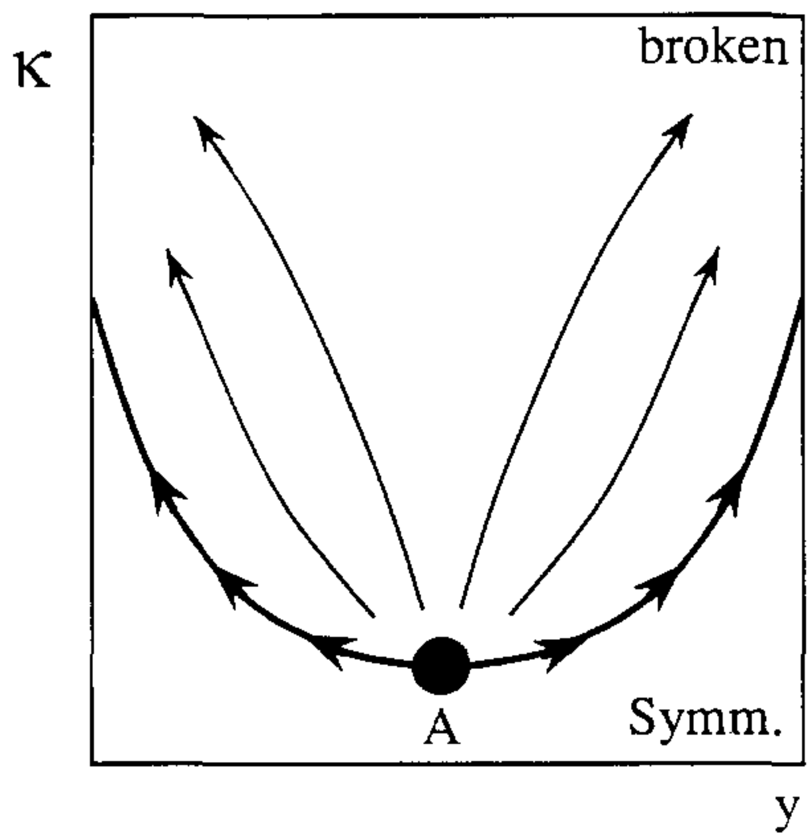

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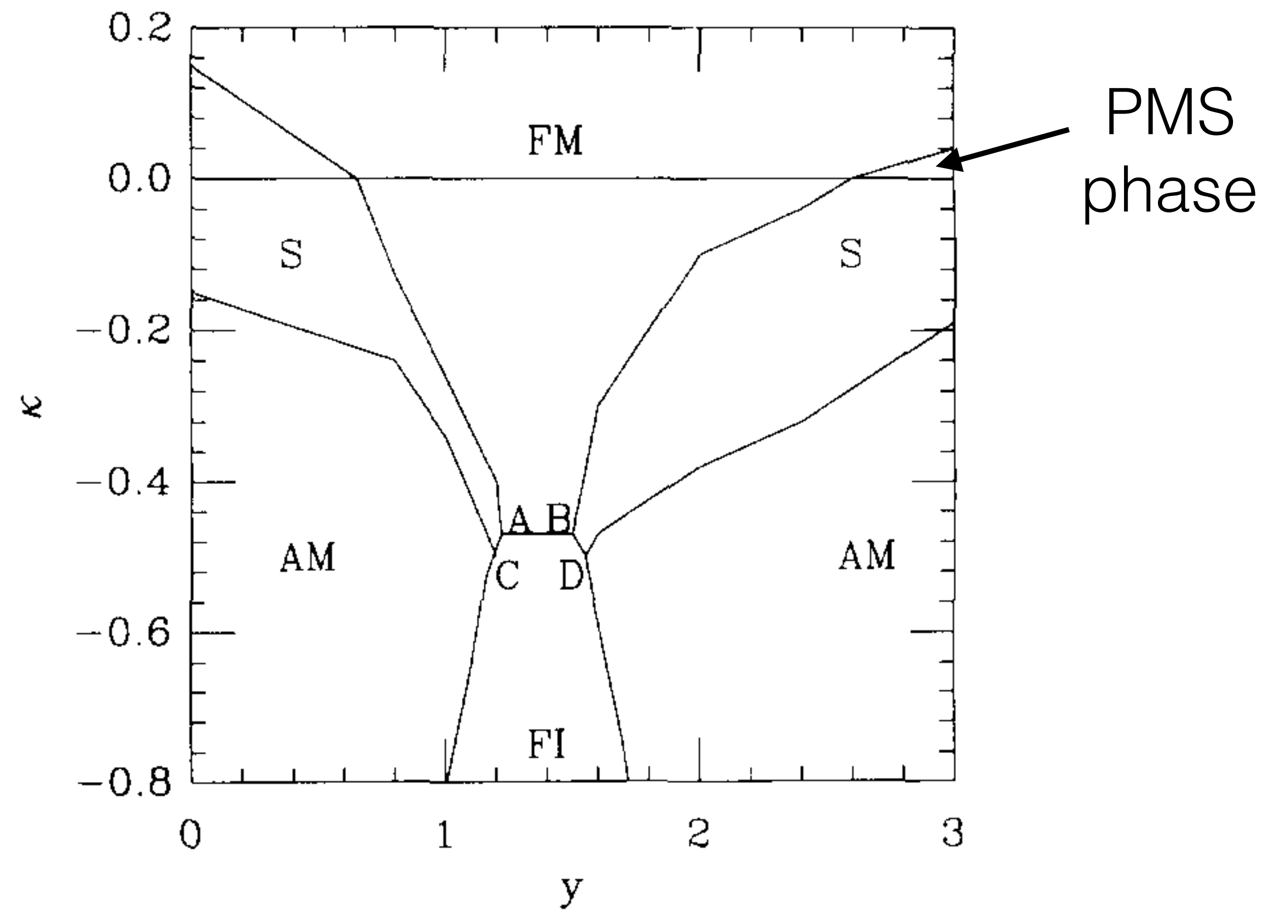

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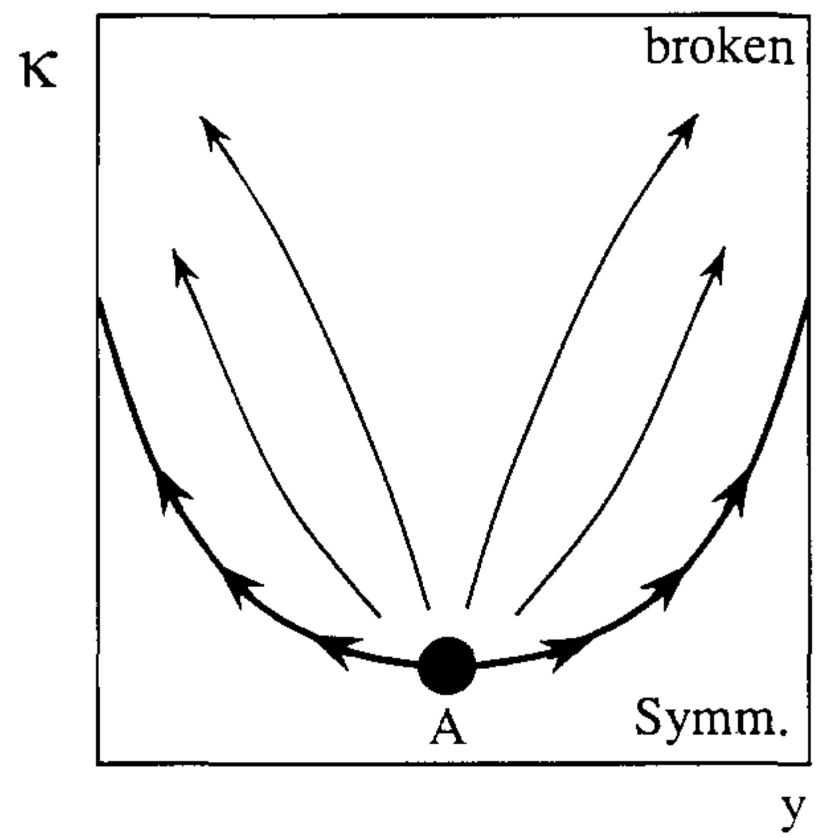

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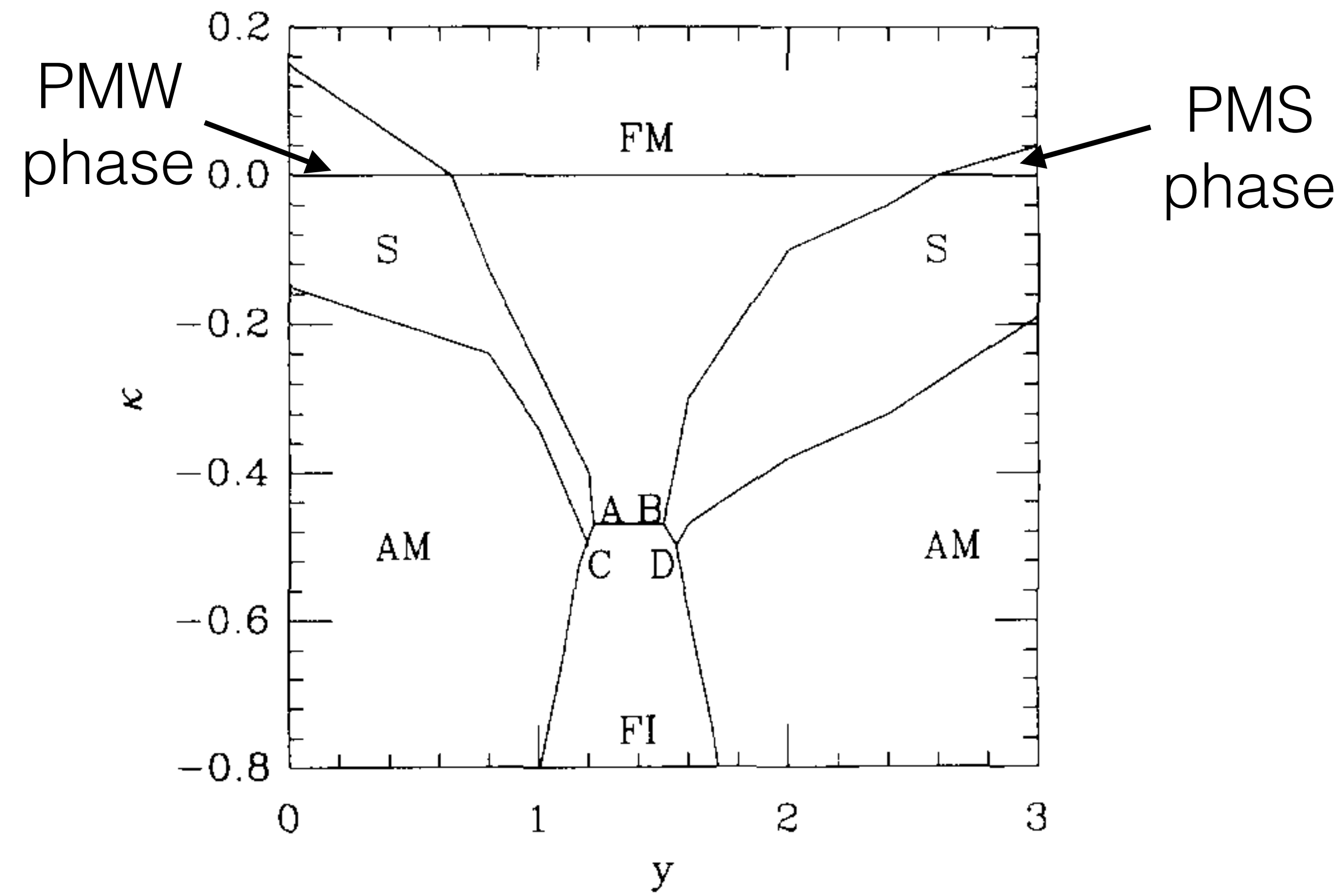

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κ=0 is a four-fermion model

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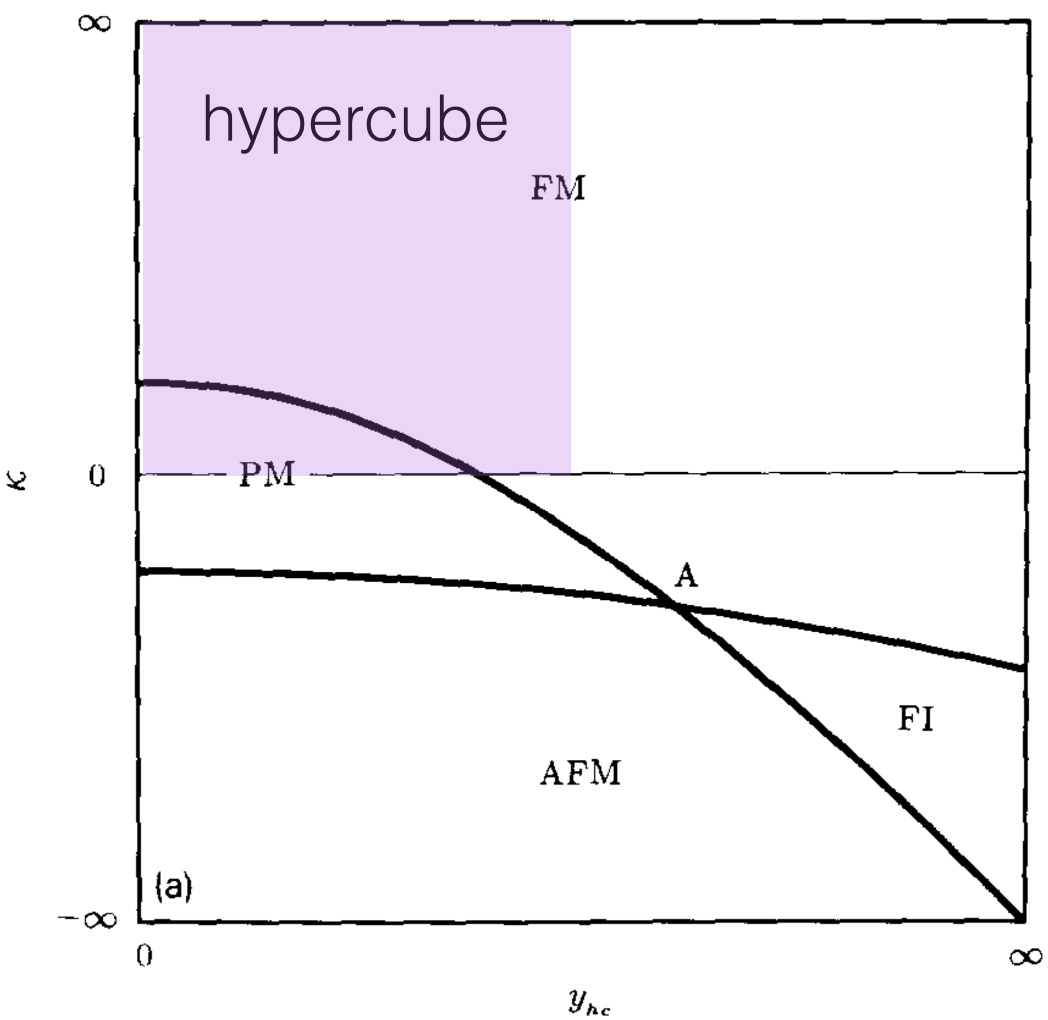

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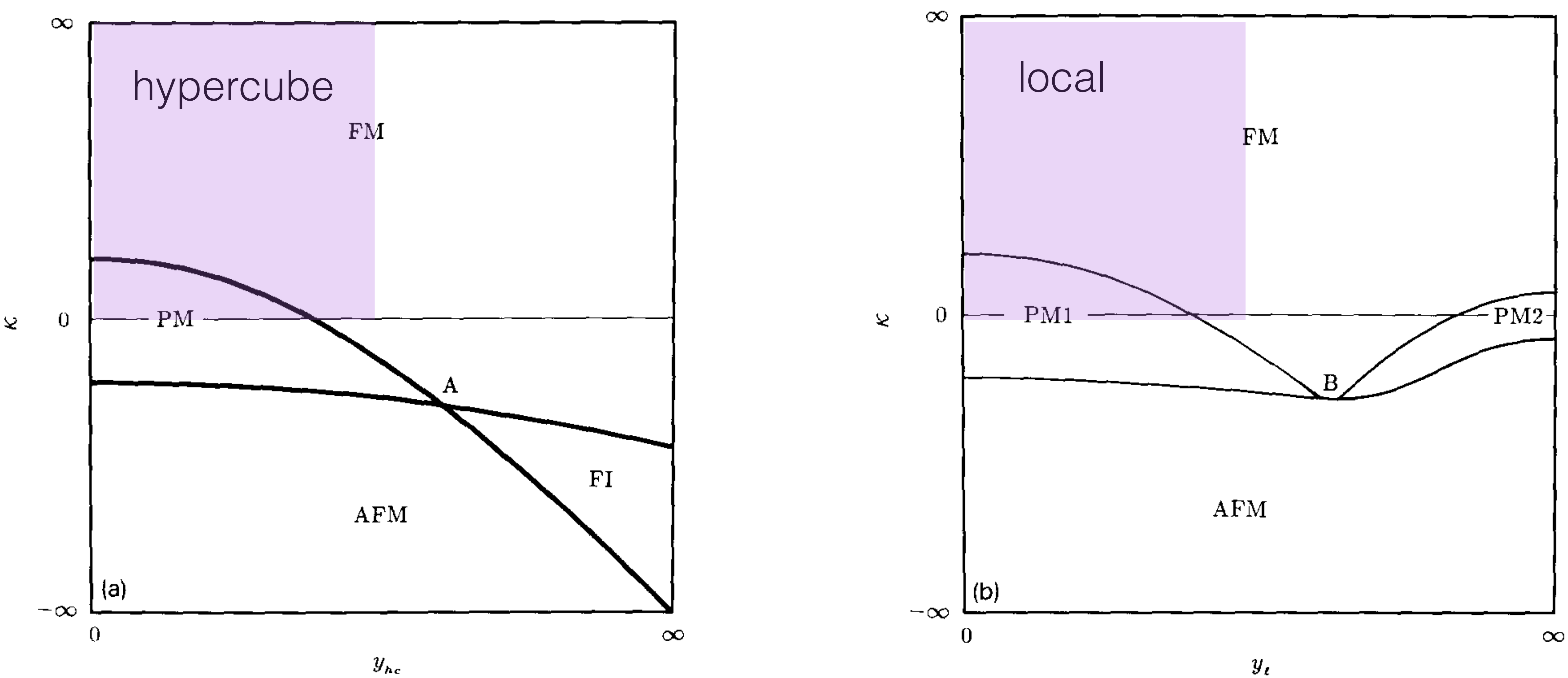

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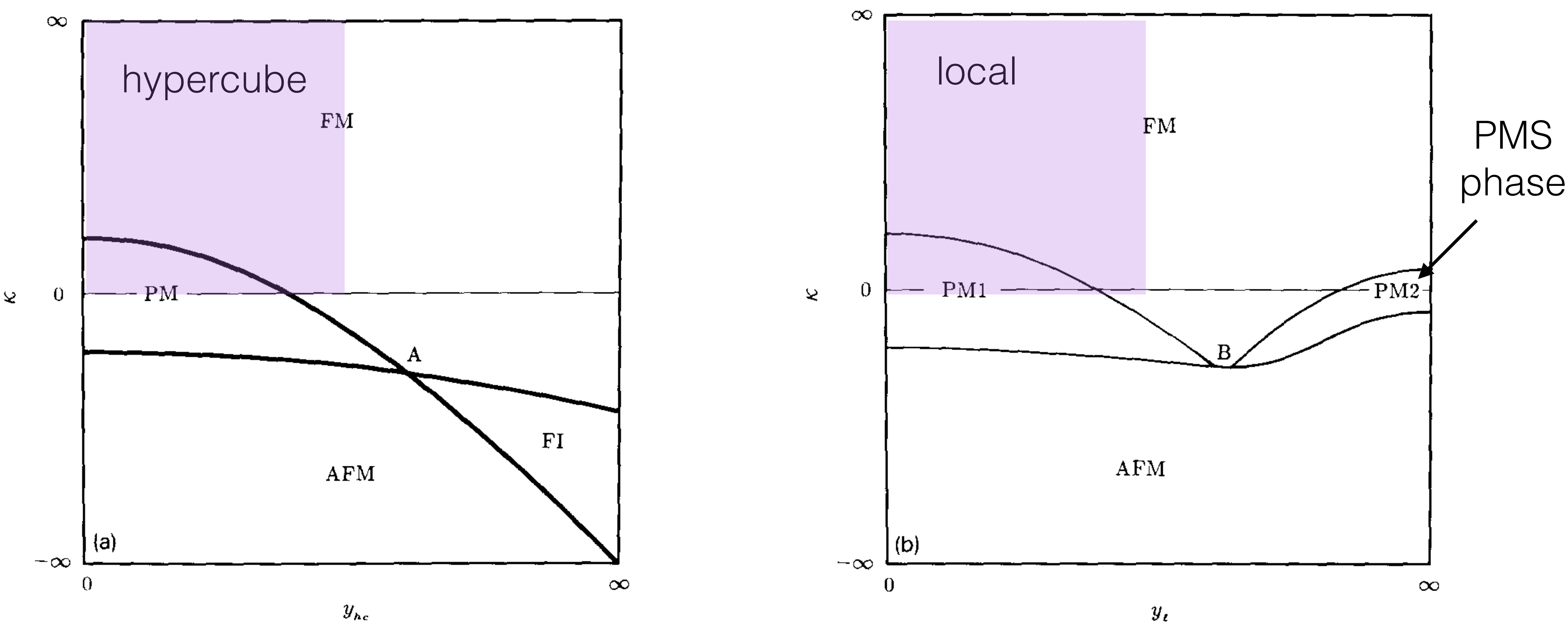

