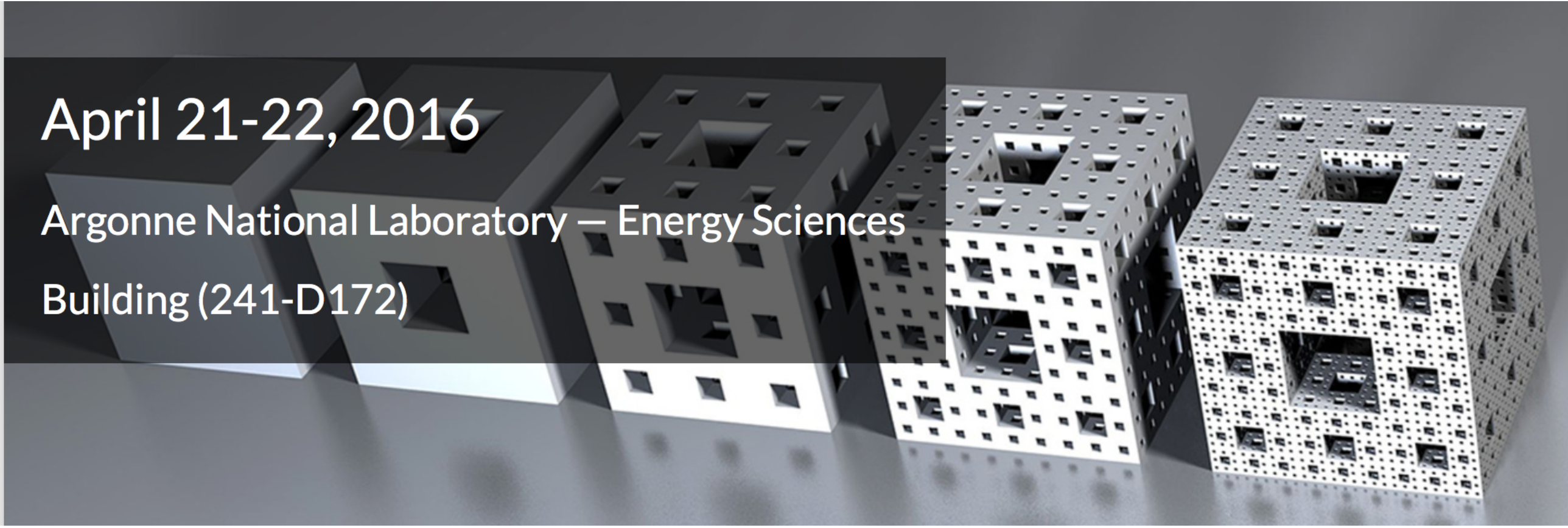


Varying Nf in QCD: scale separation, topology (and hot axions)

Maria Paola Lombardo
INFN

I. *Zero temperature:*

String tension, Critical temperature, Wilson flow

MpL, K. Miura, T. J. Nunes da Silva and E. Pallante,
Int. J. Mod. Phys. A **29**, no. 25, 1445007 (2014),
+ work in progress

II. *High temperature:*

Topological susceptibility

A. Trunin, F. Burger, E. M. Ilgenfritz, MpL and M. Müller-Preussker,
J. Phys. Conf. Ser. **668**, no. 1, 012123 (2016),
J. Phys. Conf. Ser. **668**, no. 1, 012092 (2016),
+ work in progress

Determining the topological susceptibility in lattice QCD at nonzero temperature

M. Müller-Preussker

Humboldt University Berlin, Physics Department

Gauge Field Topology:
From Lattice Simulations and Solvable Models to Experiment
Simons Center for Geometry & Physics, Stony Brook

August 17 - 21, 2015

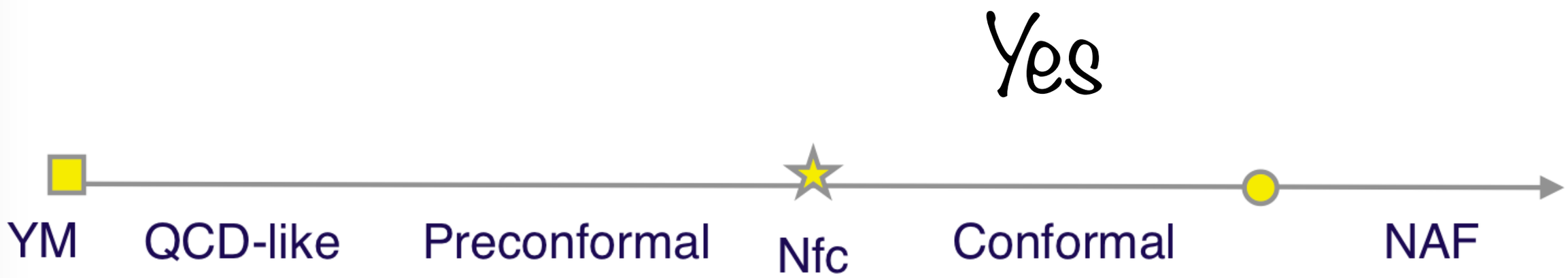
I. *Zero temperature:*

String tension, Critical temperature, Wilson flow

Phases of Strong Interactions

Which are its features?

Can we observe it?