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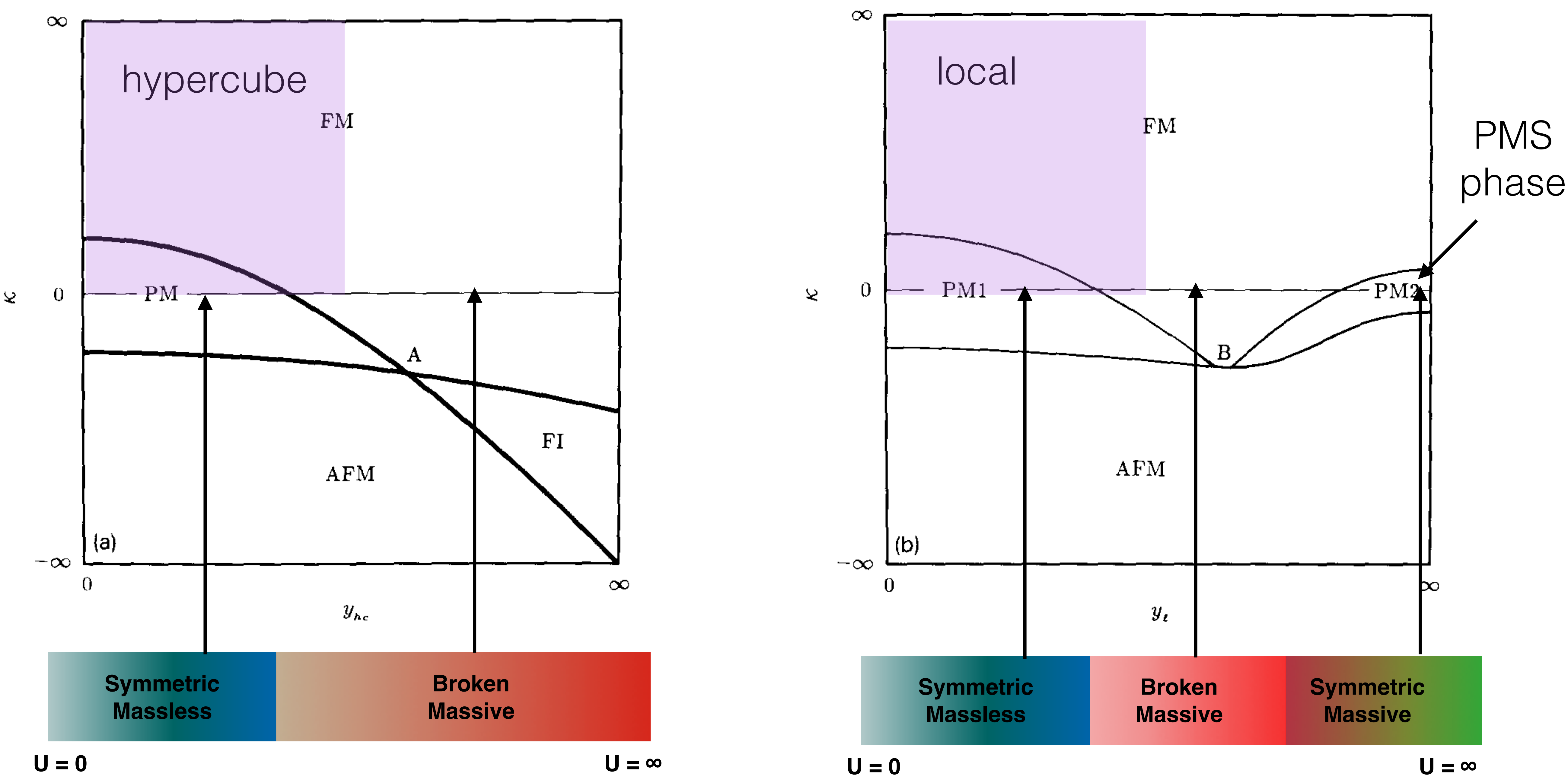

κ = 0 (four-fermion) limit

PHASE DIAGRAM OF A LATTICE SU(2) ⊗ SU(2) SCALAR-FERMION MODEL WITH NAIVE AND WILSON FERMIONS*

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[3]*Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, FRG*
[4]*Institute of Theoretical Physics, Valckenierstraat 65, NL-1018 XE Amsterdam, The Netherlands*

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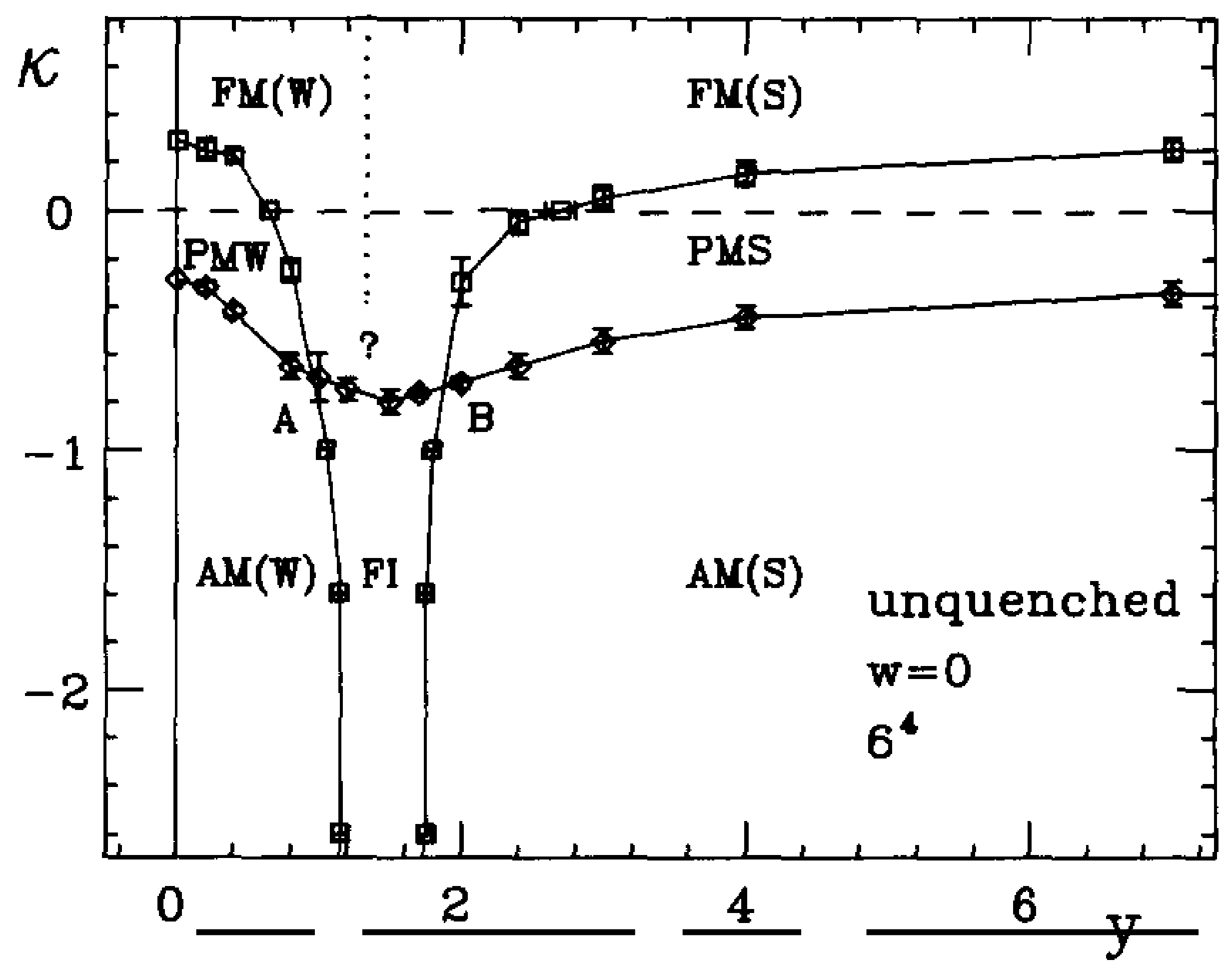

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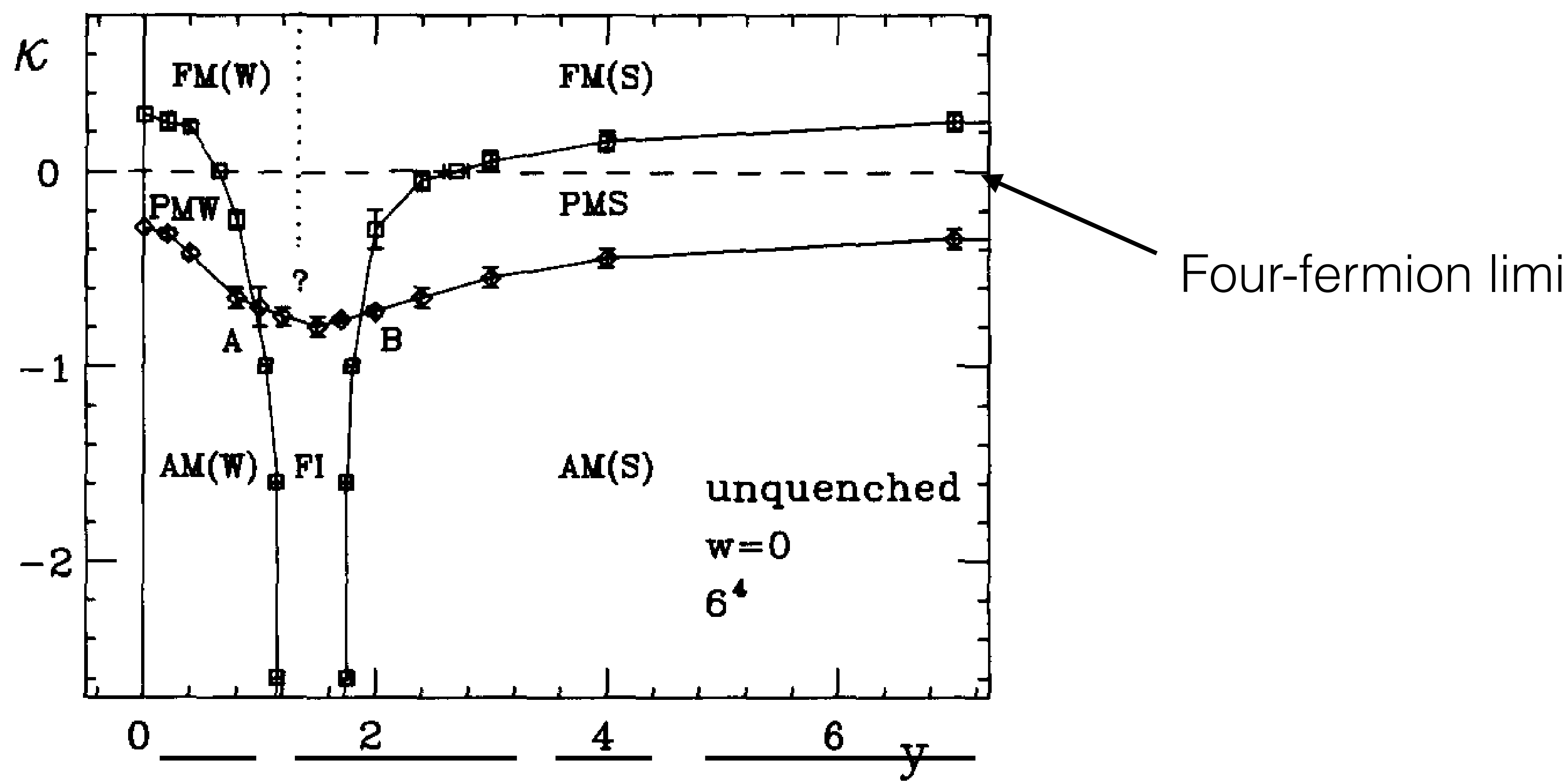

Phase diagram and quasiparticles of a lattice SU(2) scalar-fermion model in 2+1 dimensions

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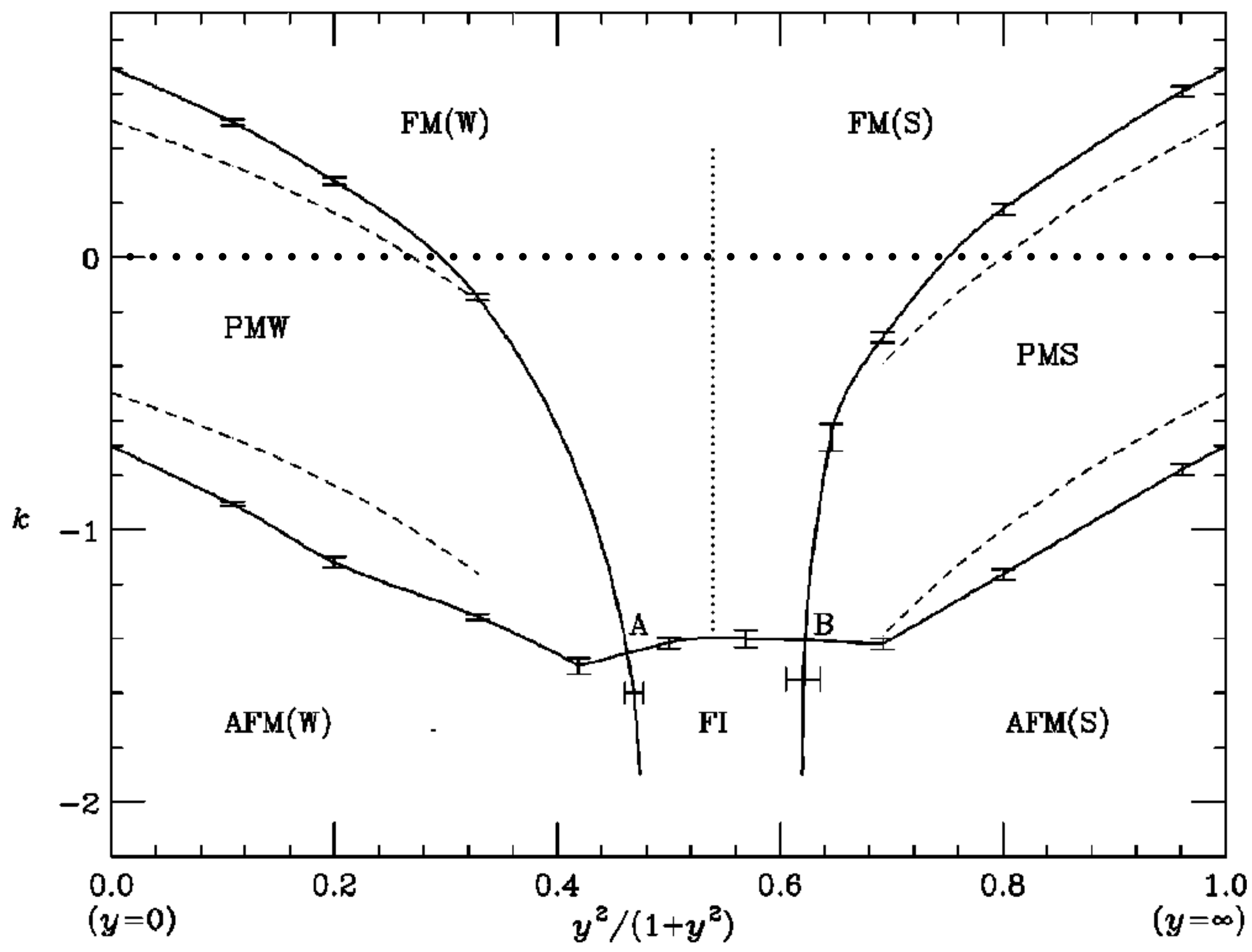

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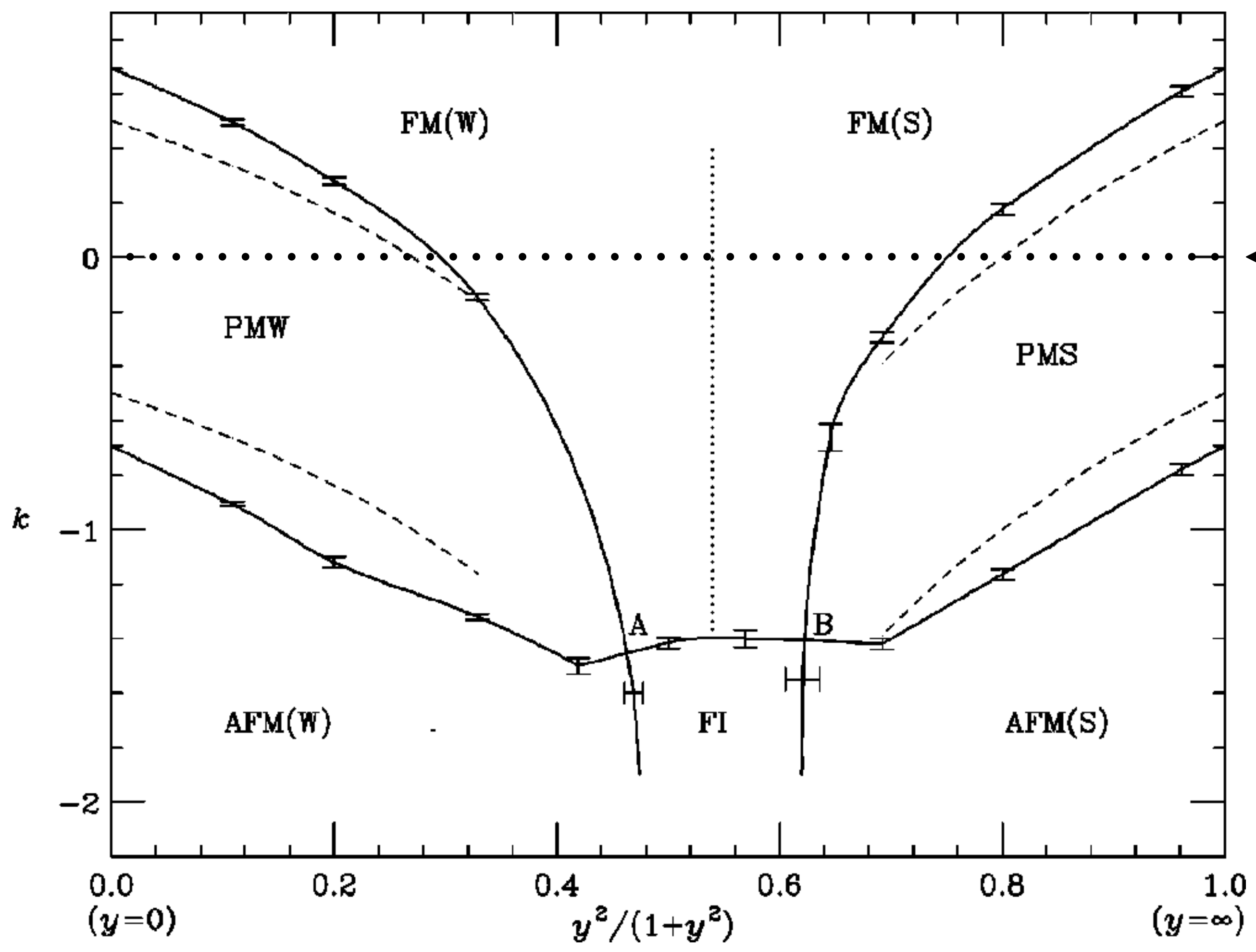

Four-fermion limit

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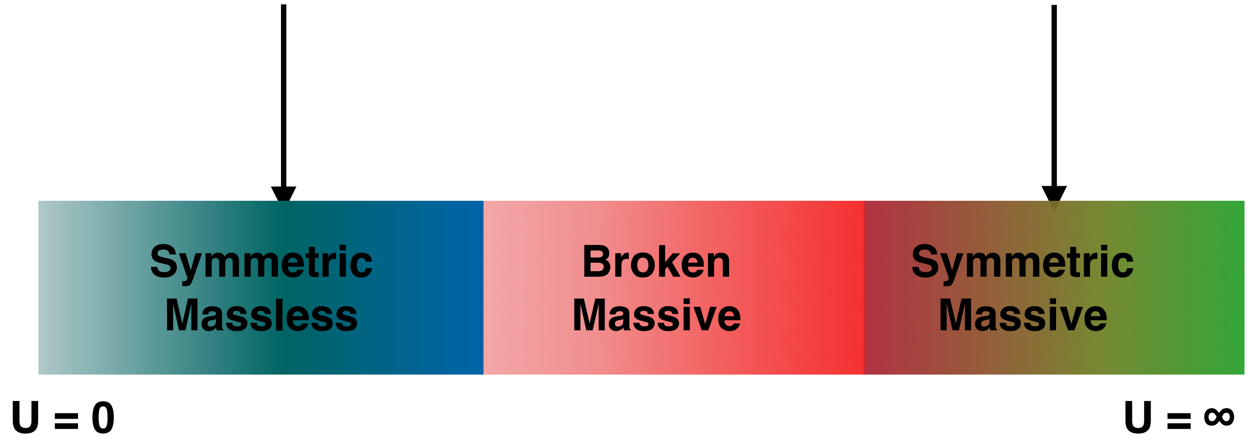

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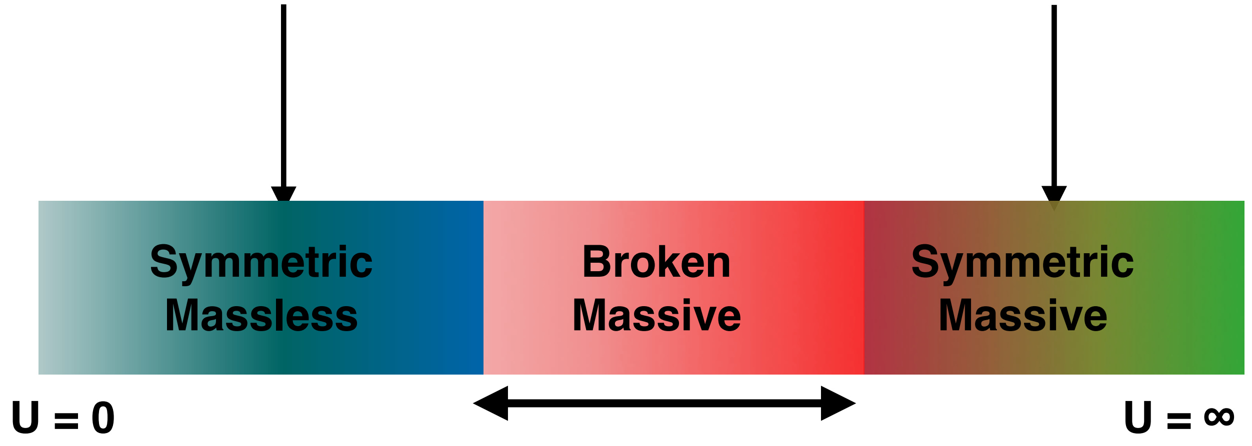

However, past studies found the broken phase to be wide

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Thus, the PMS phase seemed like a “lattice artifact”
and hence abandoned by the lattice community!