Yes
YM
QCD-like
Preconformal
Nfc
Conformal
NAF

Next
YM
QCD-like
Preconformal
Nfc
Conformal
NAF

Standard picture of scale separation

$$\Lambda_{\rm IR}/\Lambda_{\rm UV} = \mathcal{O}(1).$$

Λ_{UV}

Λ_{IR}

Nfc

In the conformal phase IR scales vanish but UV ones survive

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

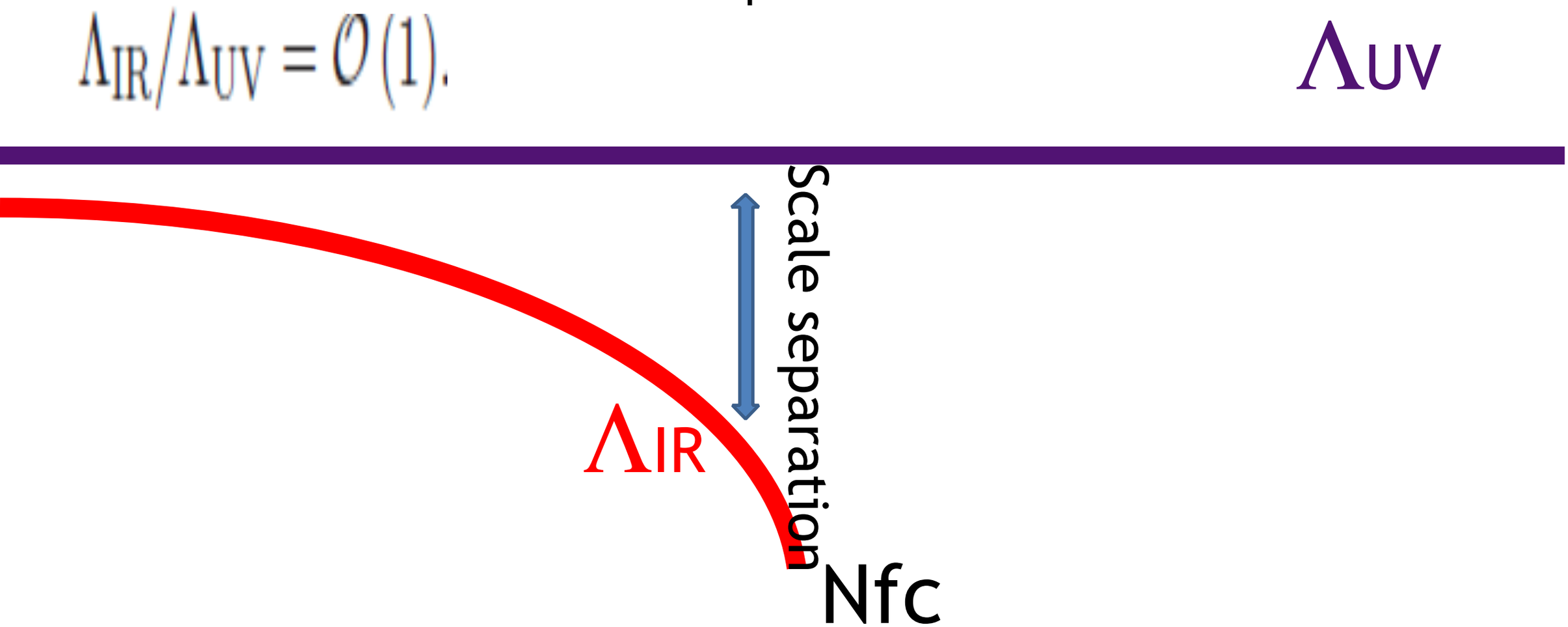
$$x = N_f/N_c$$

The coupling walks for

$$\Lambda_{\rm UV}^{-1} \ll r \ll \Lambda_{\rm IR}^{-1}$$

Standard picture of scale separation

$$\Lambda_{\rm IR}/\Lambda_{\rm UV} = \mathcal{O}(1).$$

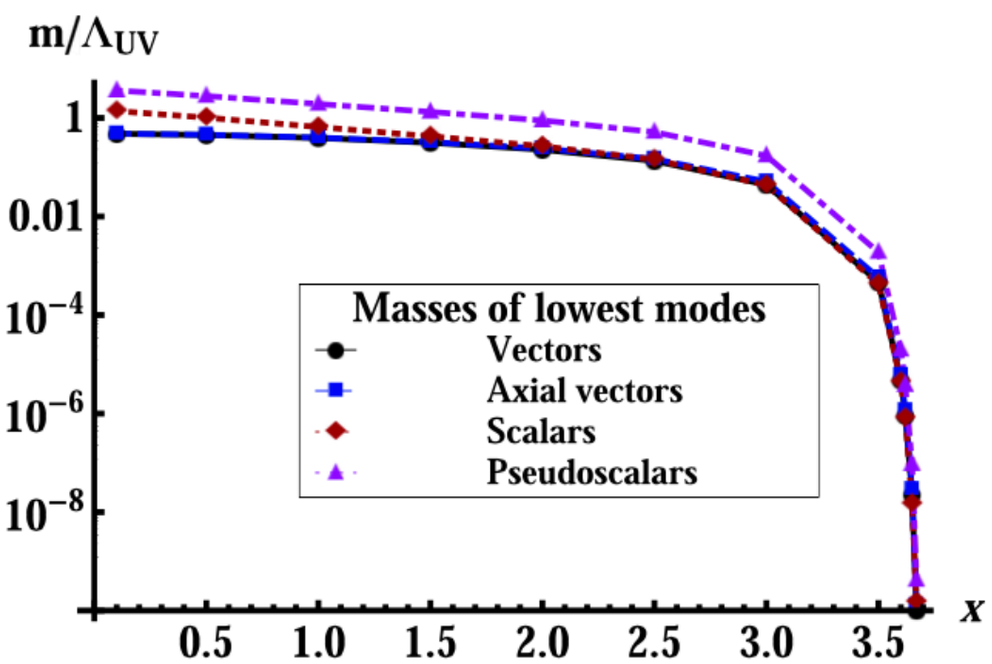


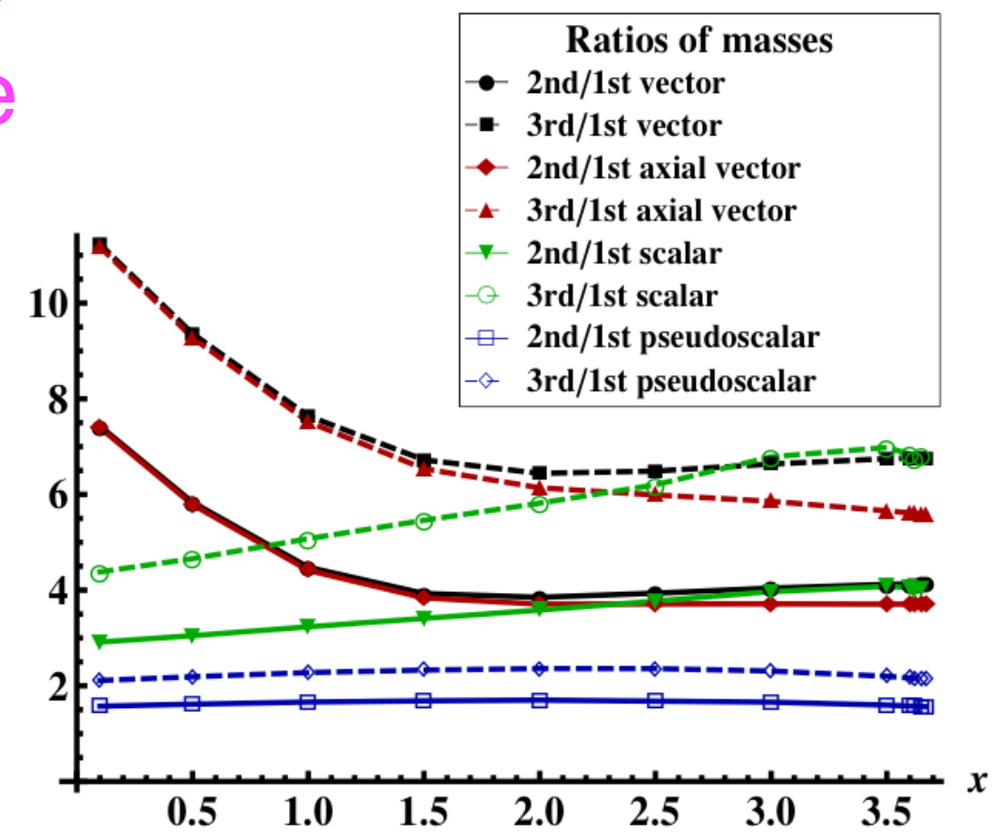
$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

(Essential) singularity in the chiral limit and mass ratios: example from holographic V-QCD

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}(n+1)}{\sqrt{x_c - x}}\right)$$

Not Unique





Arean, Iatrakis, Jarvinen, Kiritsis 2013

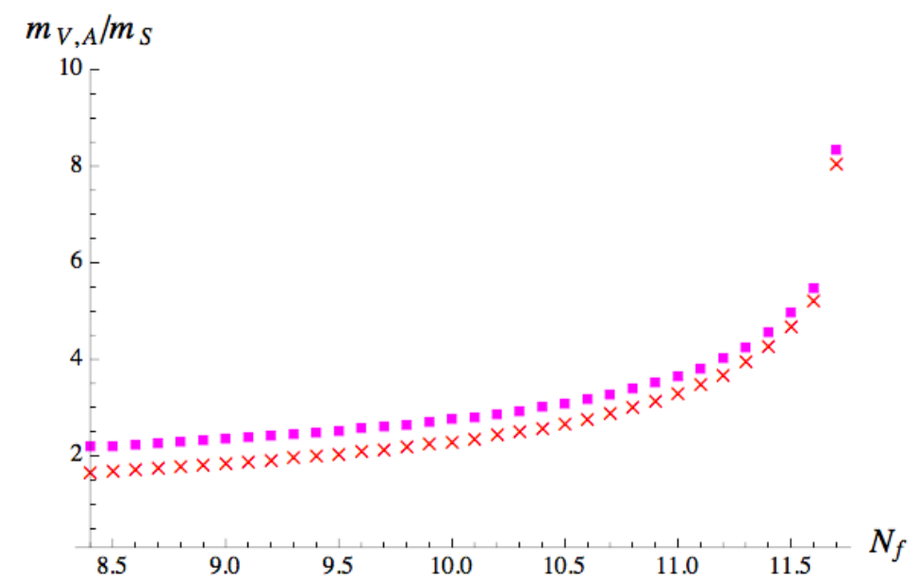
$\overline{\Lambda_{\rm IR}}$ not unique:

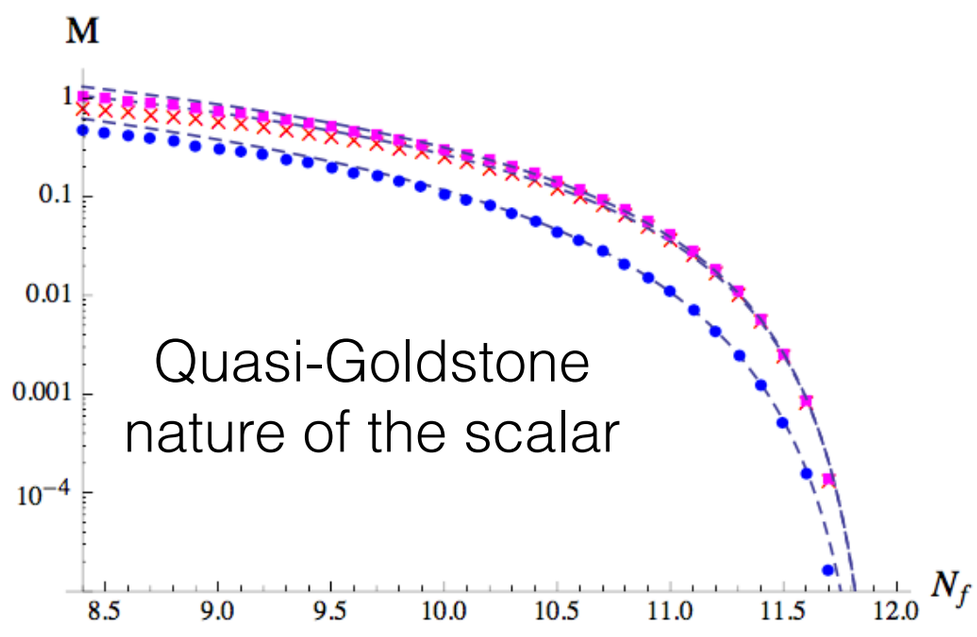
Power-law corrections to essential singularity

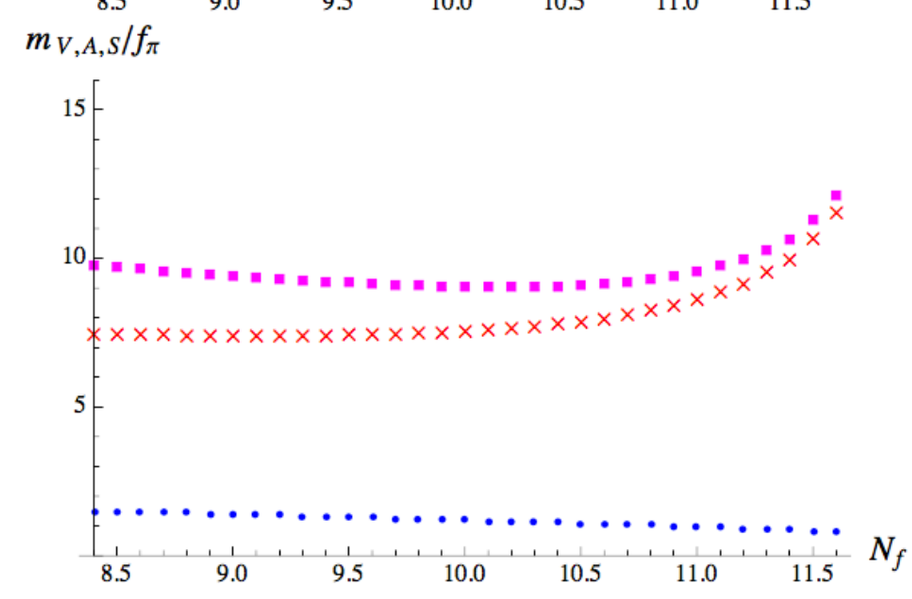
Gies et al. 2013
Alho, Evans, Tuominen 2013

$$O_i = A_i (N_f^c - N_f)^{p_i} \langle \bar{q} q \rangle^{1/3}$$

Power-law X Miranski scaling







Mass deformed theory: EoS approach for IR quantities

$y = f(x)$
$y = m/ < \bar{\psi}\psi >^{\delta}$ $\qquad \delta = \frac{6-\eta}{2-\eta}$

Second order transition:
$x = ({N_f}^c - N_f)/ < \bar{\psi}\psi >^{\frac{1}{\beta}}$ $\qquad < \bar{\psi}\psi >= ({N_f}^c - N_f)^{\beta}$

Essential singularity: Nogawa, Hasegawa, Nemoto, 2012
$x = e^{\sqrt{({N_f}^c - N_f)}}/ < \bar{\psi}\psi >$ $\qquad < \bar{\psi}\psi >= e^{\sqrt{({N_f}^c - N_f)}}$

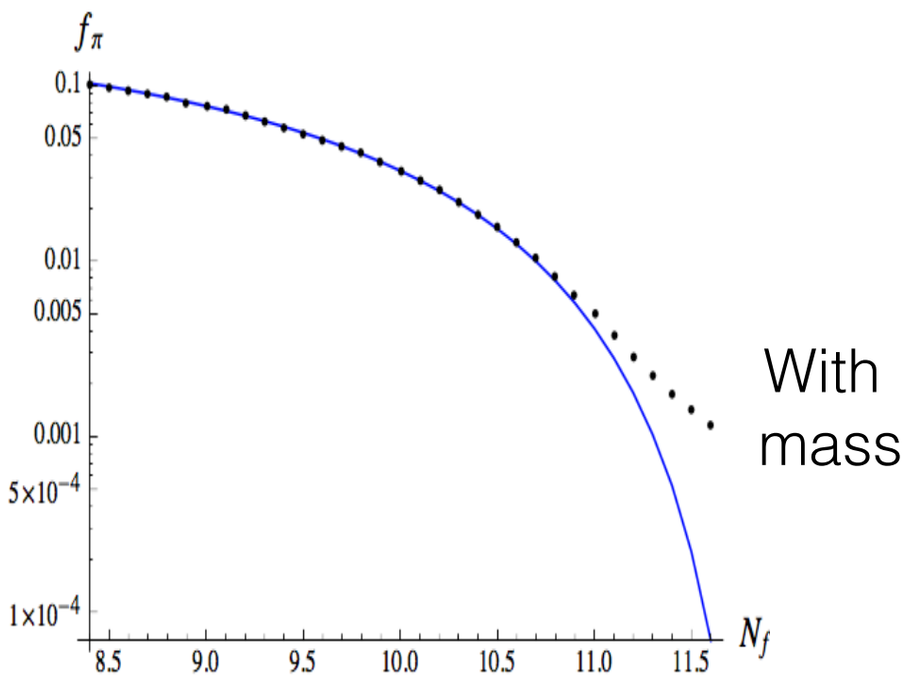
Continuity of $f(x)$ plus asymptotic forms for $m \to 0$ and $N_f \to {N_f}^c$ imply
$< \bar{\psi}\psi > \propto e^{\sqrt{({N_f}^c - N_f)}}$ for m *smallish* and $({N_f}^c - N_f)$ *largish*
$< \bar{\psi}\psi > \propto m^{1/\delta}$ for m *largish* and $({N_f}^c - N_f)$ *smallish*

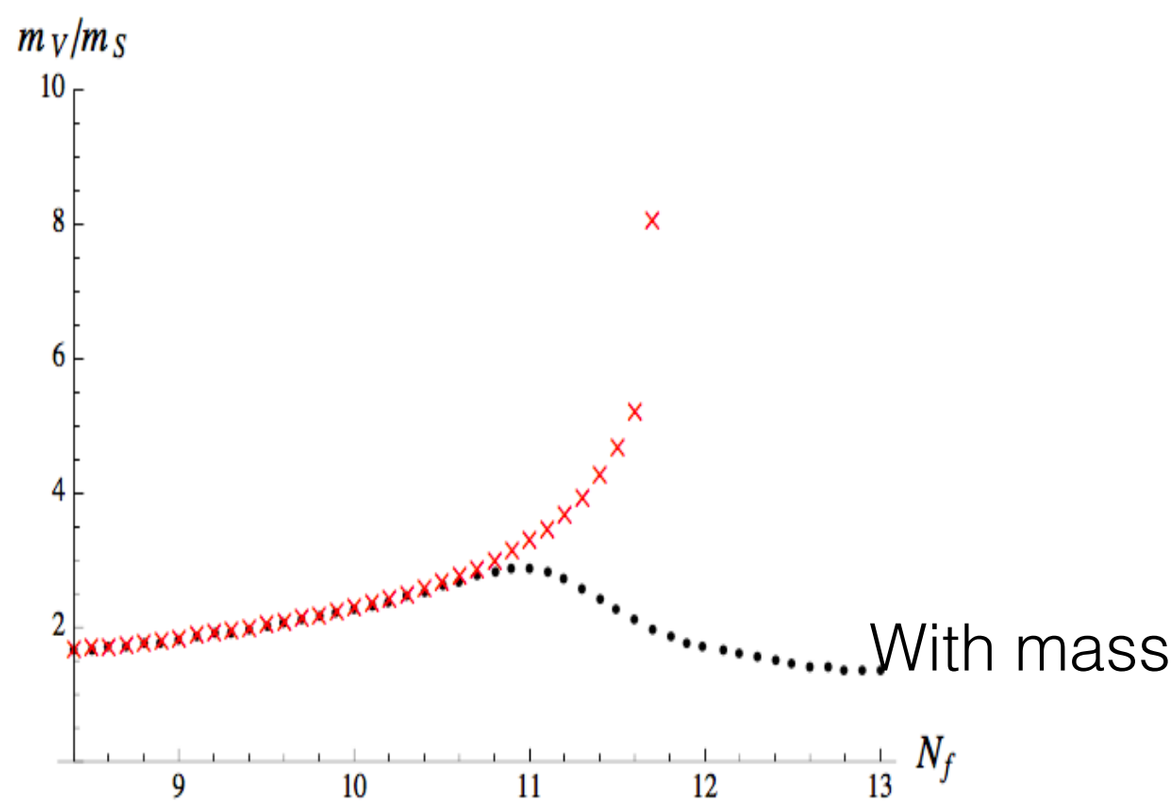
Anomalous dimension appears naturally below Nfc
Scaling limited by Goldstone singularities in the chiral limit (Wallace Zia