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↓

"Continuum limit" of the PMS phase!

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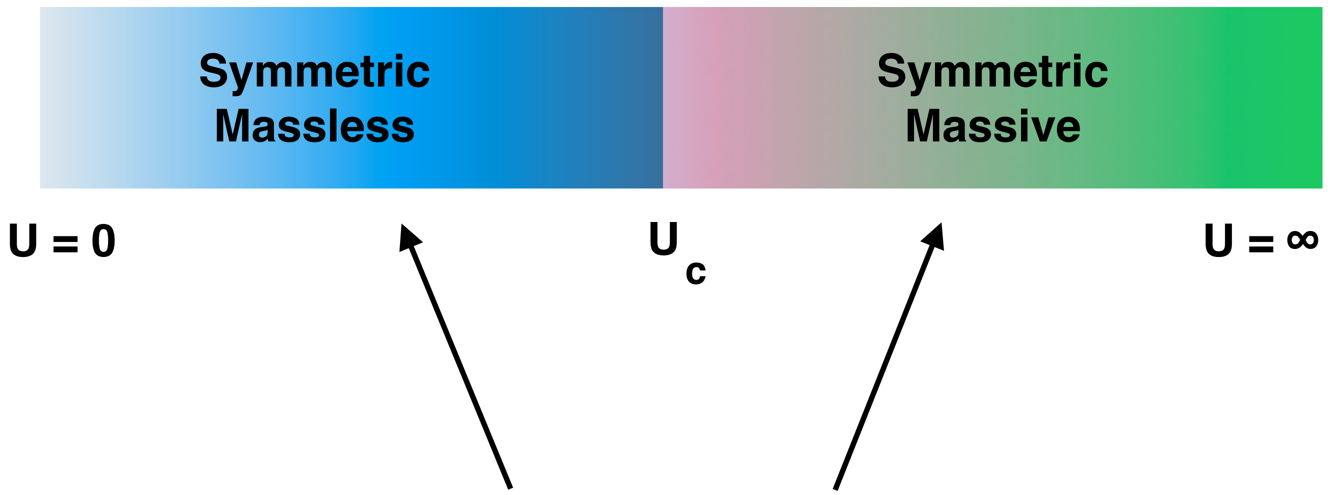

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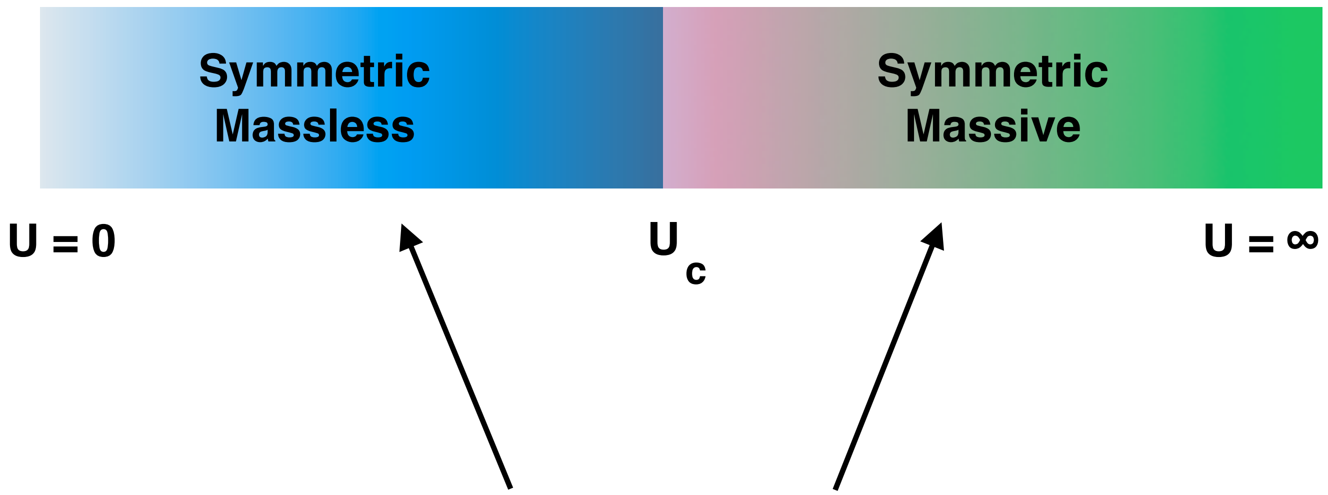
Continuum Limit?

A direct second order phase transition between the PMW and PMS phases



phases with same lattice symmetries!

A non-Landau Ginzburg type, “exotic” transition!

Such transitions also seem to be at the heart of formulating chiral fermions on the lattice!

The Eichten-Preskill approach to chiral lattice fermions is related to this exotic phase transition

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Absence of chiral fermions in the Eichten–Preskill model

Maarten F.L. Golterman [1], Donald N. Petcher [2] and Elena Rivas [3]

Department of Physics, Washington University, St. Louis, MO 63130-4899, USA

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6. Conclusion

When Eichten and Preskill originally presented their model, a clear element of the scenario they envisaged for it to successfully produce a continuum theory of (asymptotically free) chiral fermions entailed the existence of a phase transition for which the fermion mass was an order parameter, and over which no symmetry breaking occurred.

We have analyzed the model in several regions of the phase diagram, and all indications are that no such phase transition exists. Indeed, we do find phases with massive and massless fermions, but always a broken phase appears in between. In the symmetric phase with massive fermions (a paramagnetic phase in strong Yukawa coupling, or PMS phase), bound states are formed which pair up with the original chiral fields to form Dirac representations, all of which are massive (although one massless Dirac fermion can be arranged by tuning). The fermions remain massive across the symmetry breaking phase boundary to the broken phase (ferromagnetic or FM phase), and finally across the phase boundary to the second symmetric phase (paramagnetic phase at weak Yukawa coupling, or PMW phase), all fermions become massless, including the doublers. The crucial ingredient for the failure of the emergence of a chiral theory of fermions as originally imagined is the existence of the broken phase separating the two symmetric phases. Through

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Instead of a local order parameter, some topological order parameter, governed by symmetries distinguishes the phases.

Topological Insulators and Trivial Insulators
are typically examples of two different SPT states of matter

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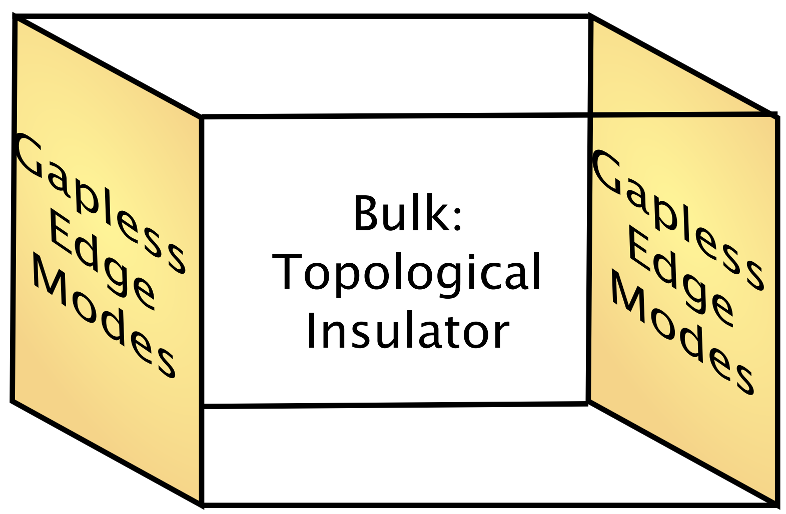

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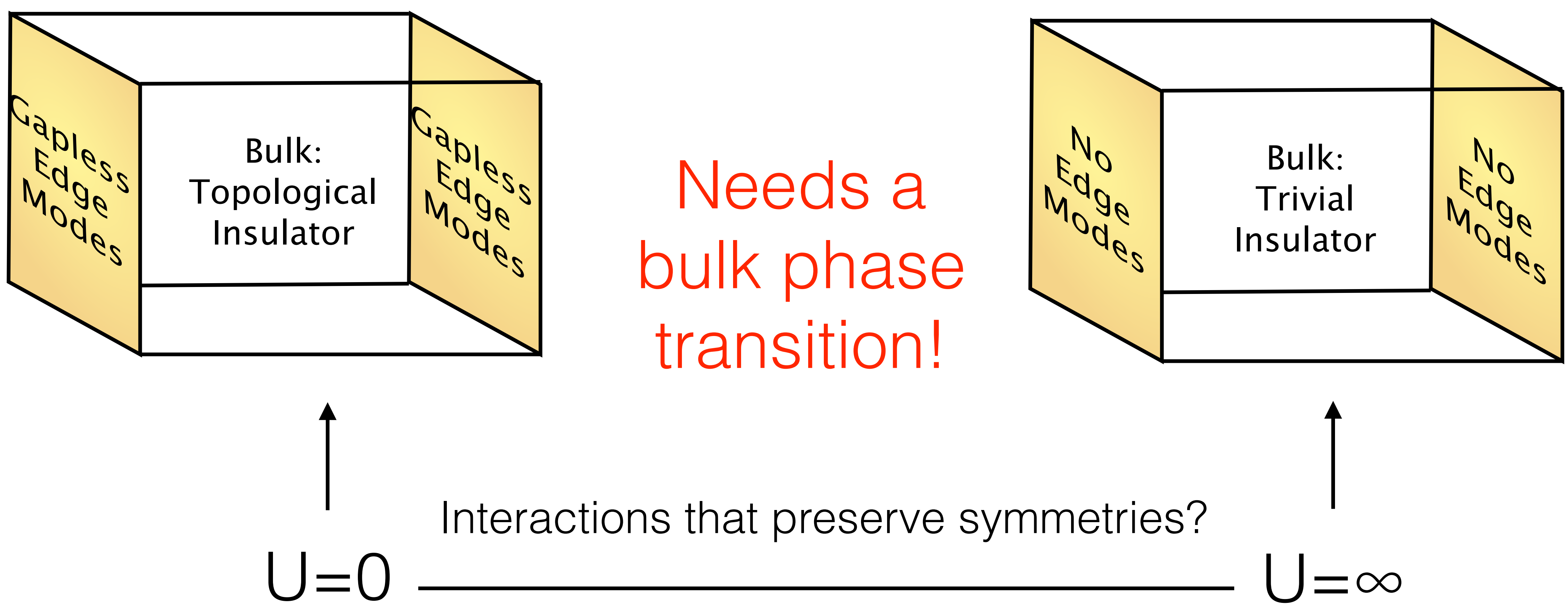

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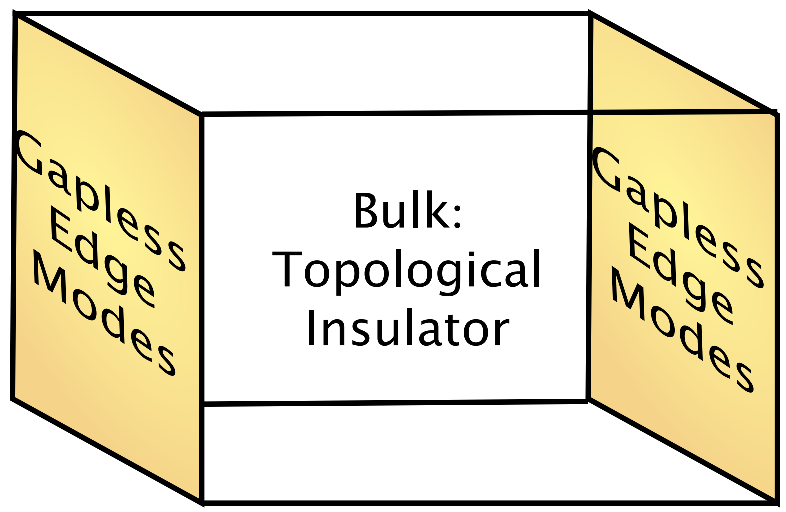

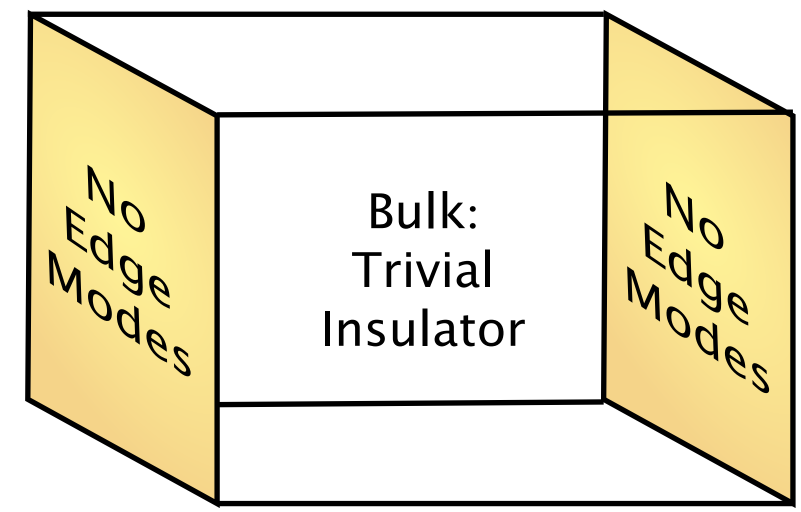

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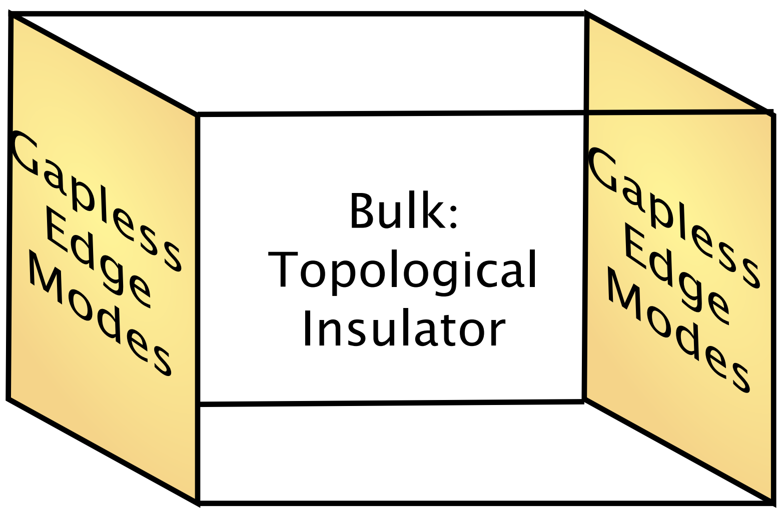
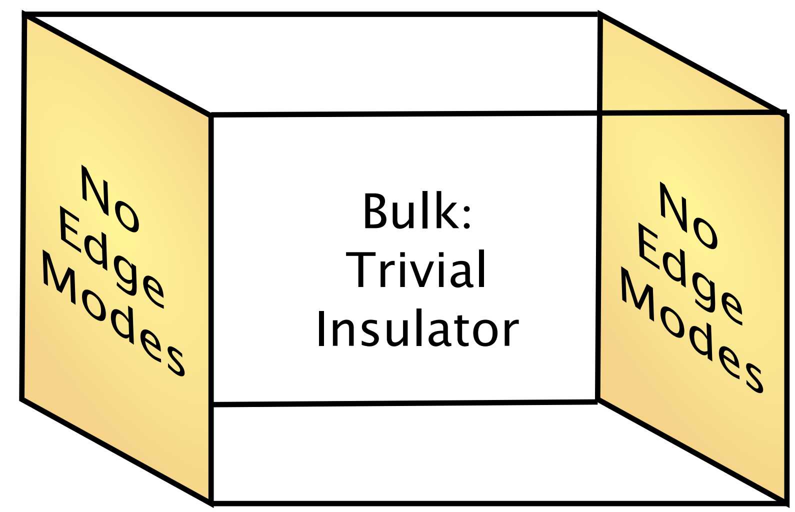


No bulk phase transition necessary!



Interactions that preserve symmetries?

U=0 ——— U=∞

with 16 Majorana (2-component) fermions
in 2+1d and 3+1d

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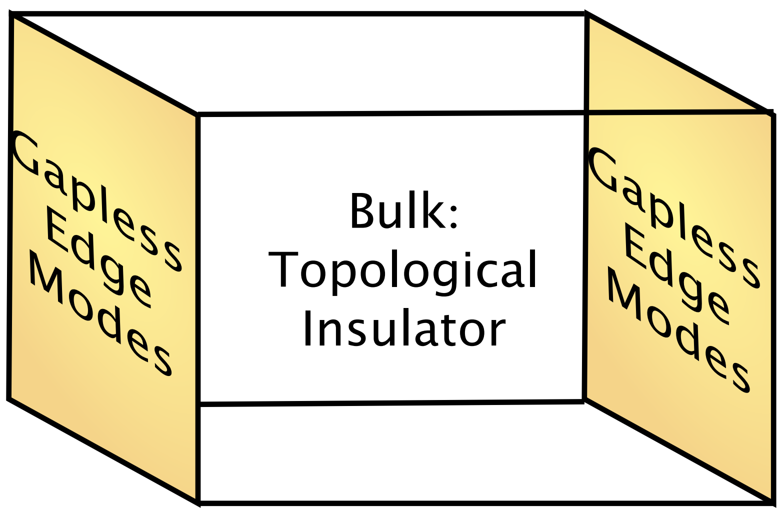

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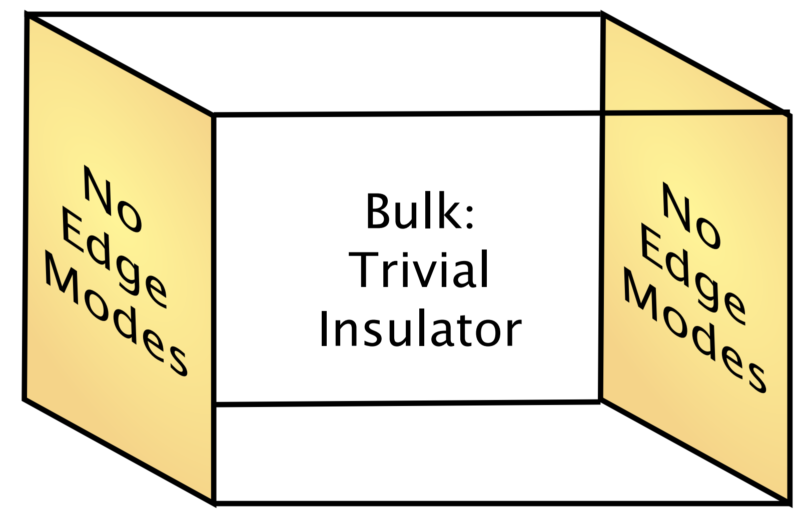

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Kitaev-Wen Mechanism for fermion mass generation on the edge

Kitaev, AIP Conf. Proc. 1134 (2009)
Wen, Chin.Phys.Lett. 30 (2013)

Can we test the Wen-Kitaev mechanism within a simple lattice field theory model?

$$S \;=\; \frac{1}{2}\sum_{x,y}\chi_x^T\; M_{x,y}\;\chi_y \;-\; U\sum_x \chi_x^4\;\chi_x^3\;\chi_x^2\;\chi_x^1$$

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No past work in 3d!

Fermion Bag Approach

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V. Ayyar and SC, Phys.Rev. D91 (2015) 6, 065035

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Partition function

$$Z = \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T M\chi + U\sum_x \chi^4_x\chi^3_x\chi^2_x\chi^1_x}$$

$$= \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T M\chi}\ \prod_x (1 + U\chi^4_x\chi^3_x\chi^2_x\chi^1_x)$$

$$= \sum_{[n]}\ U^k\ \int [d\chi]\ \mathrm{e}^{-\frac{1}{2}\chi^T W\chi}$$

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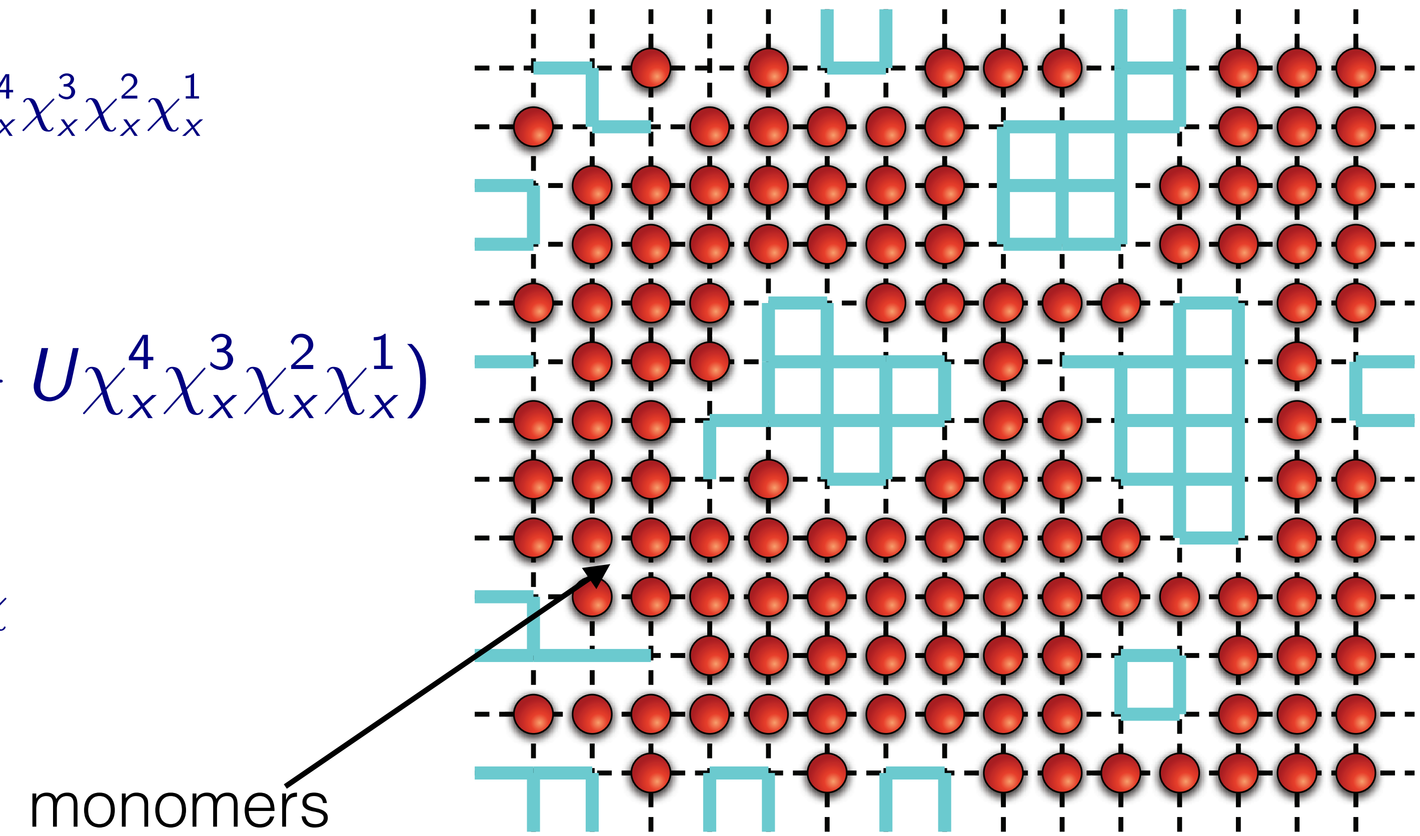
Illustration of configuration [n]



k refers to the number of monomers

Monte Carlo Results in 2+1D

Observables:

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Correlation Function

$$C_1(0,x) \;=\; \left\langle \chi_0^1\chi_0^2 \;\; \chi_x^1\chi_x^2 \right\rangle$$

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Correlation Ratios

$$R_1 = C_1(0, L/2-1)/C_1(0,1)$$

$$R_2 = C_2(0, L/2)/C_2(0,0)$$

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Susceptibilities

$$\chi_a \;=\; \sum_x C_a(0,x)$$

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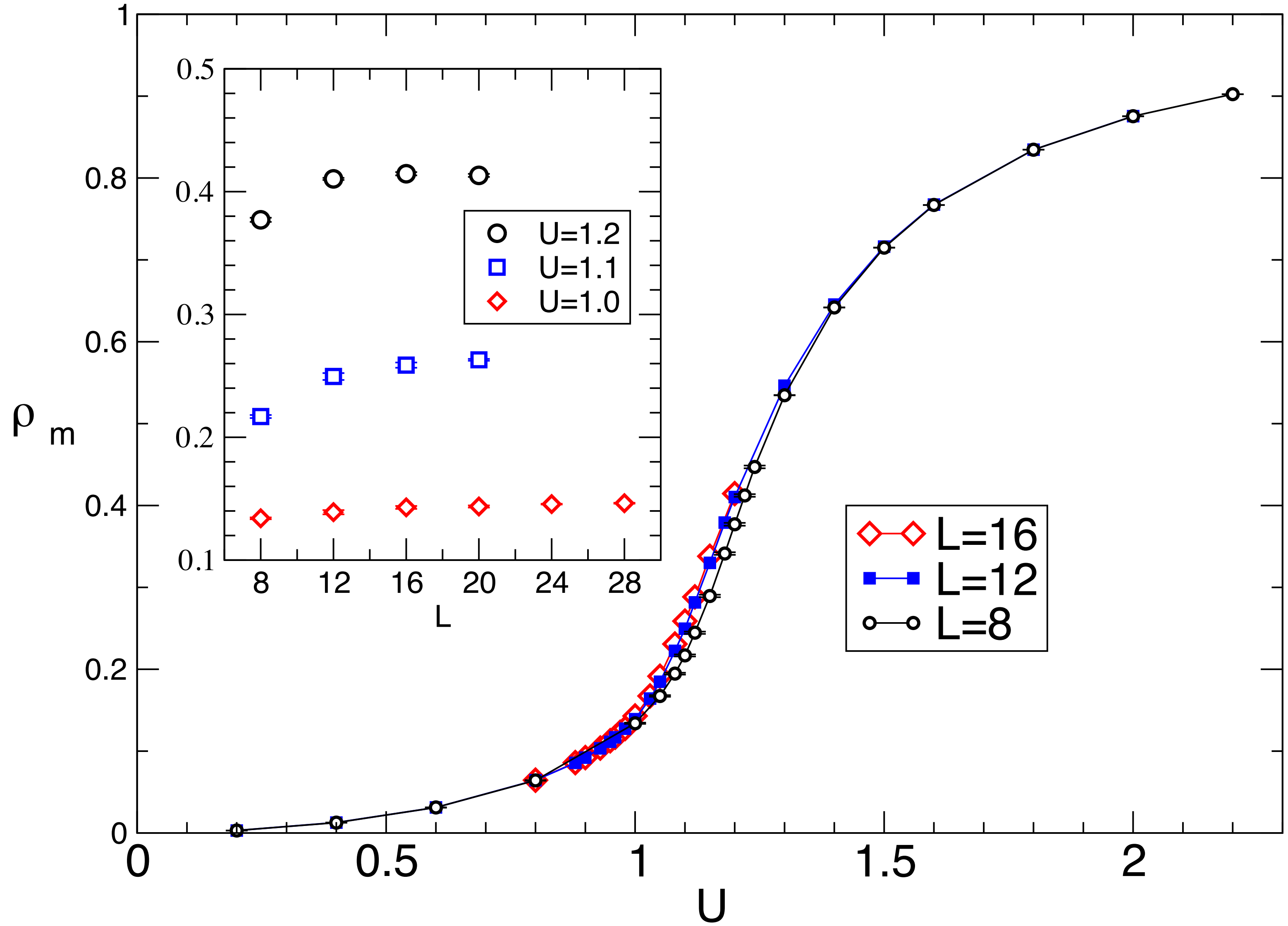

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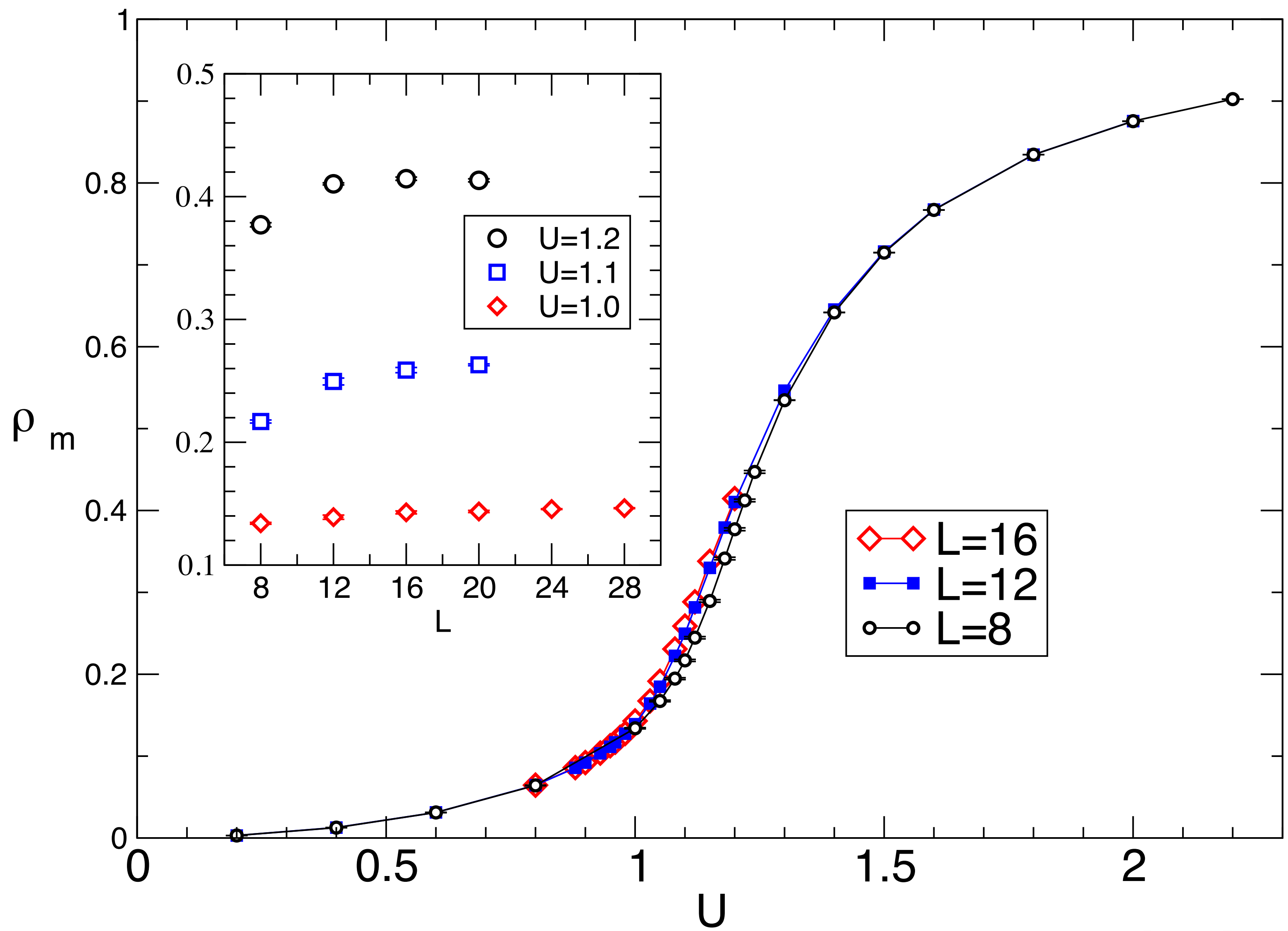

No indication of a strong first order transition

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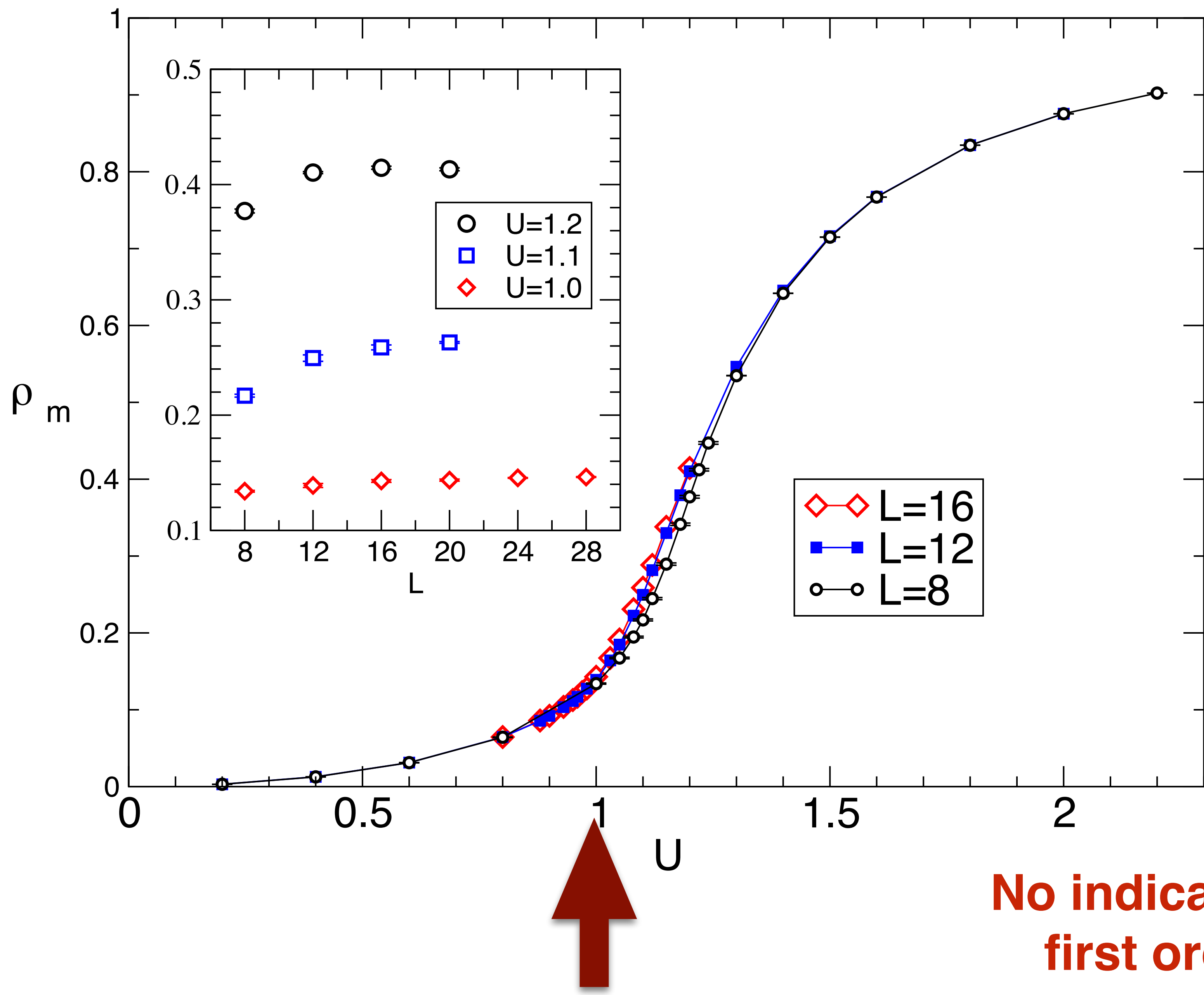

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critical point $\approx$ 1?

Susceptibility

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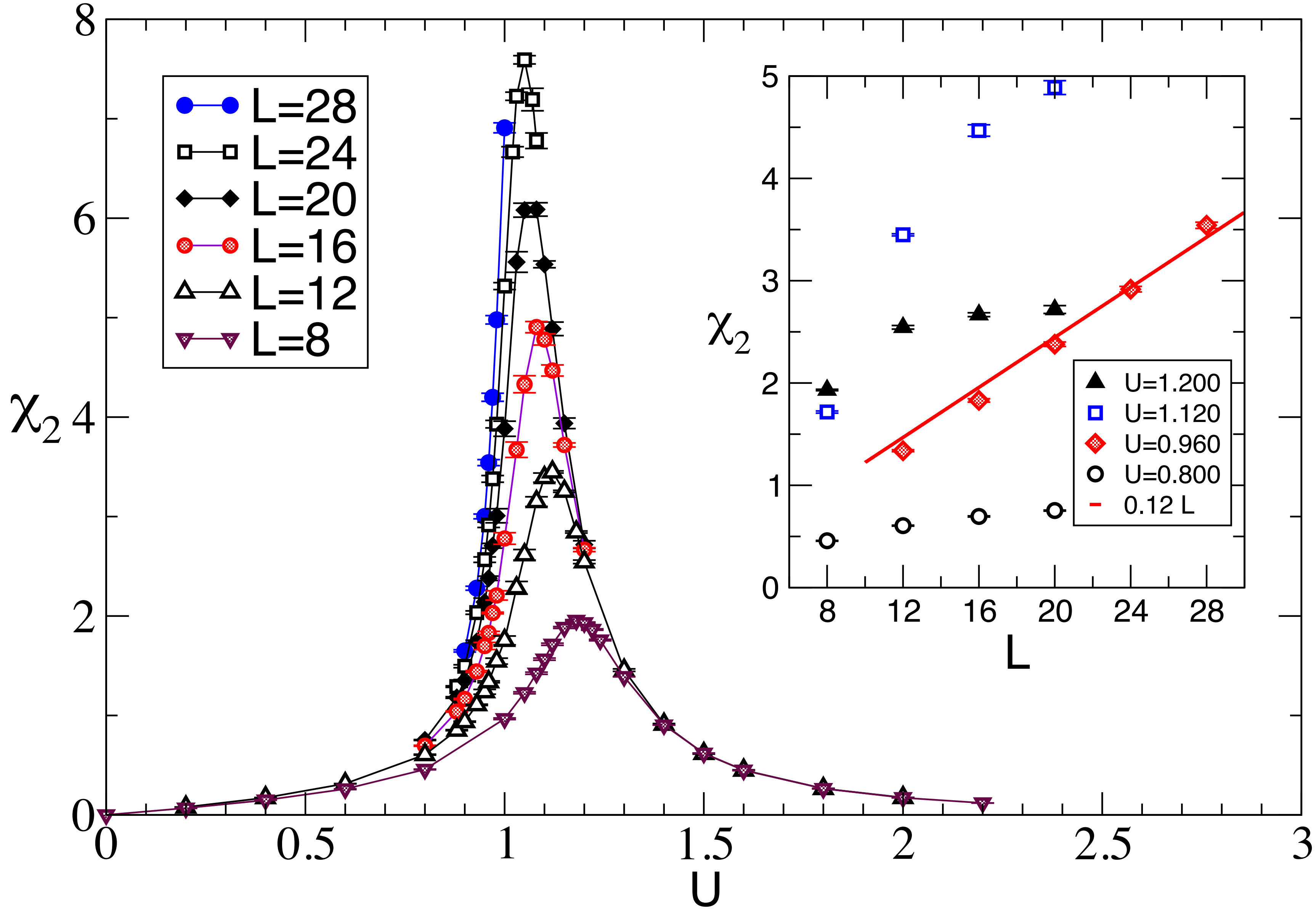

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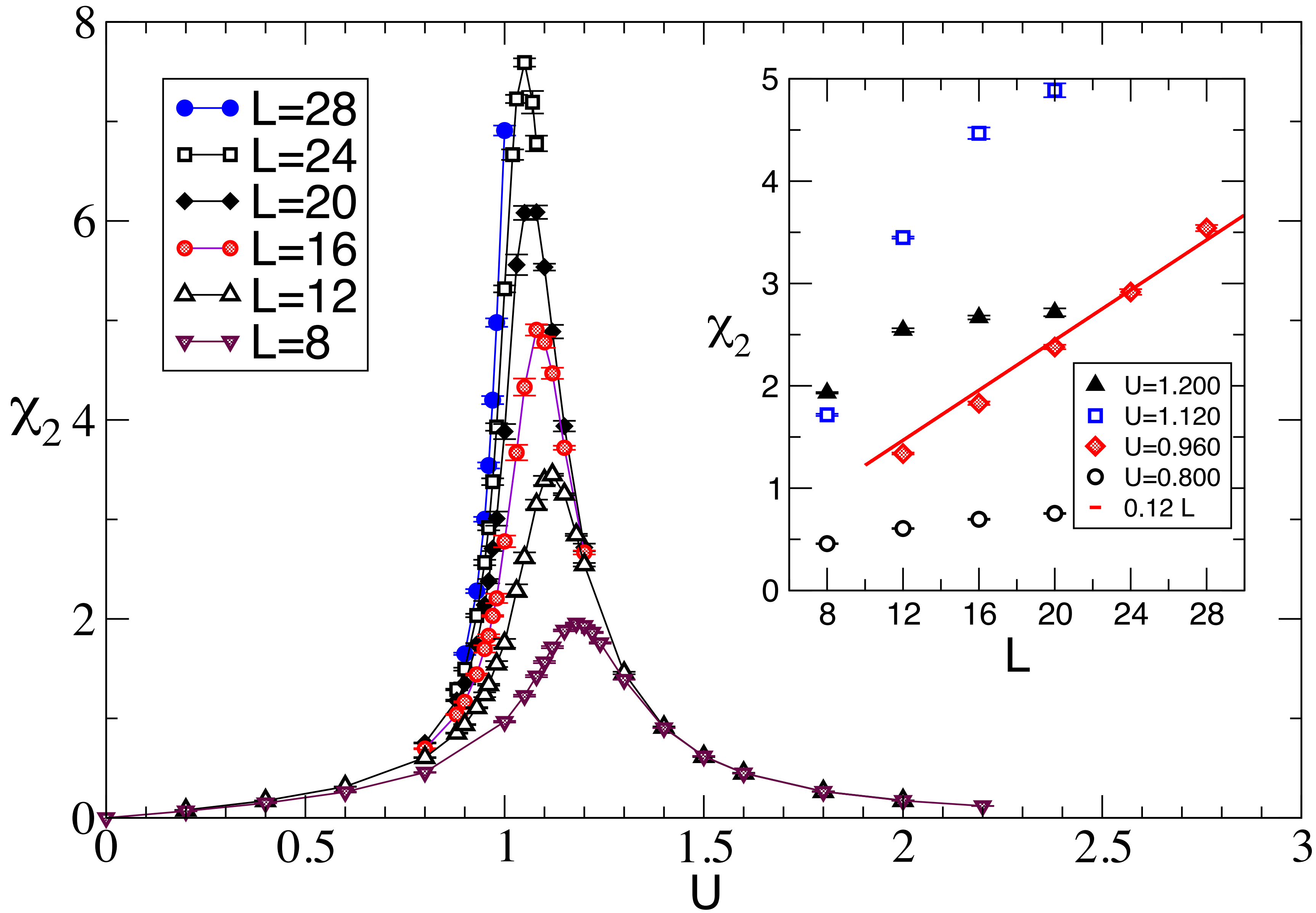

In the presence of a non-zero fermion bilinear condensate we expect $\chi_2 \sim L^3$.