These features are seen in model calculations:

Alho, Evans, Tuominen 2013

Mass deformed theory



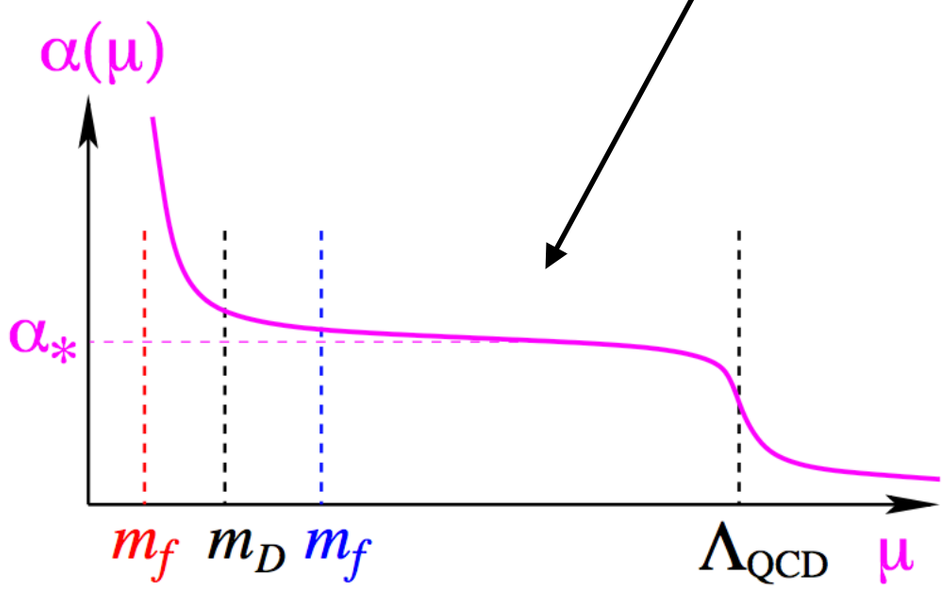


Analogous to KMI, LSD

Mass deformed theory II: KMI discussion

Mutatis mutandis,
Eos approach reproduces KMI scenario:

Scaling with anomalous dimension



KMI 2013

Approaching conformality from below and above

Essential sing.

IR IR IR…..

(X power-law)

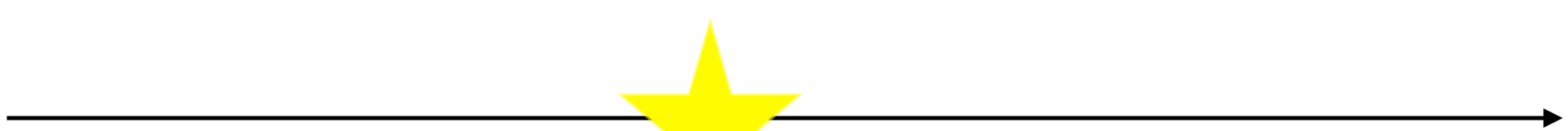
Conformal scaling

Analogies
in the broken phase

Differences
in the symmetric phase

EoS

IR



2nd order transition

Griffith's analyticity

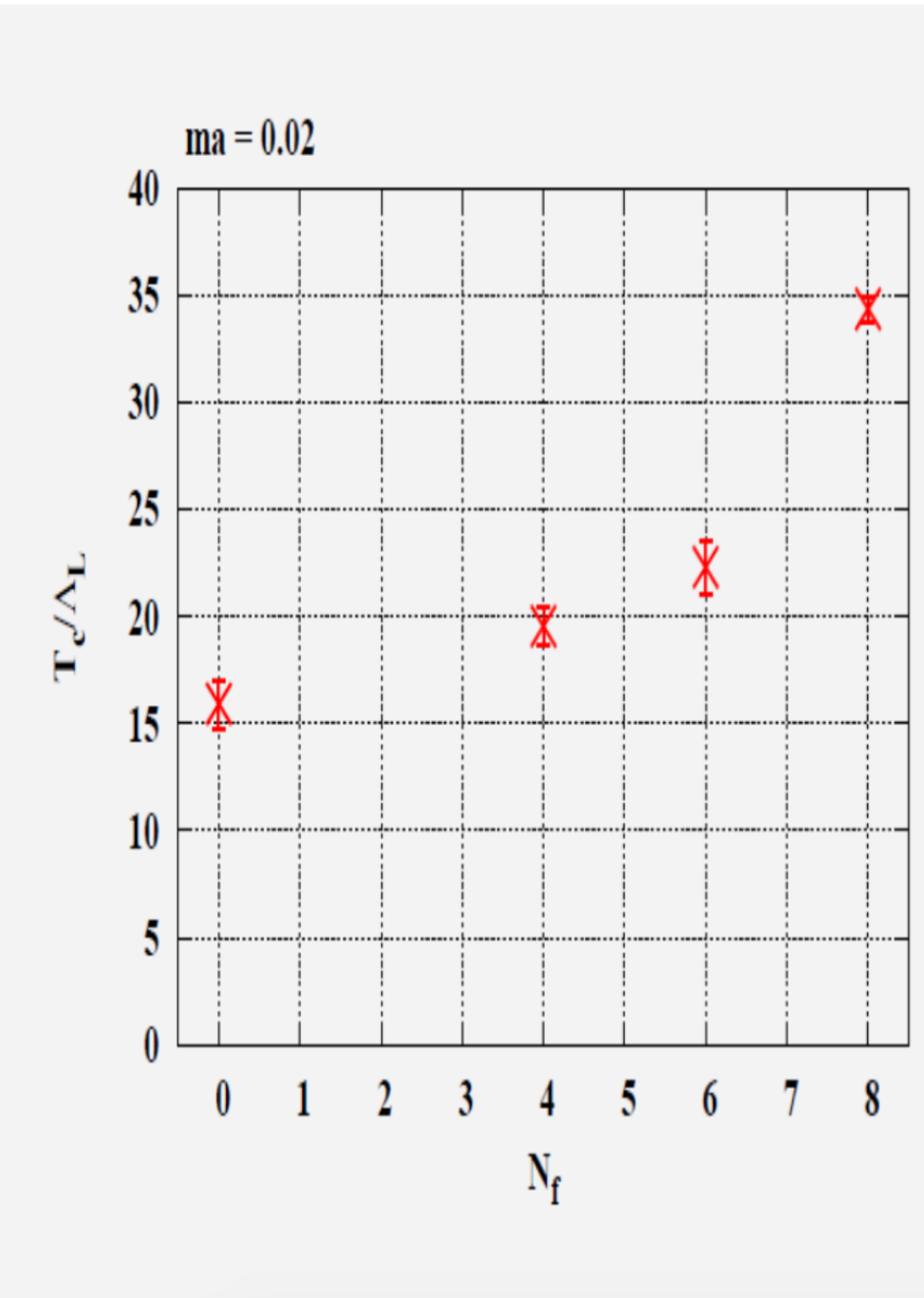
Observables:

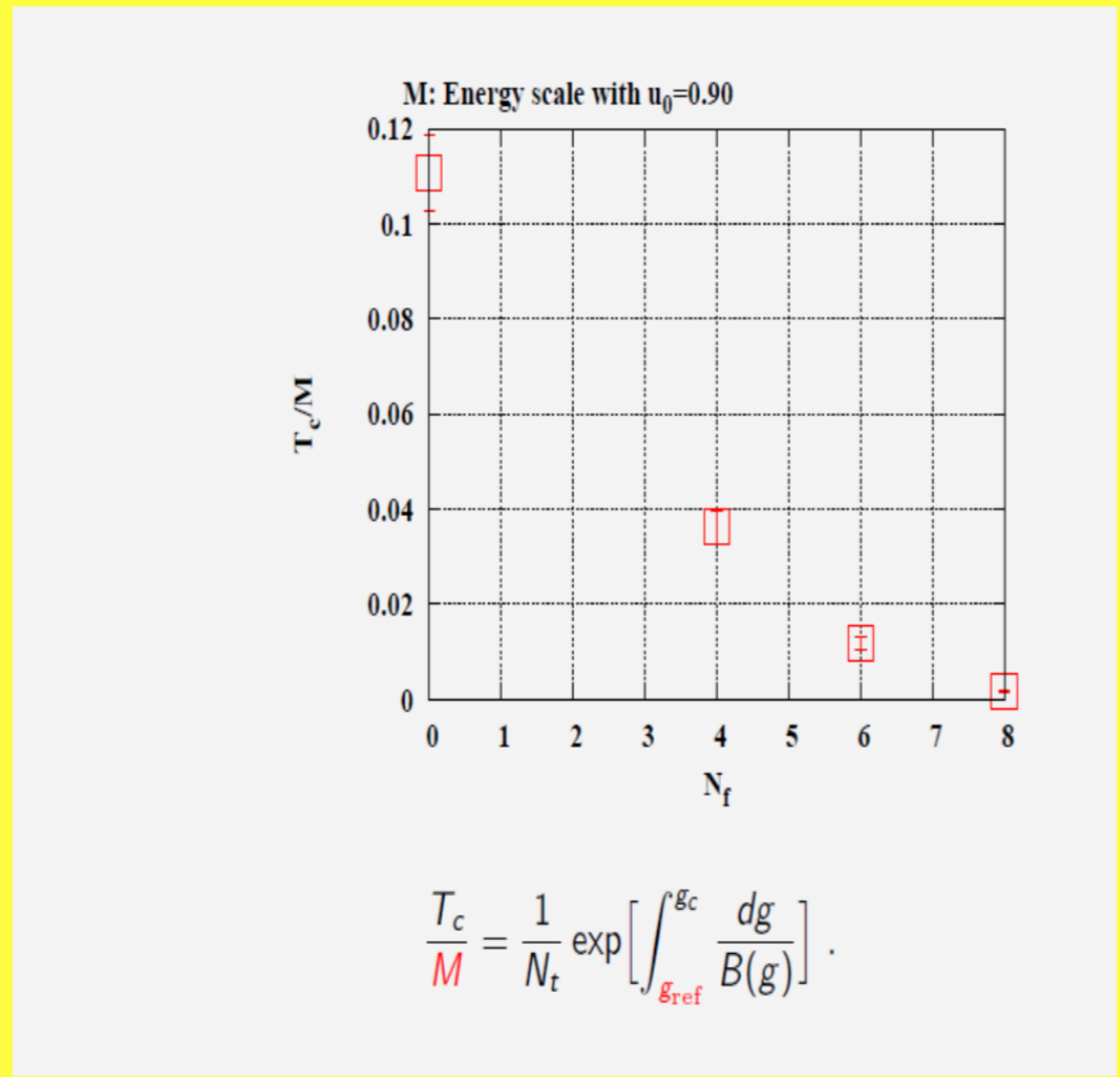
Critical temperature
W0, W1, … induced by Wilson flow Preliminary
String tension Preliminary
Technical lattice scale defined at one lattice spacing
Λ_{LAT}

Strategy: consider dimensionless ratios
R = O1/O2
When O2 is UV this is the facto a conventional scale setting
Observation: R relatively stable wrt mass variations