Correlation Ratios

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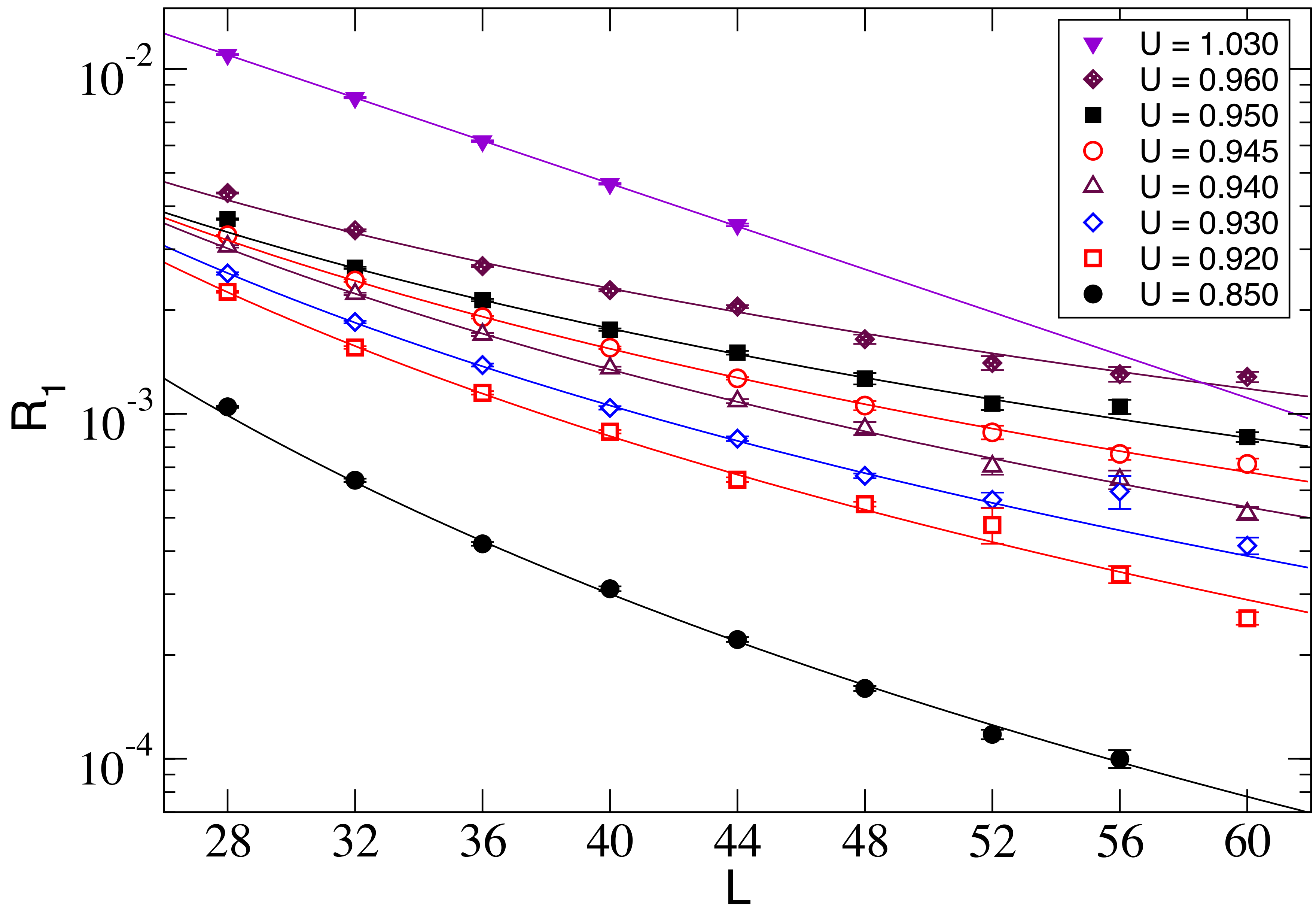

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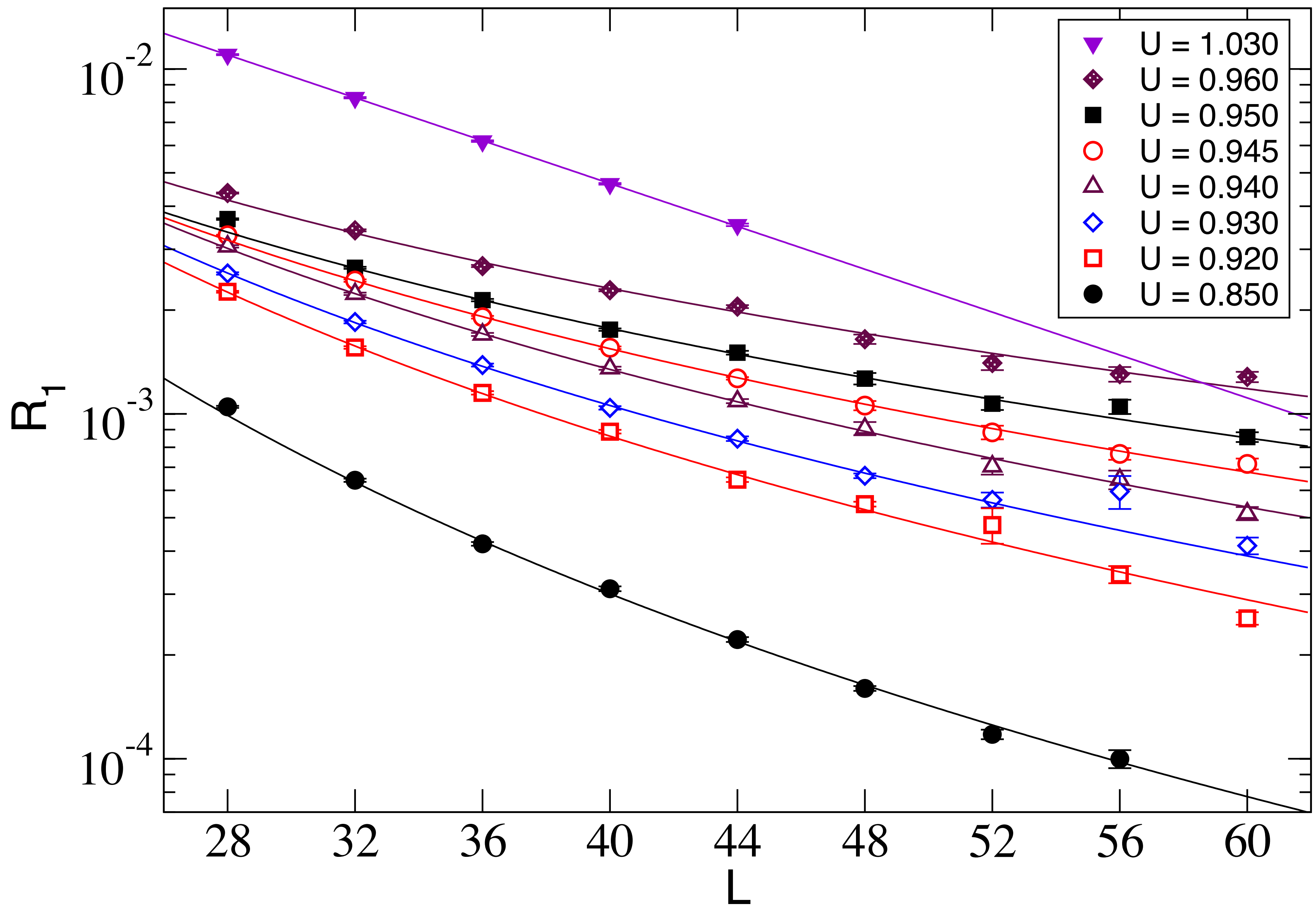
$$R_1 \sim \begin{cases} 1/L^4 & U < U_c \\ 1/L^{1+\eta} & U = U_c \\ exp(-mL) & U > U_c \end{cases}$$



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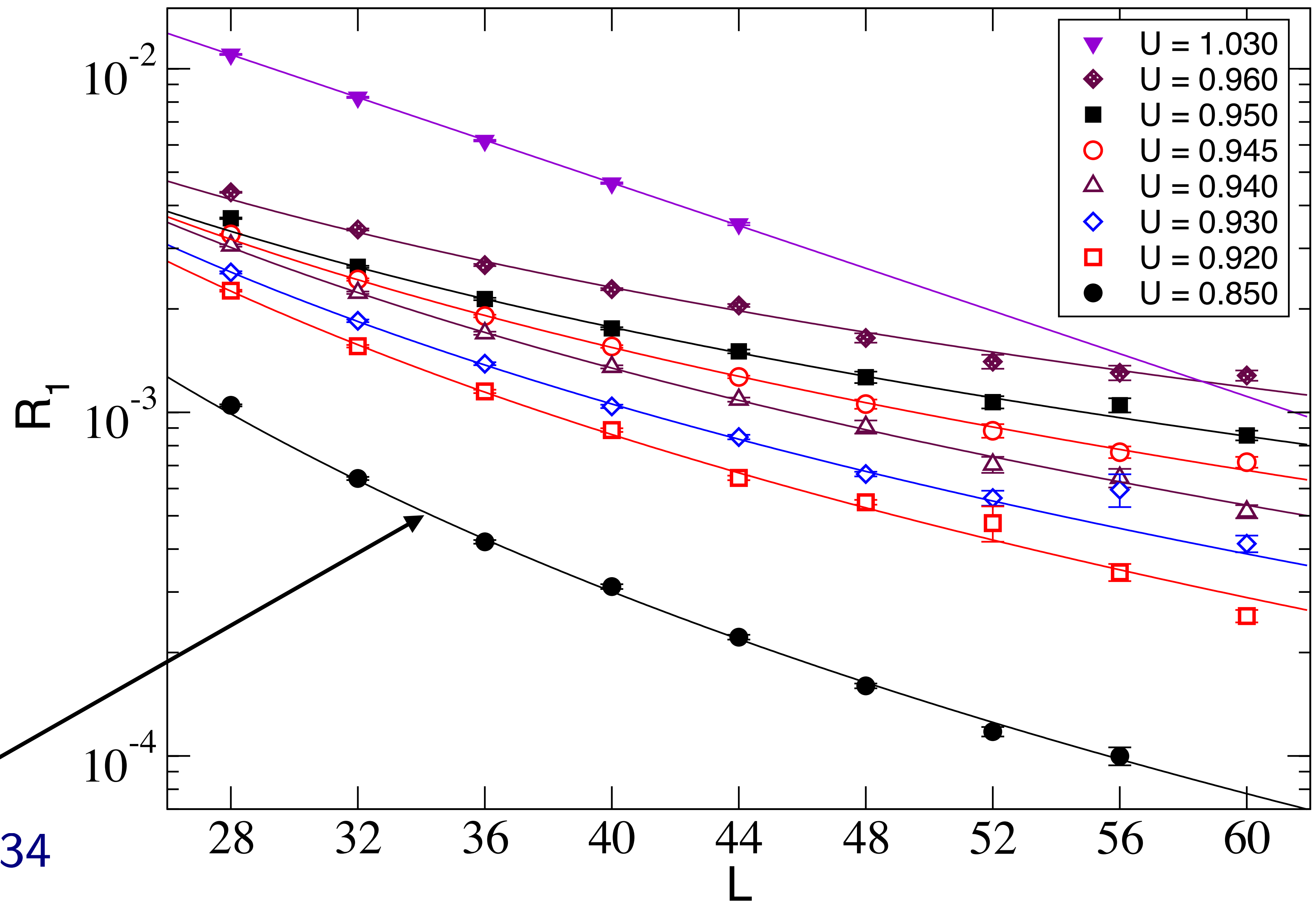

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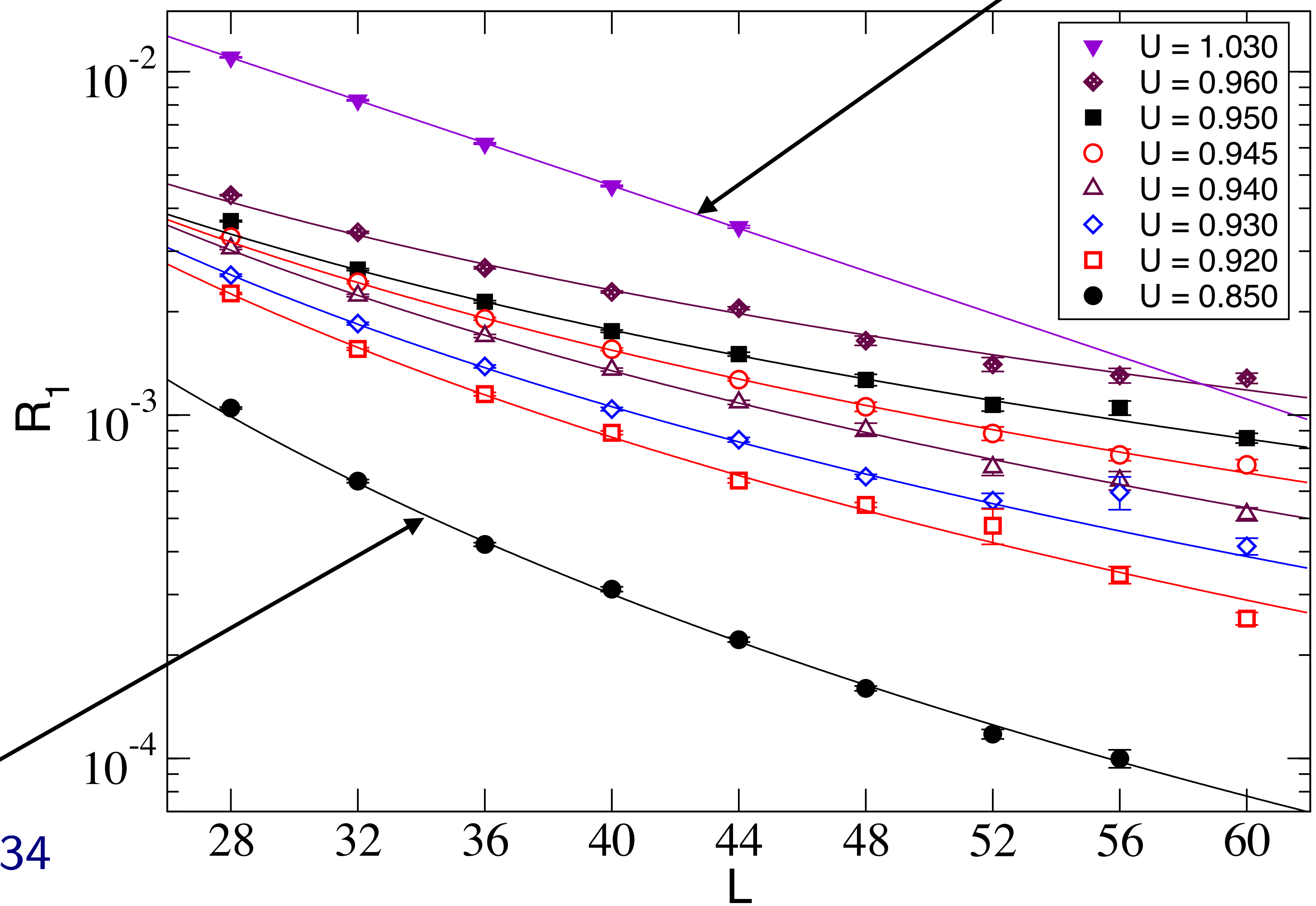

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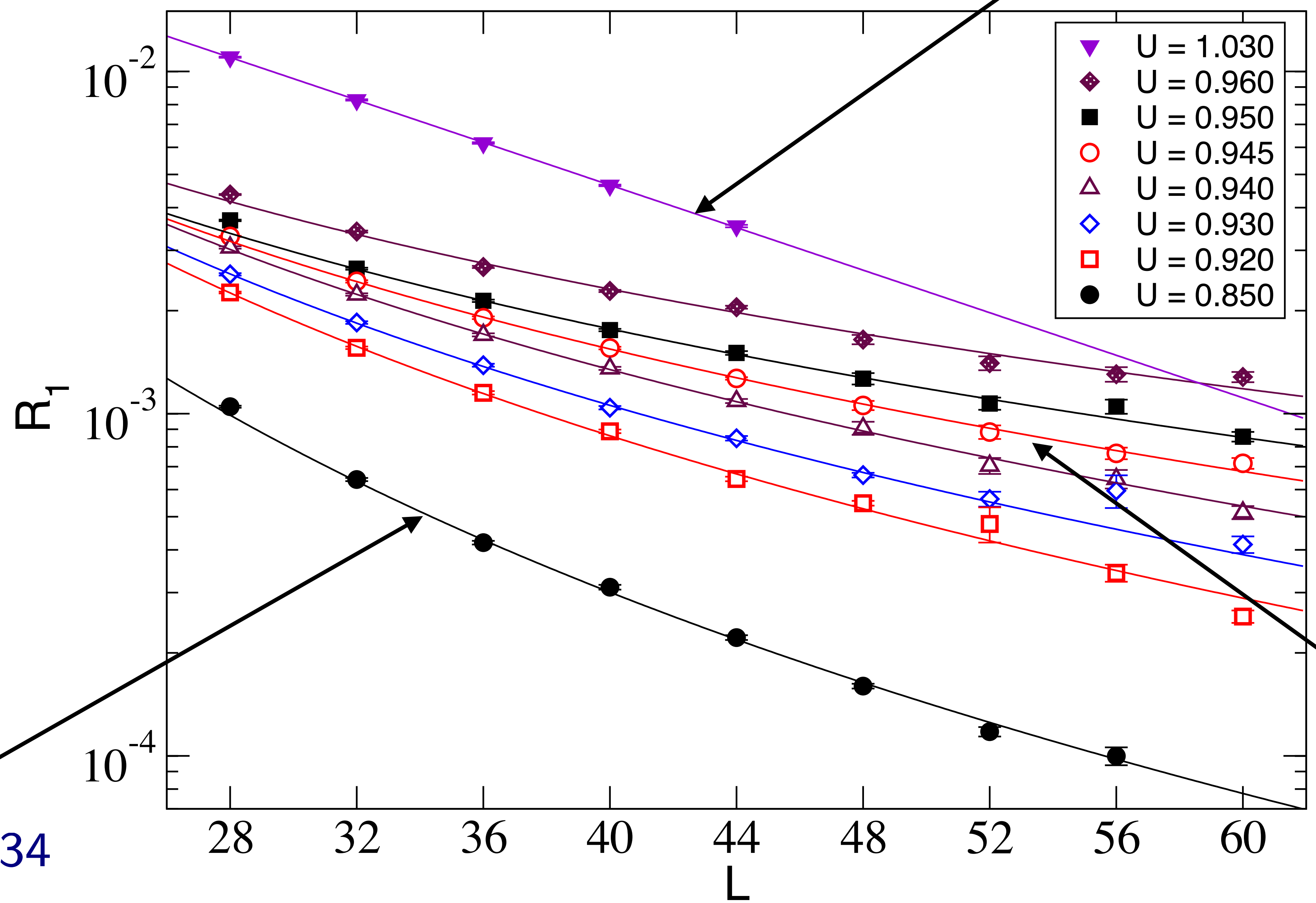
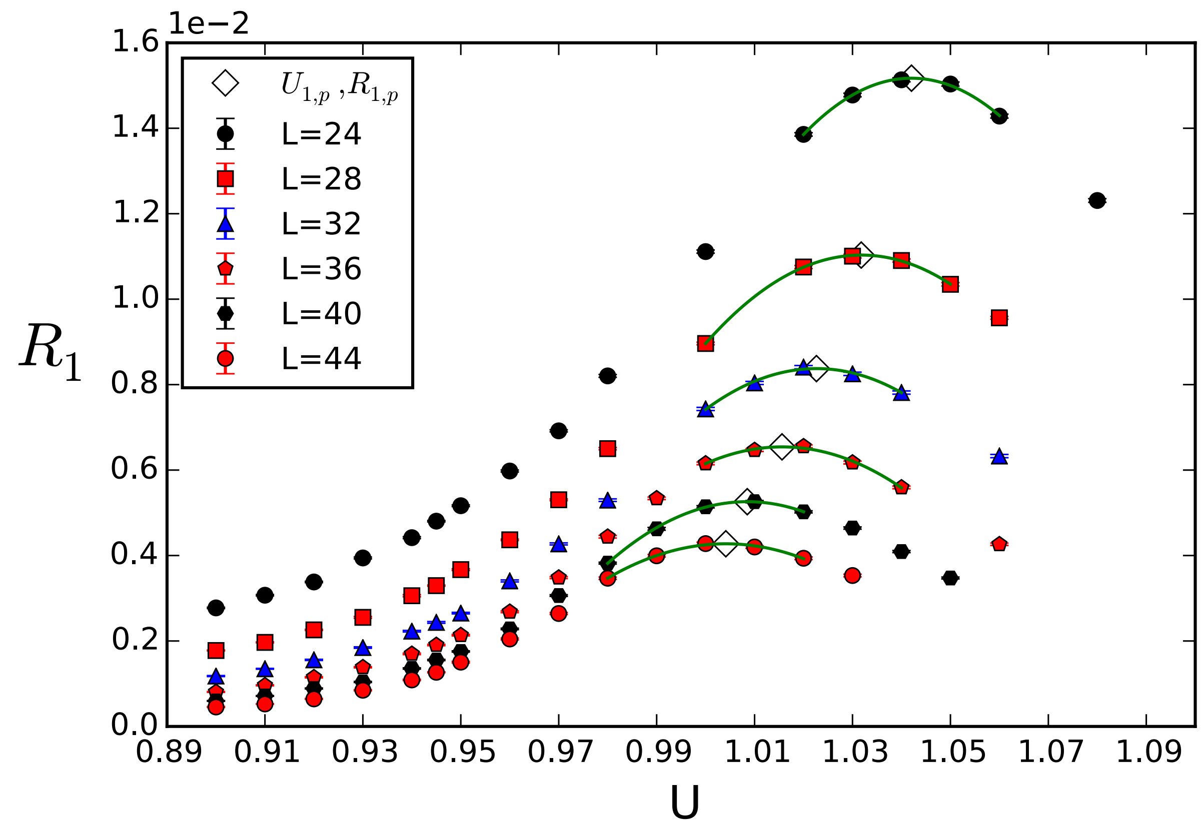


$1/L^2$

$1/L^{3.34}$

Scaling of the Peaks of Correlation Ratios



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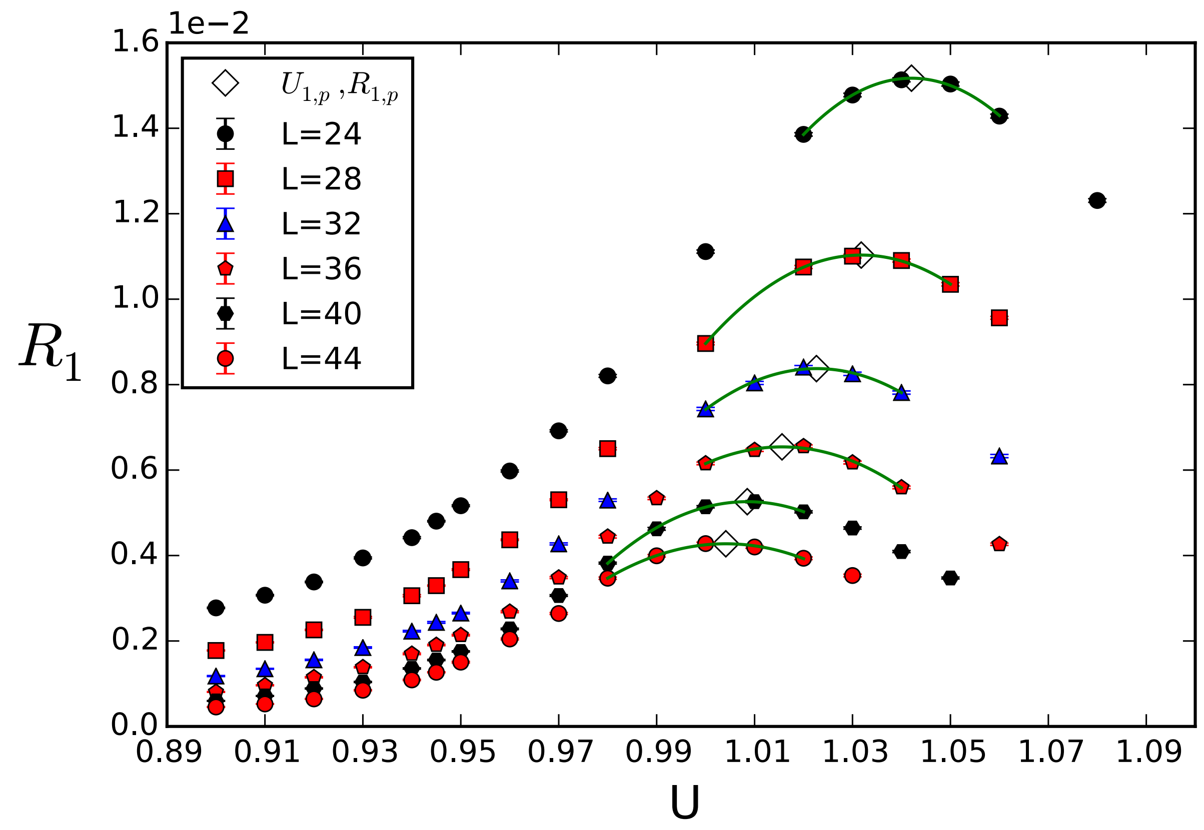
$$R_{a,\text{peak}} = b_a / L^{1+\eta}$$

$$U_{a,\text{peak}} = U_c + d_a L^{1/\nu}$$



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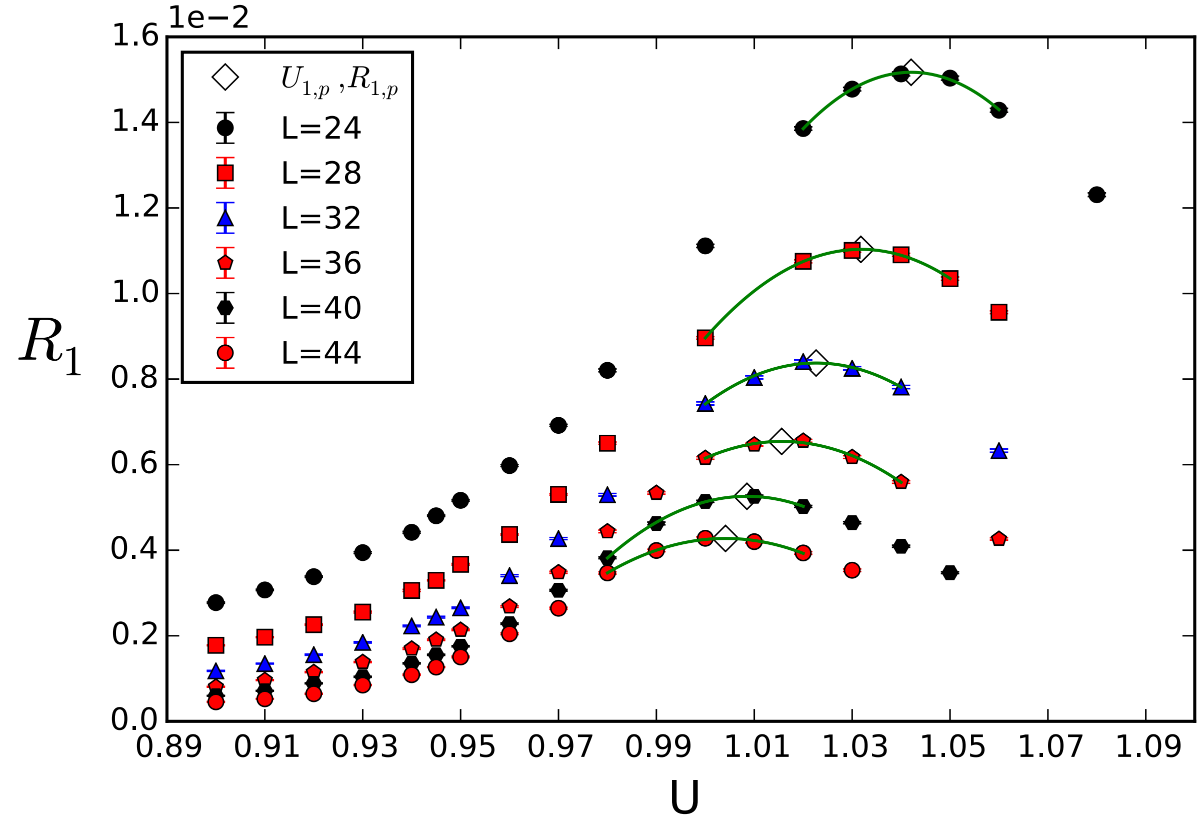

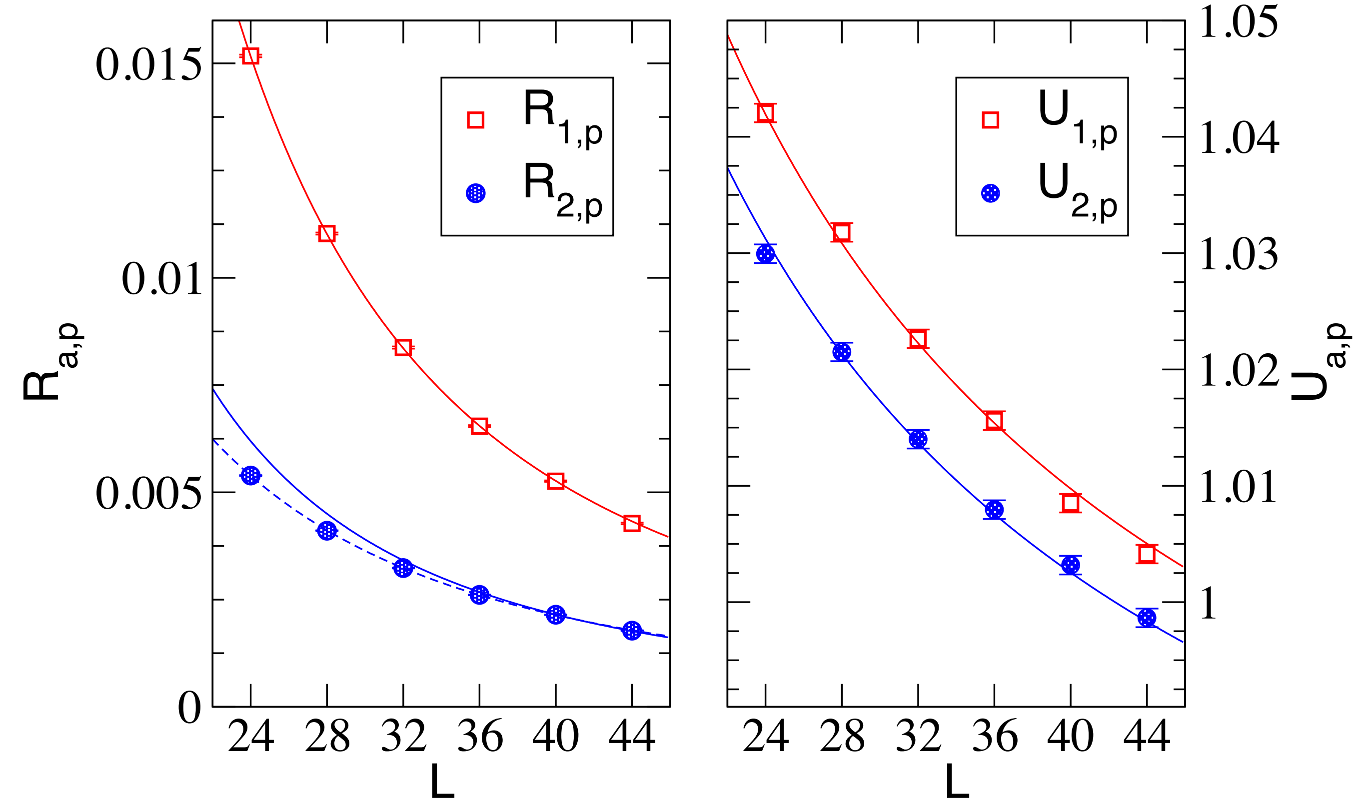

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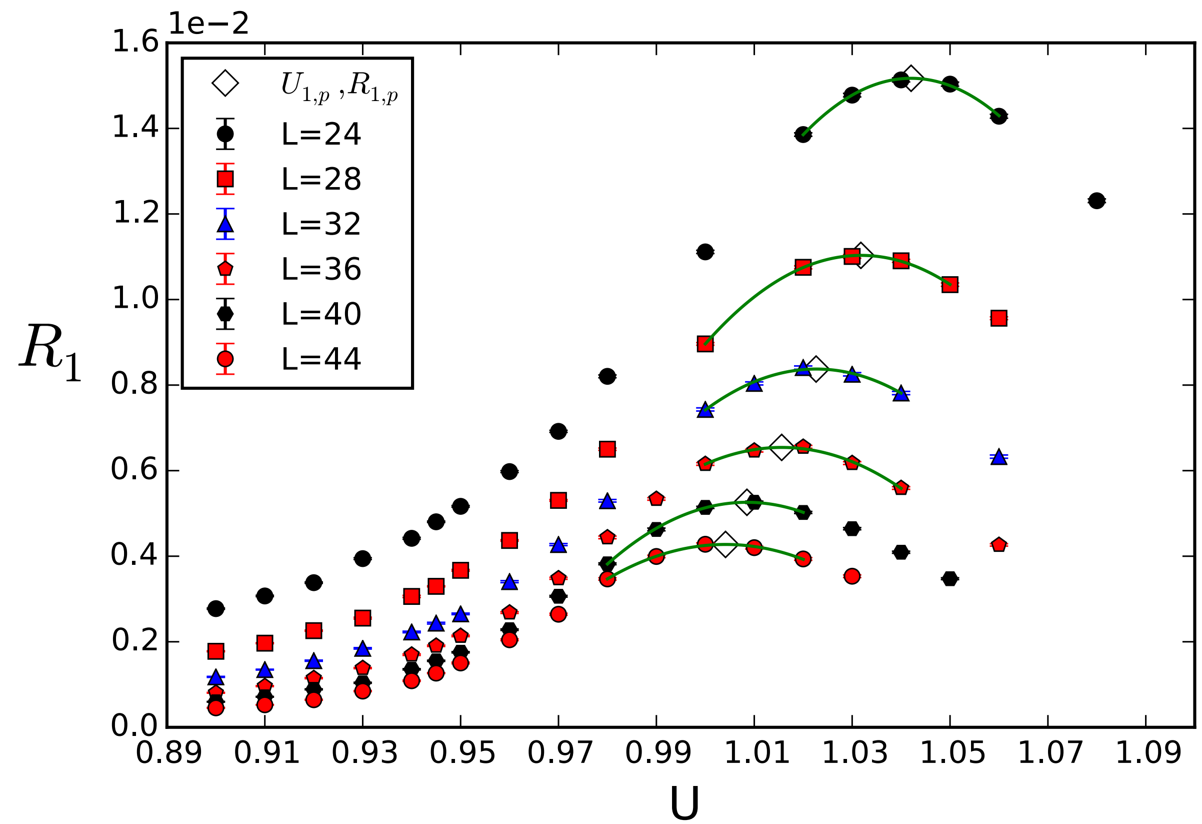
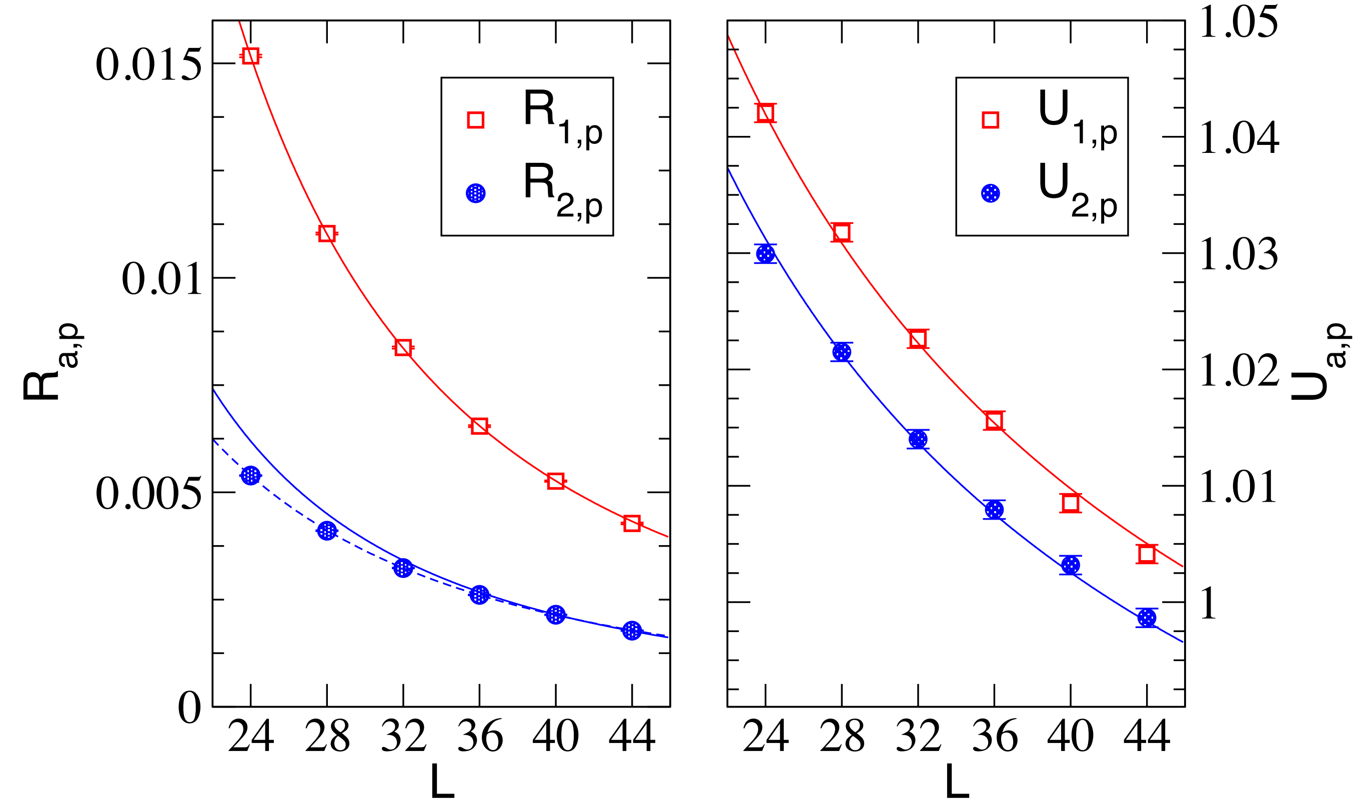
$$U_c = 0.943(2)$$

$$\eta = 1.05(5)$$

$$\nu = 1.30(7)$$





Critical Scaling

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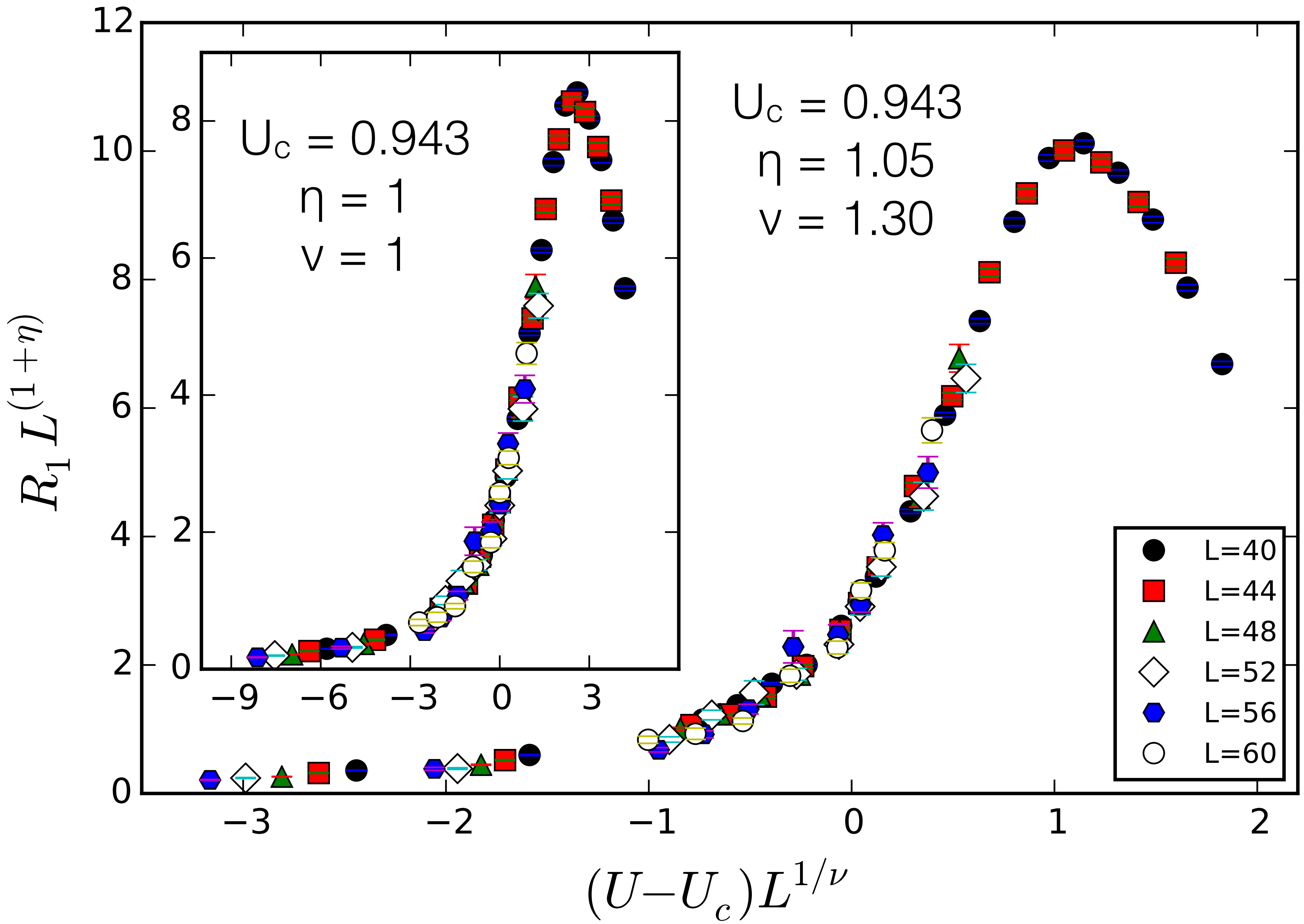