Results

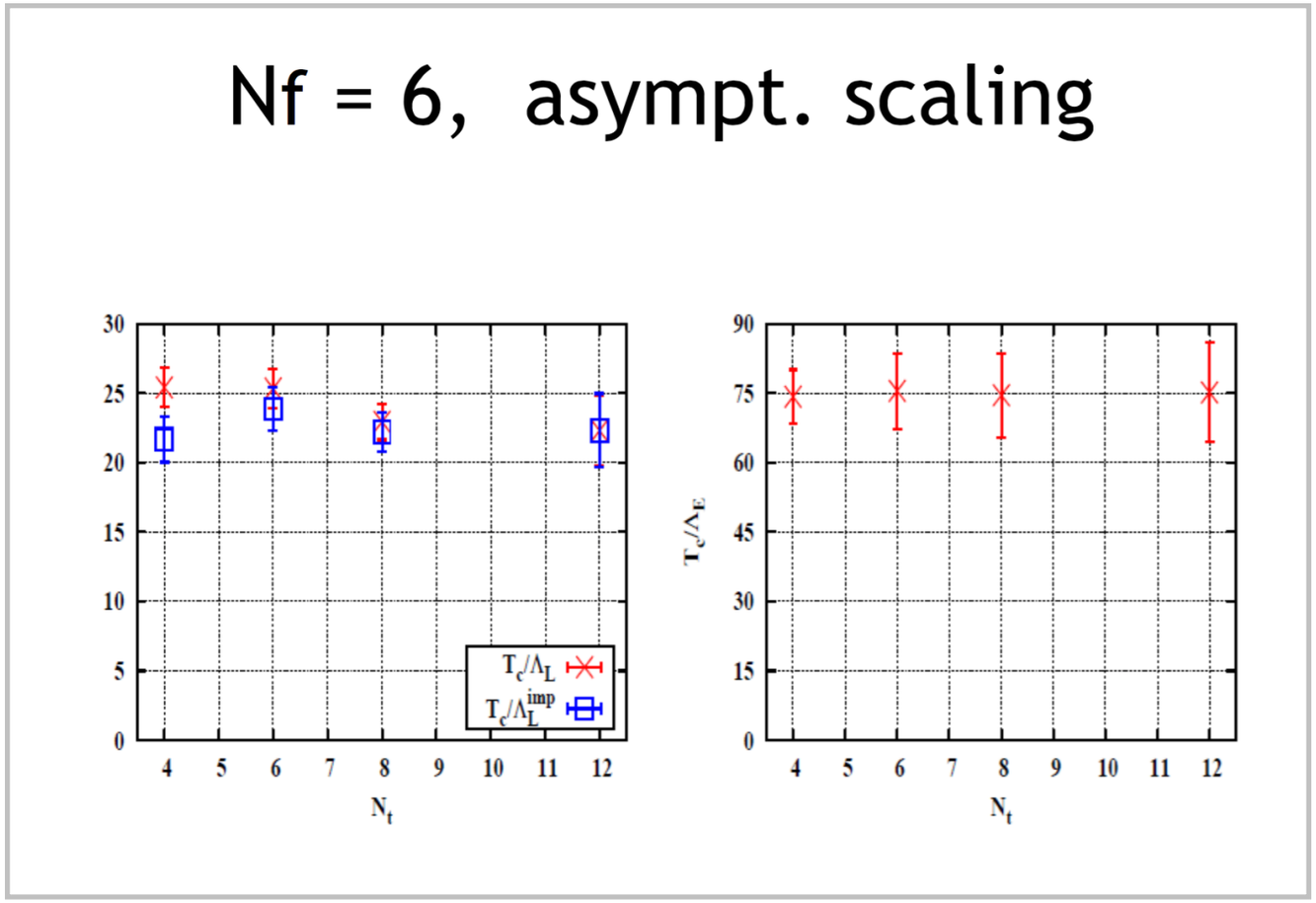
From an IR to a UV scale: Tc decreases





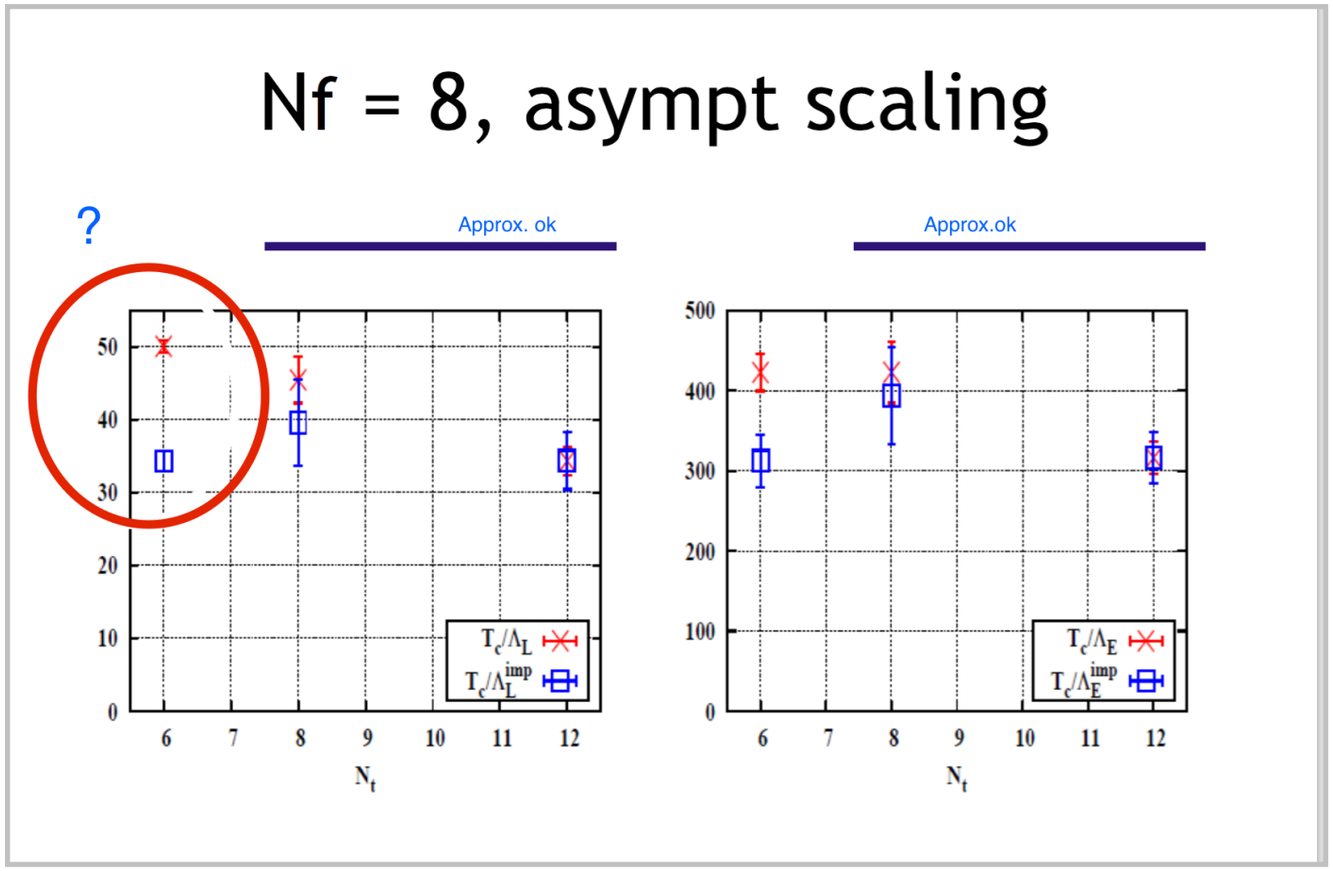
$$\frac{T_c}{M} = \frac{1}{N_t} \exp\left[\int_{g_{\rm ref}}^{g_c} \frac{dg}{B(g)}\right].$$

KM, MpL, EP 2012



Asymptotic scaling of Tc gives

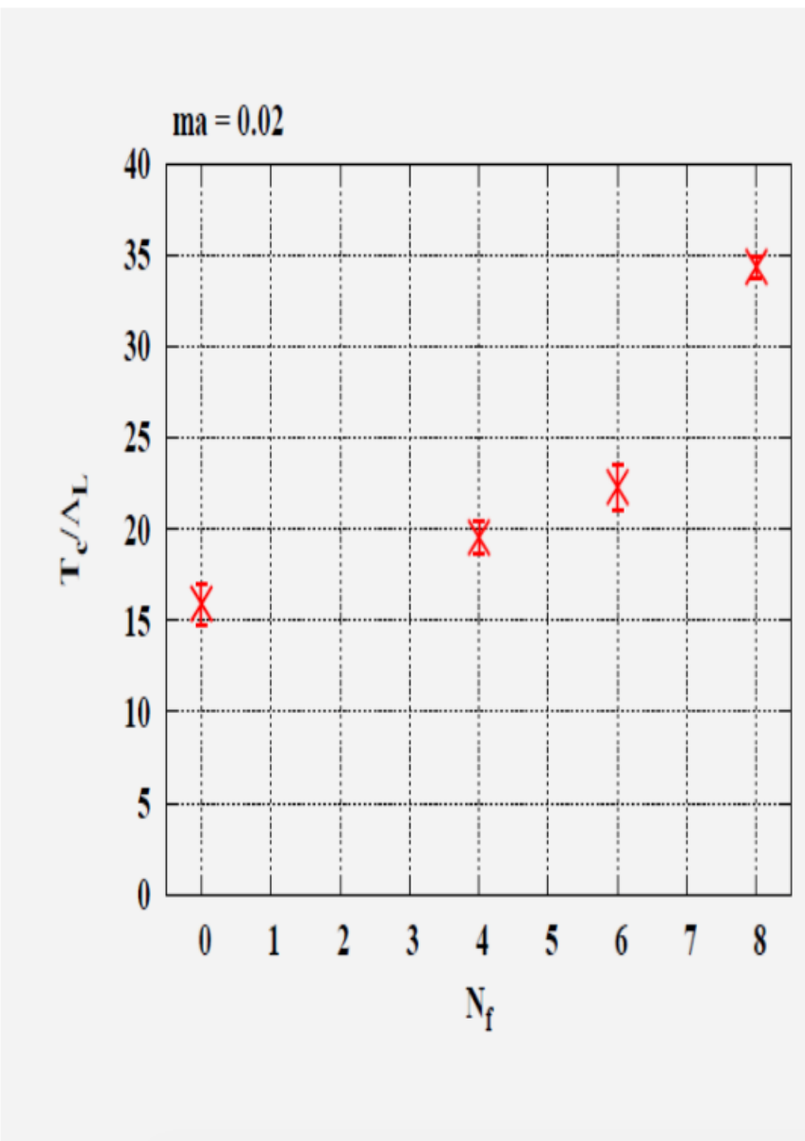
T_c/Λ



More difficult to reach for Nf=8

Towards a quantitive comparison with holography

$$\frac{2\pi T_c}{M_{KK}} = 1 - \frac{1}{126\pi^3}\lambda_4^2 \frac{N_f}{N_c}\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right)$$

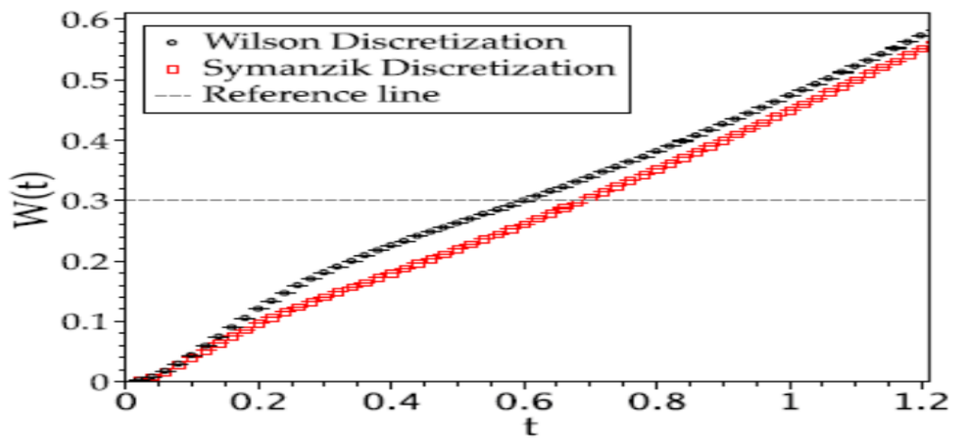


Bigazzi and Cotrone, JHEP 2015

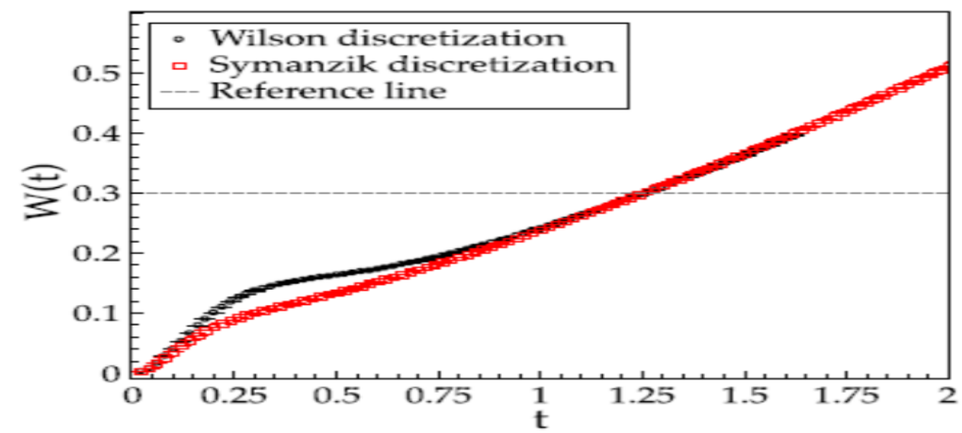
$$\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right) \approx -1.987$$