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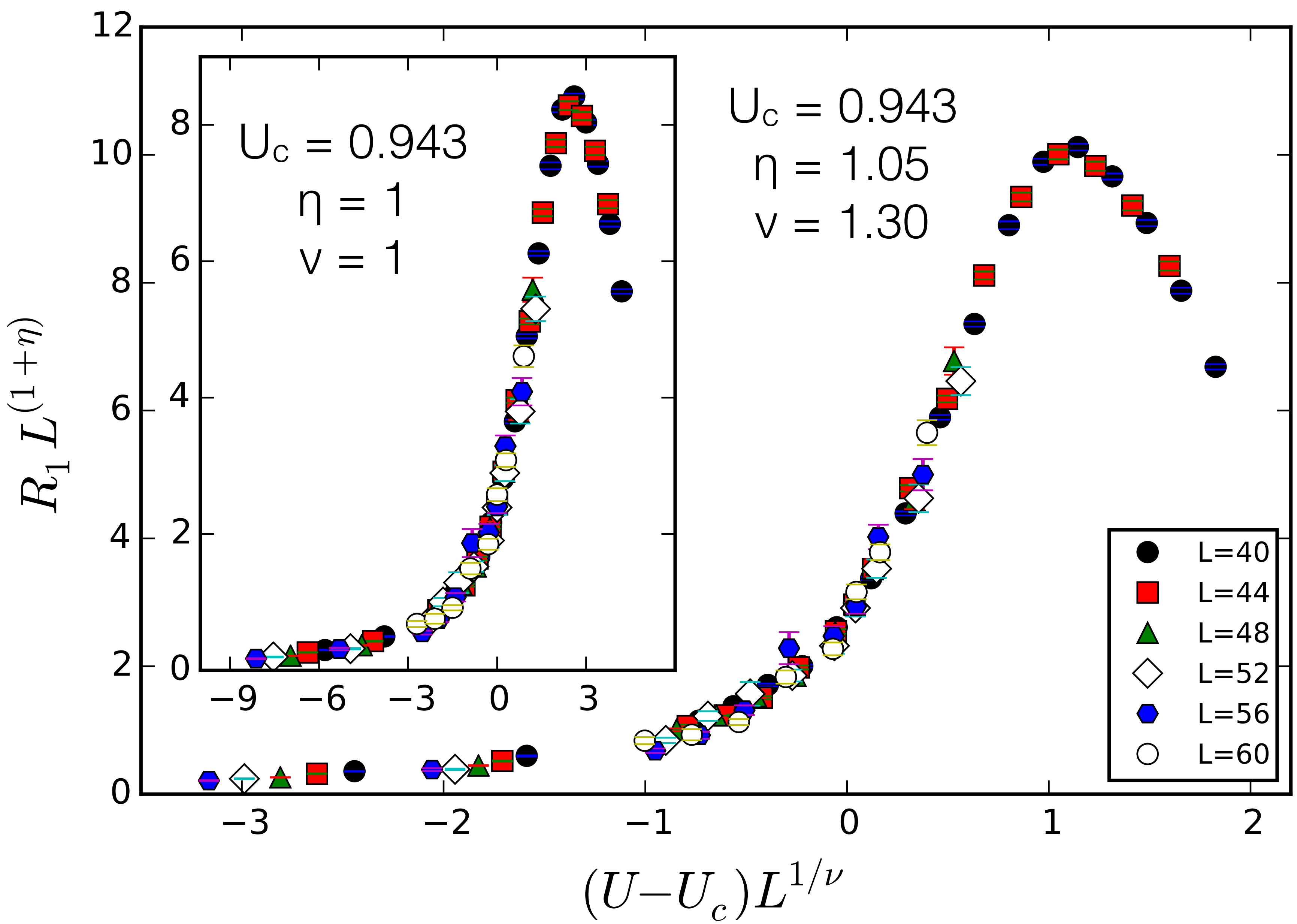

Strong evidence for the exotic transition!

Monte Carlo Results in 3+1D

Summary of Past Work:

Lee,Shigemitsu and Shrock
NPB 334 (1990) 265

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Found a wide intermediate FM phase.

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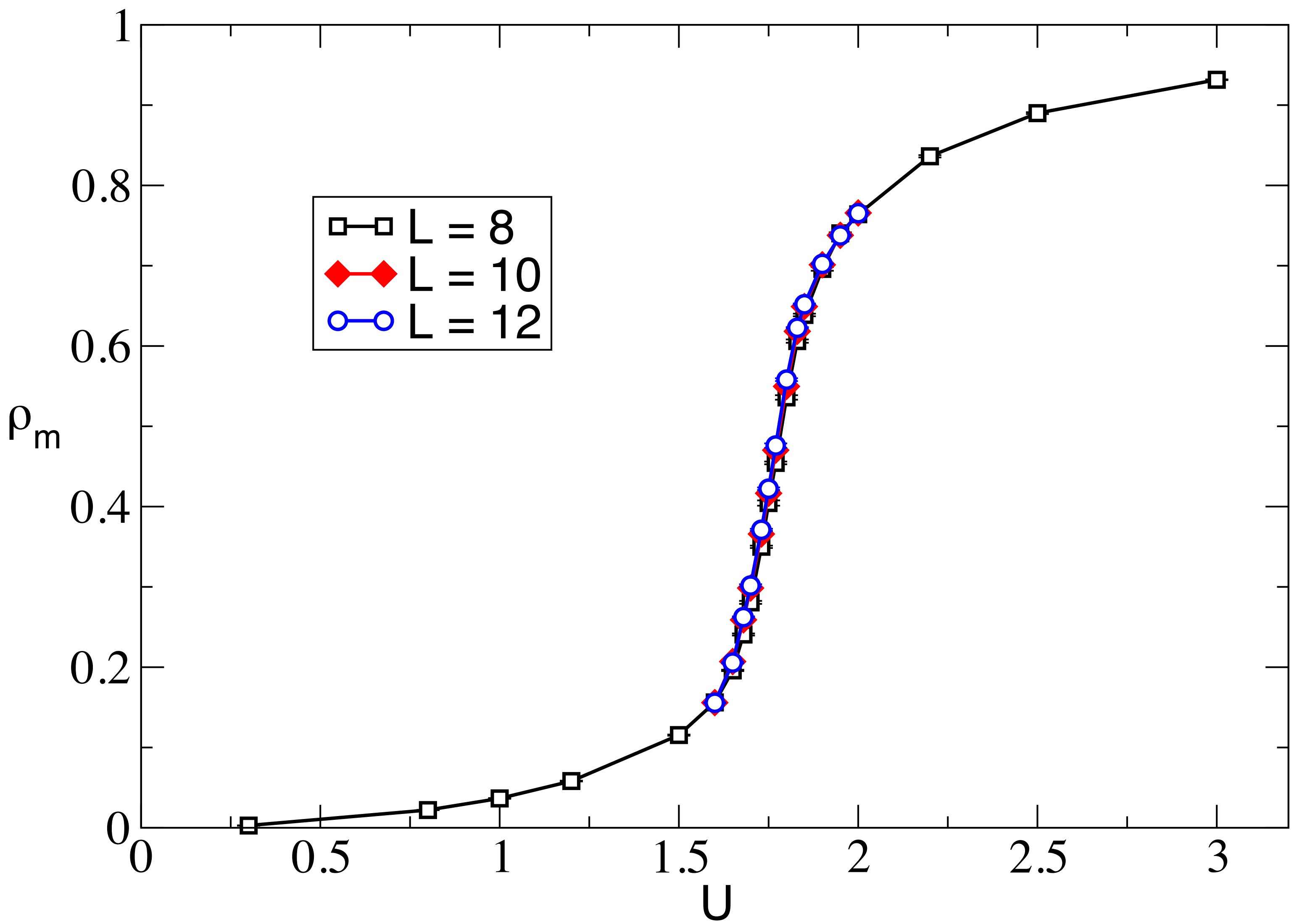

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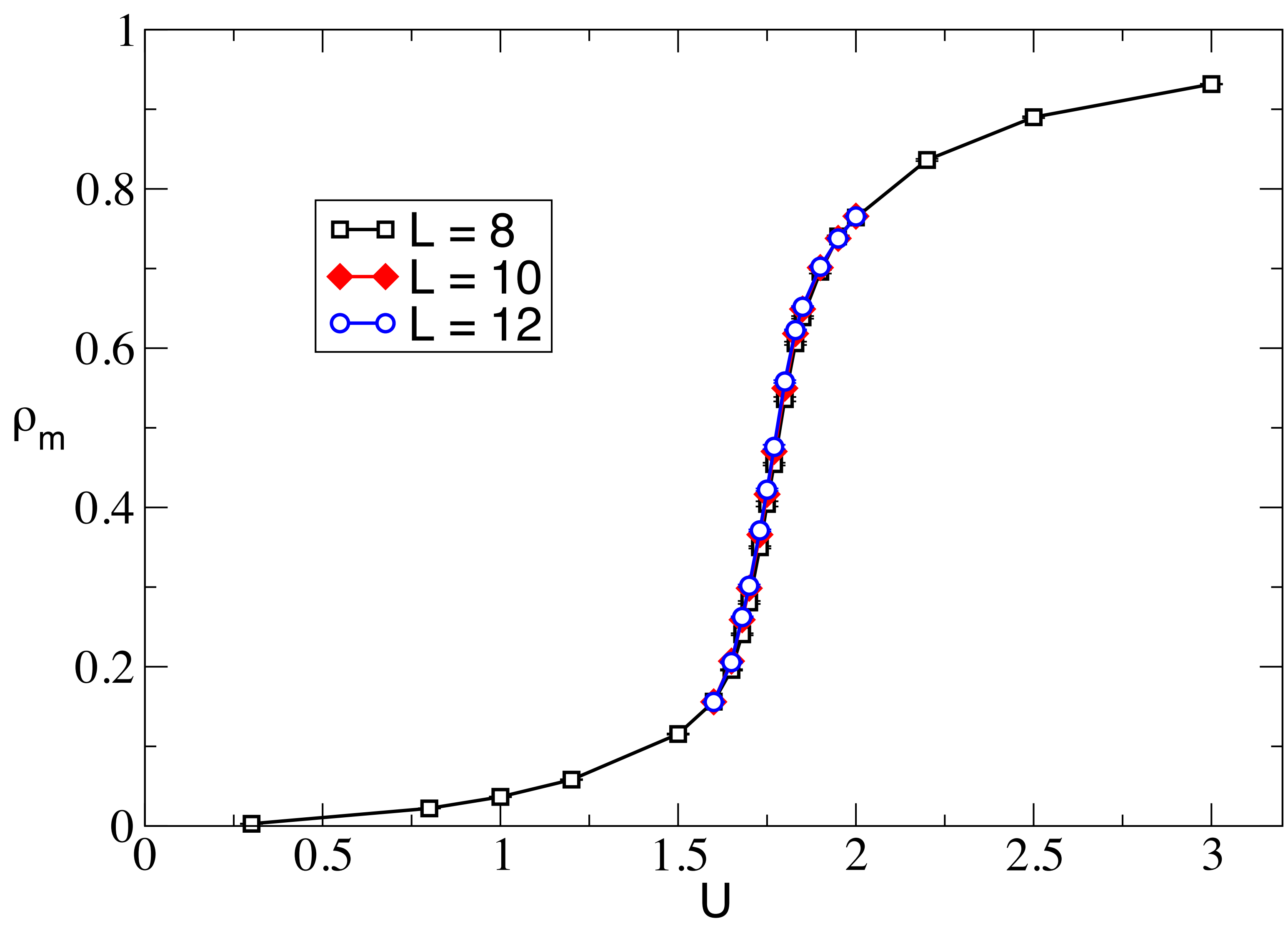

No indication of a strong first order transition

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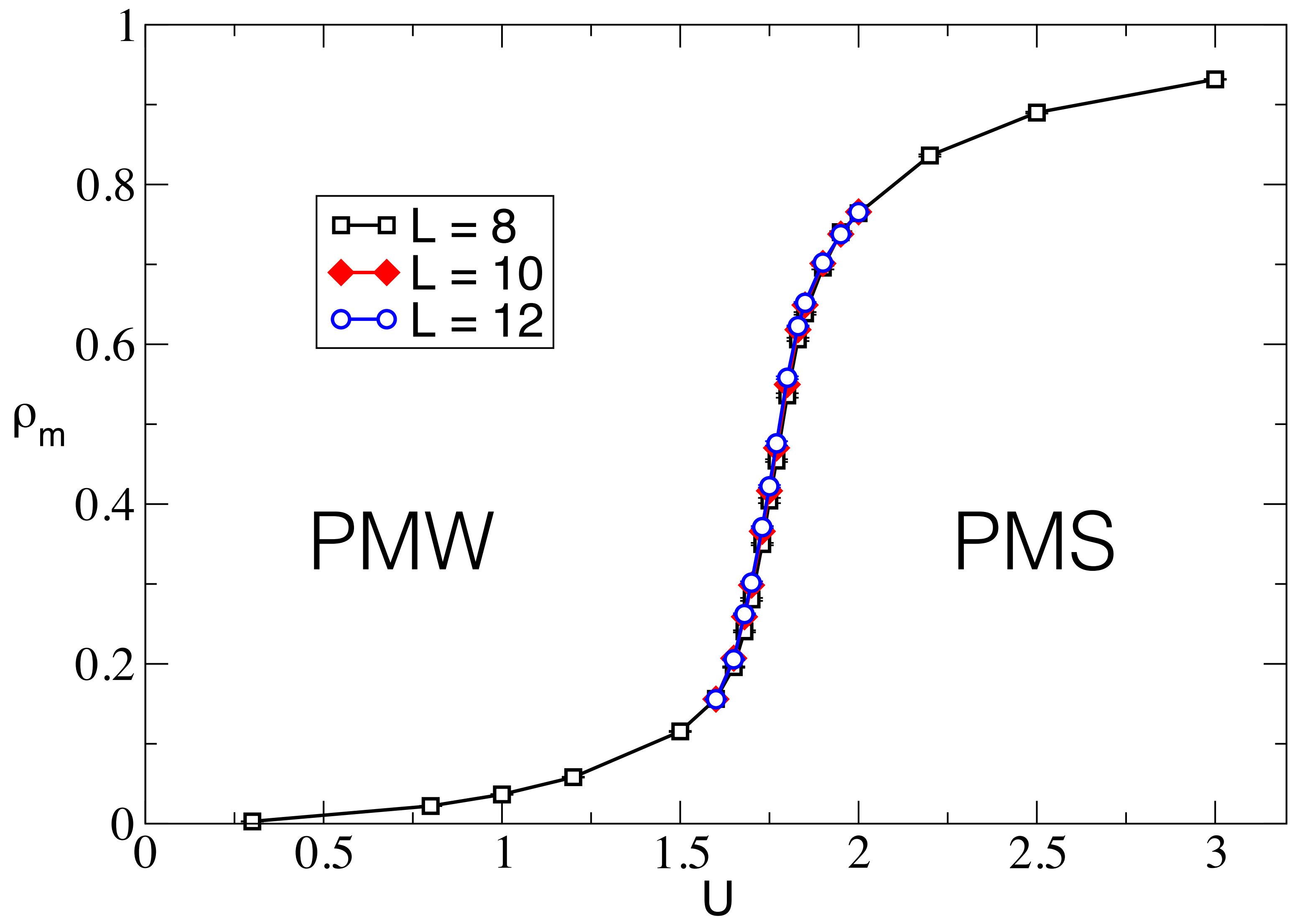

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Very narrow FM phase

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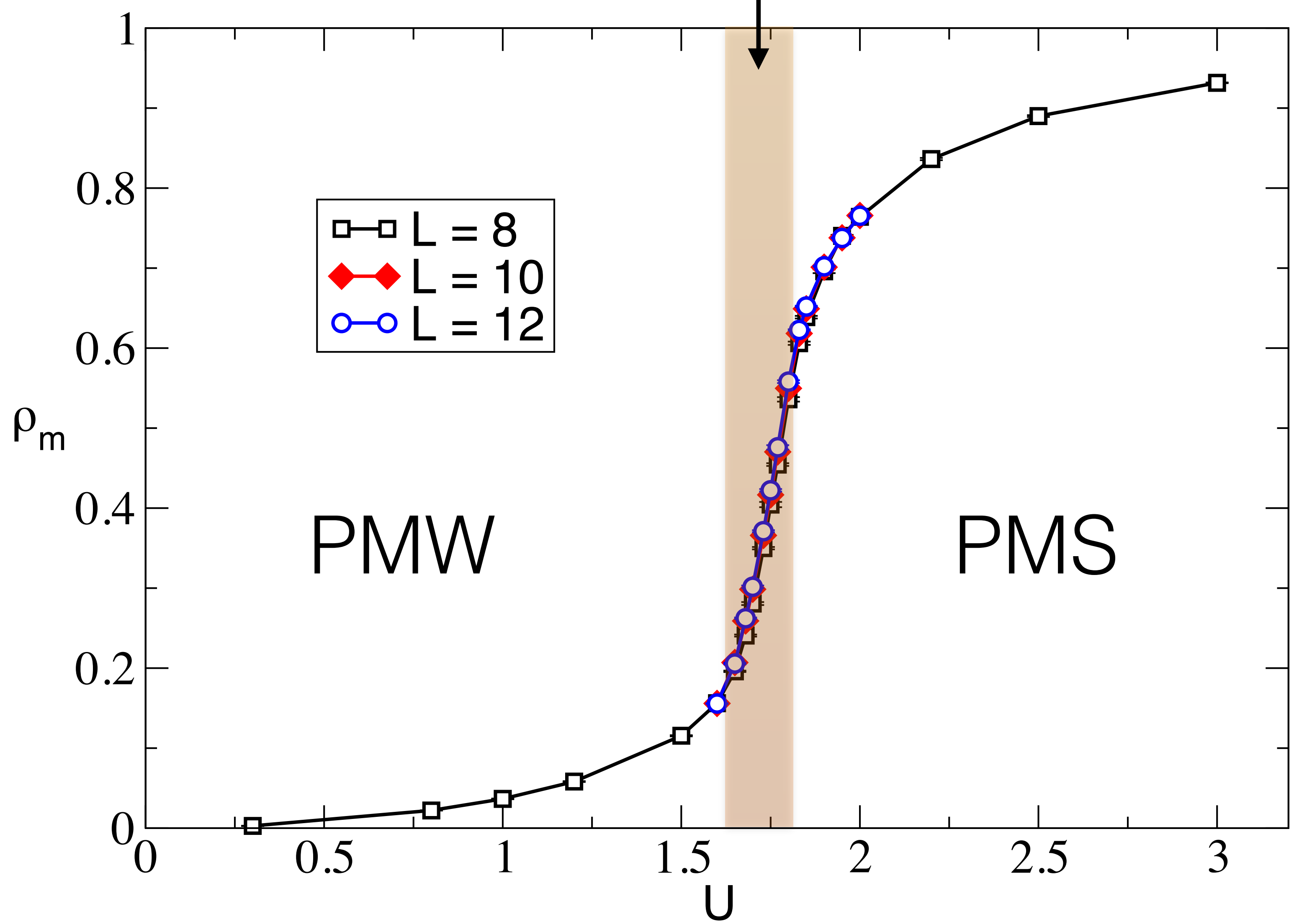

Possible FM phase is quite narrow
1.6 < U < 1.8

No indication of a strong first order transition

Susceptibility

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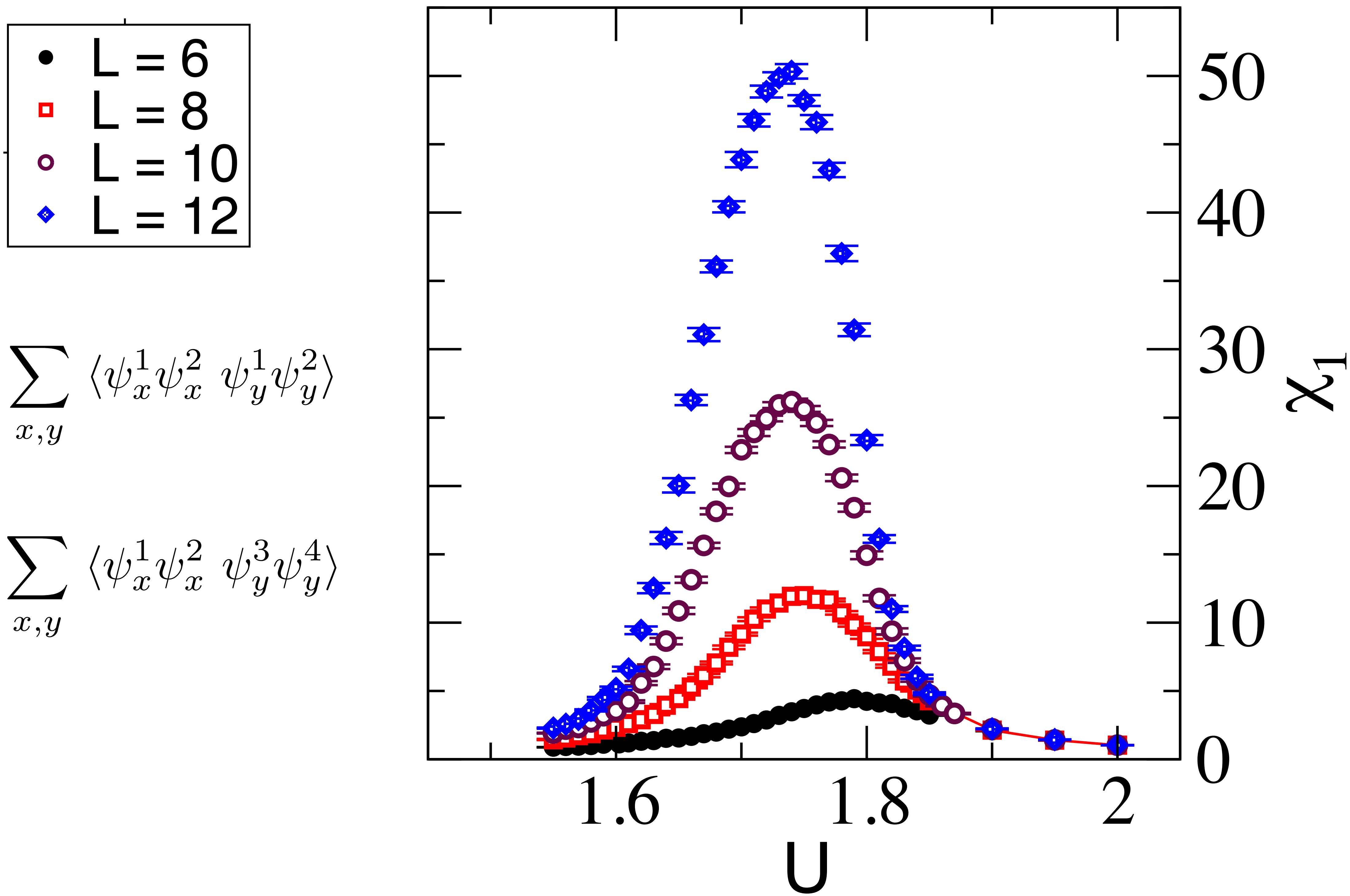

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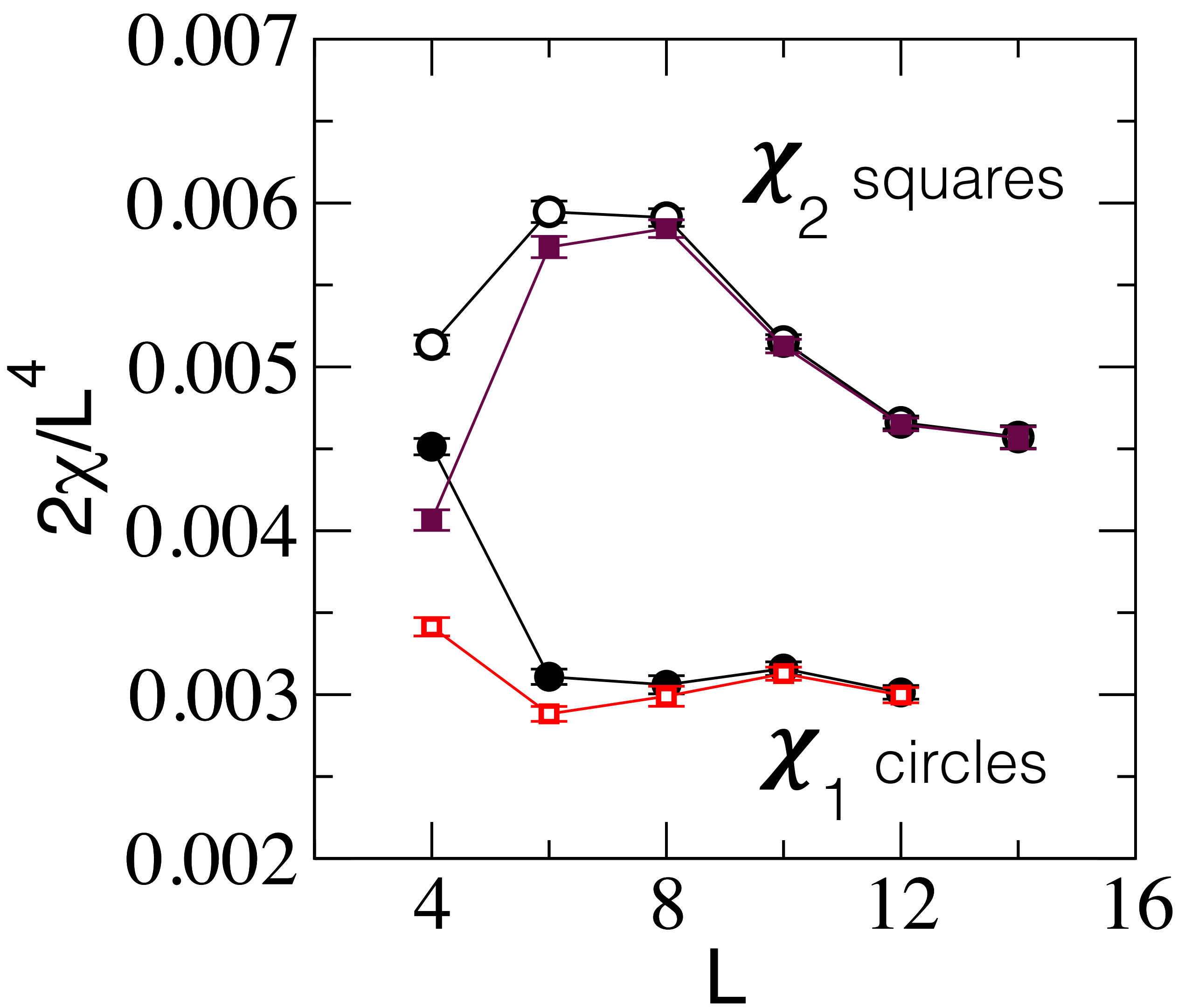

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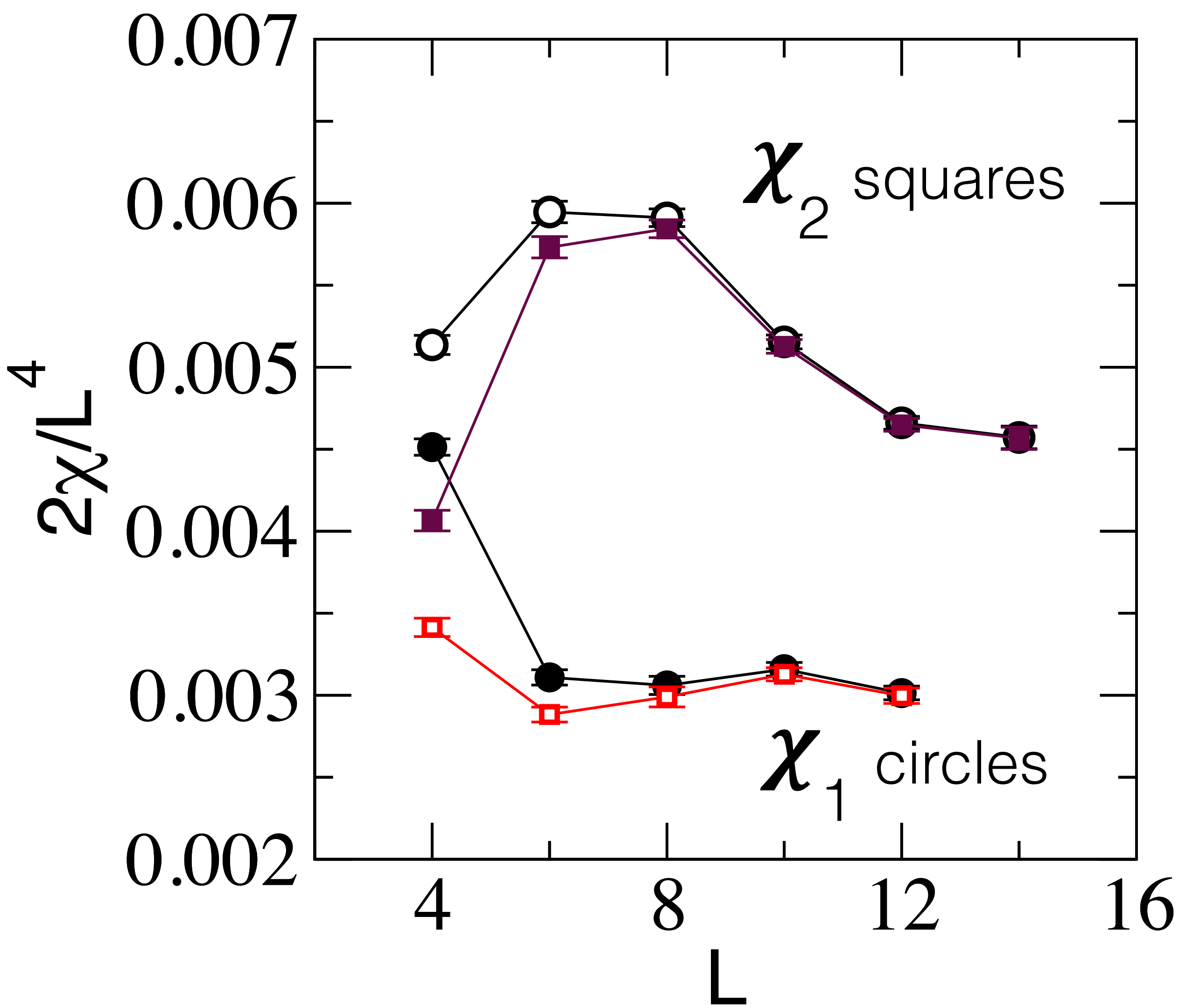
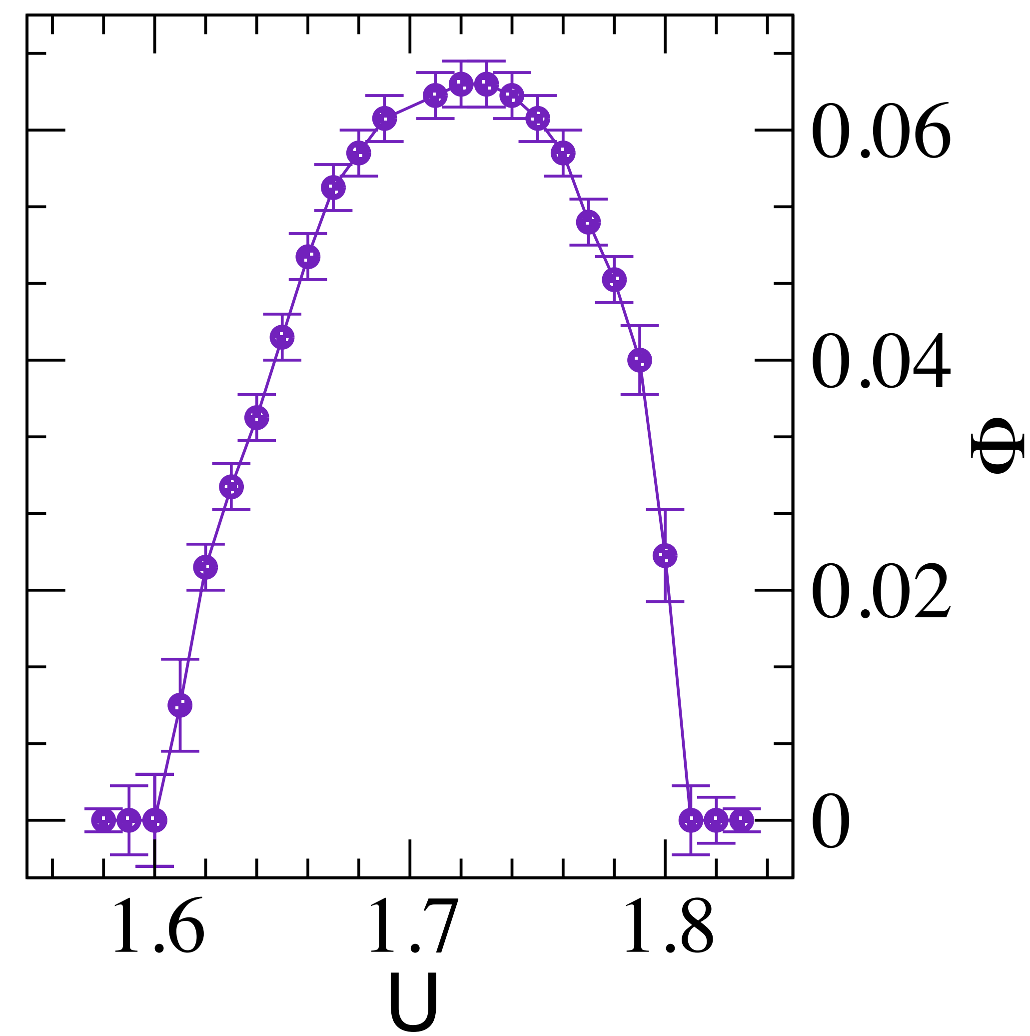
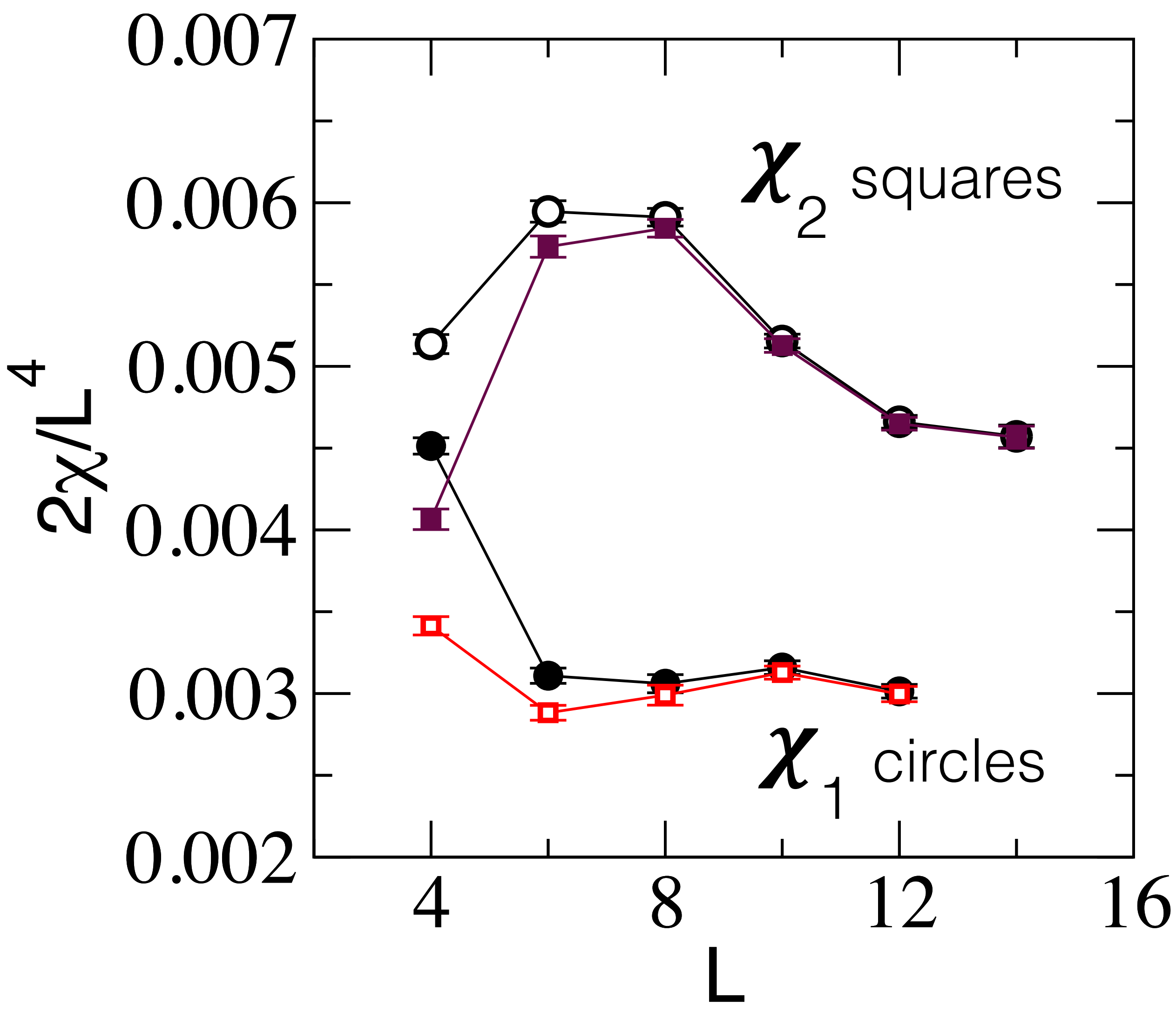
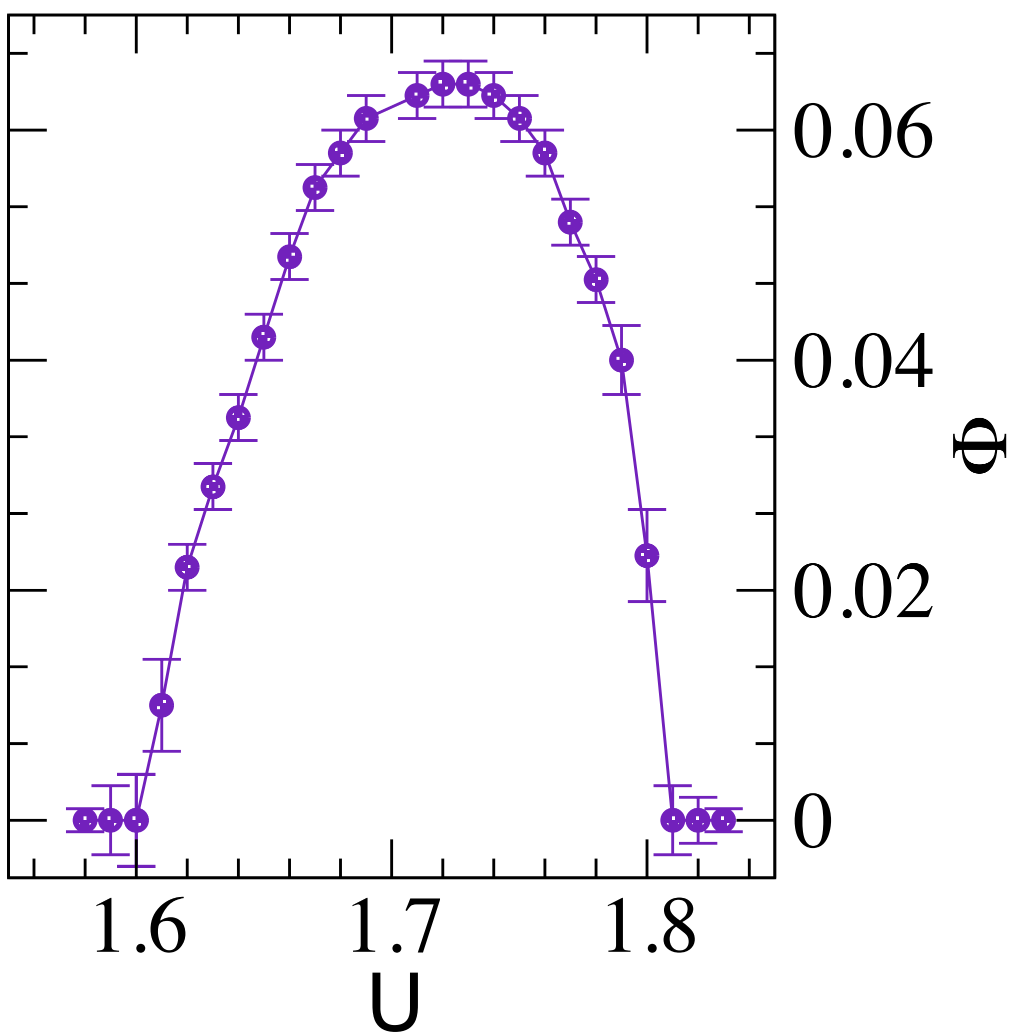




Condensate

$$\chi \sim \frac{1}{2}\,\Phi^2\,L^4$$





Condensate is naturally small in lattice units!

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    - Are there applications to BSM phenomenology?