T increases with Nf on the scales used in these two studies

Nf = 6, Wilson Flow

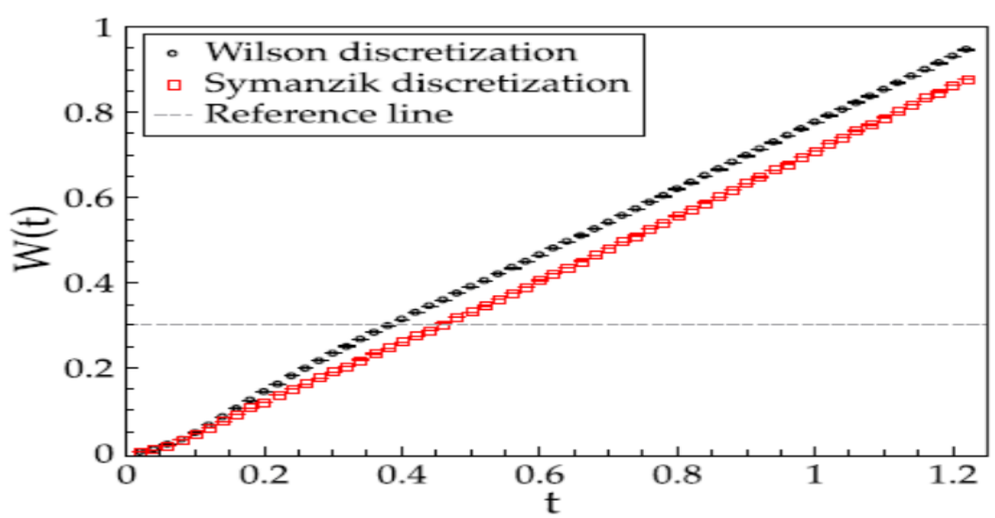


$\beta = 5.025$

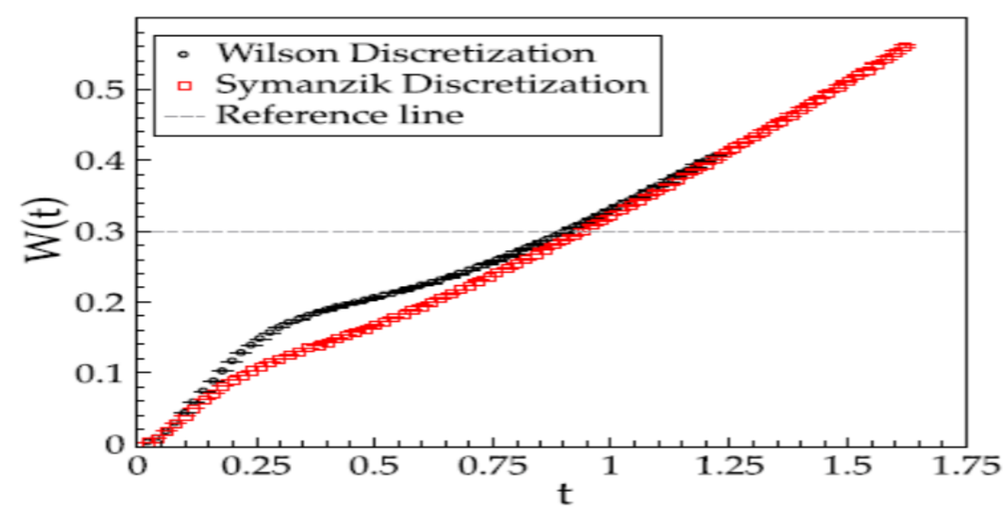


$, \beta = 5.200$

Nf = 8, Wilson Flow



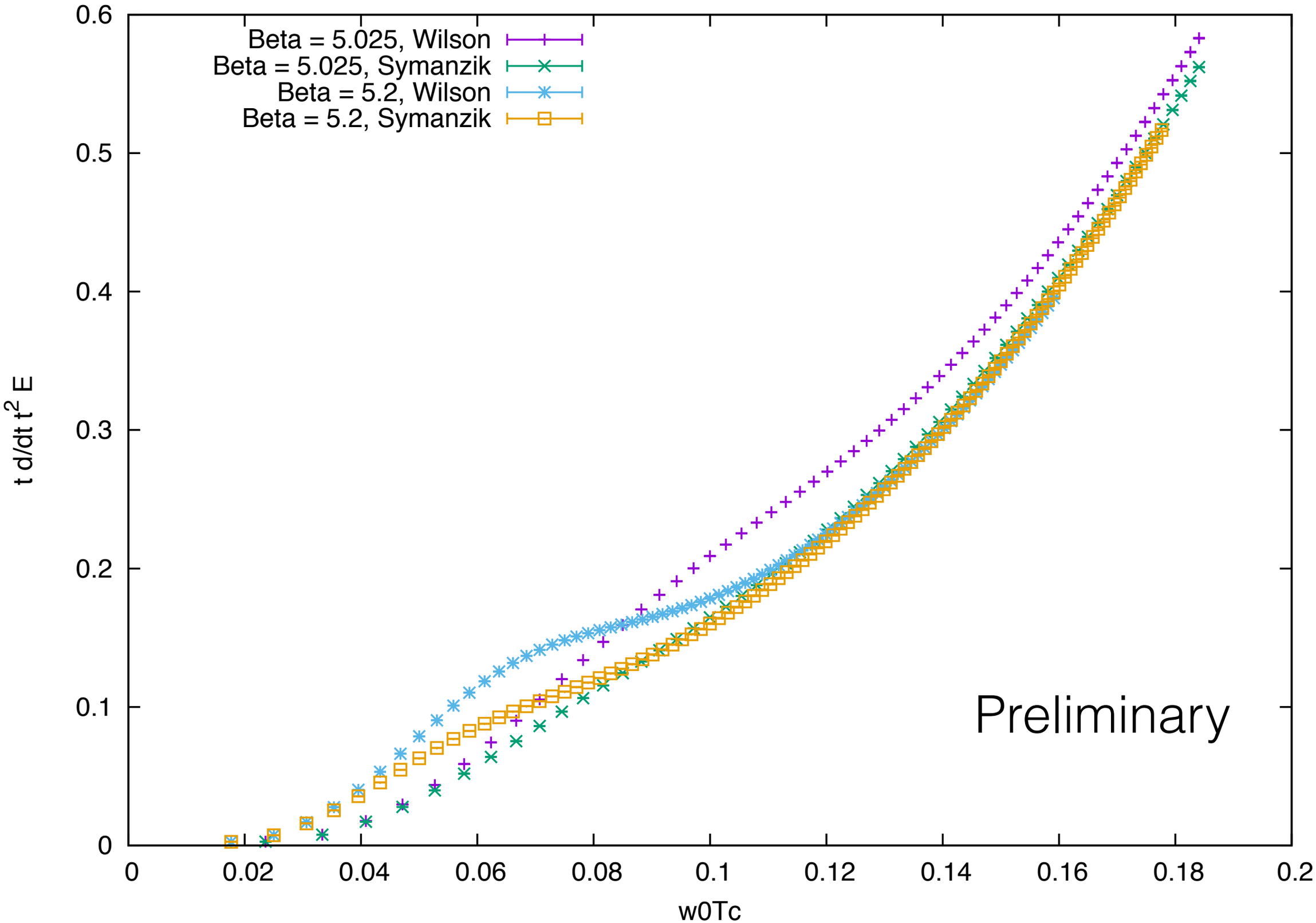
$, \beta = 4.1125$



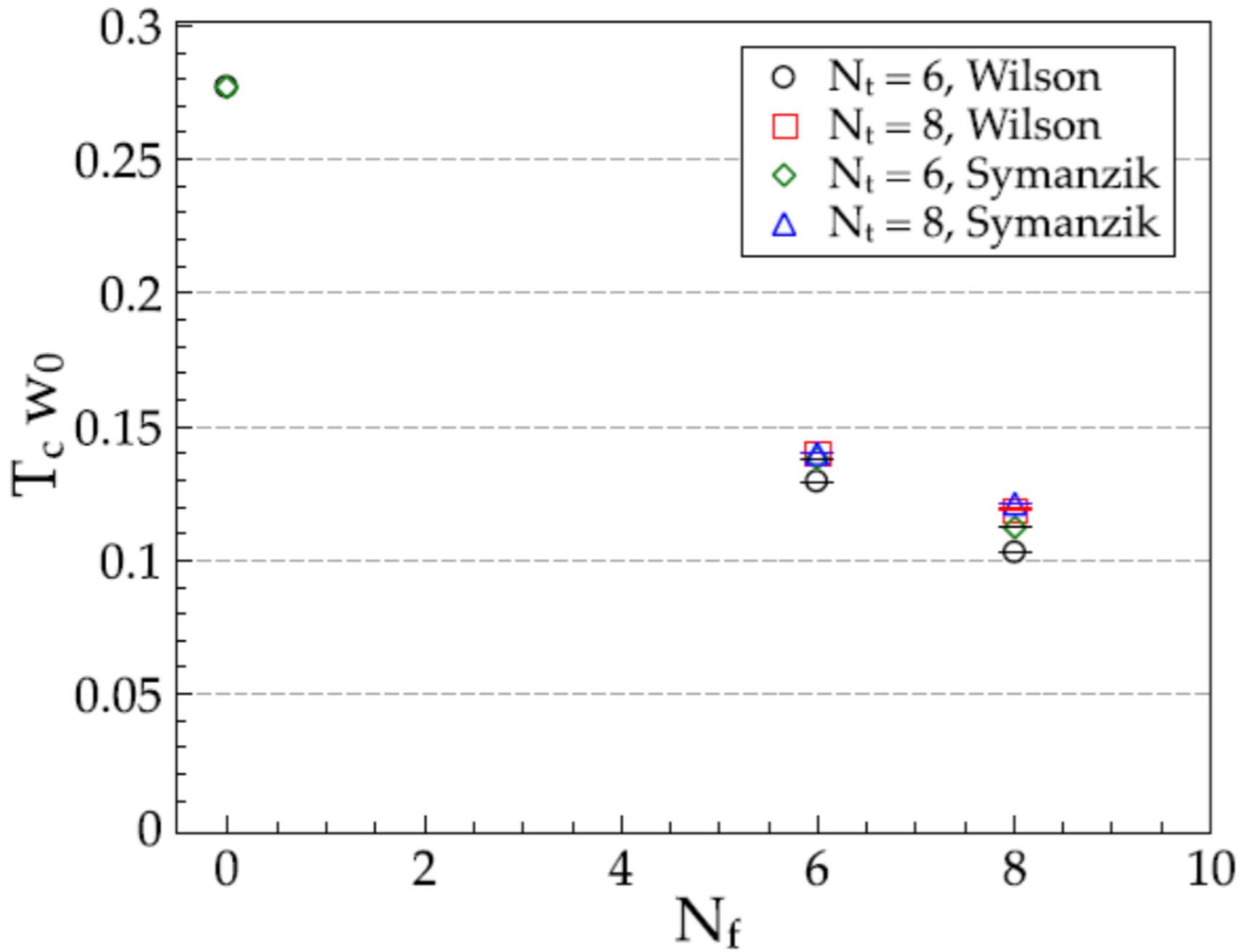
$, \beta = 4.275$

Scale from Wilson flow

Nf=6

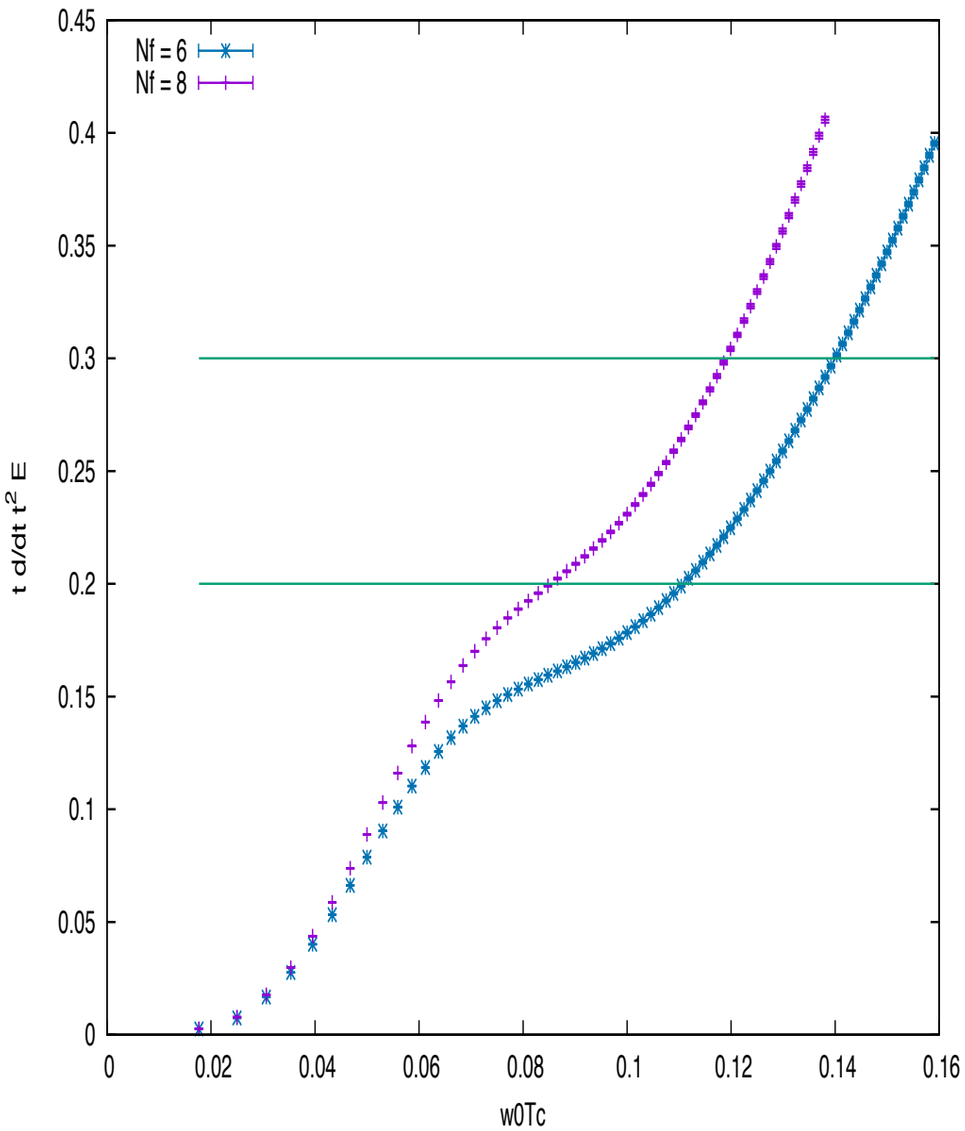


Tc on the 1/w0 scale

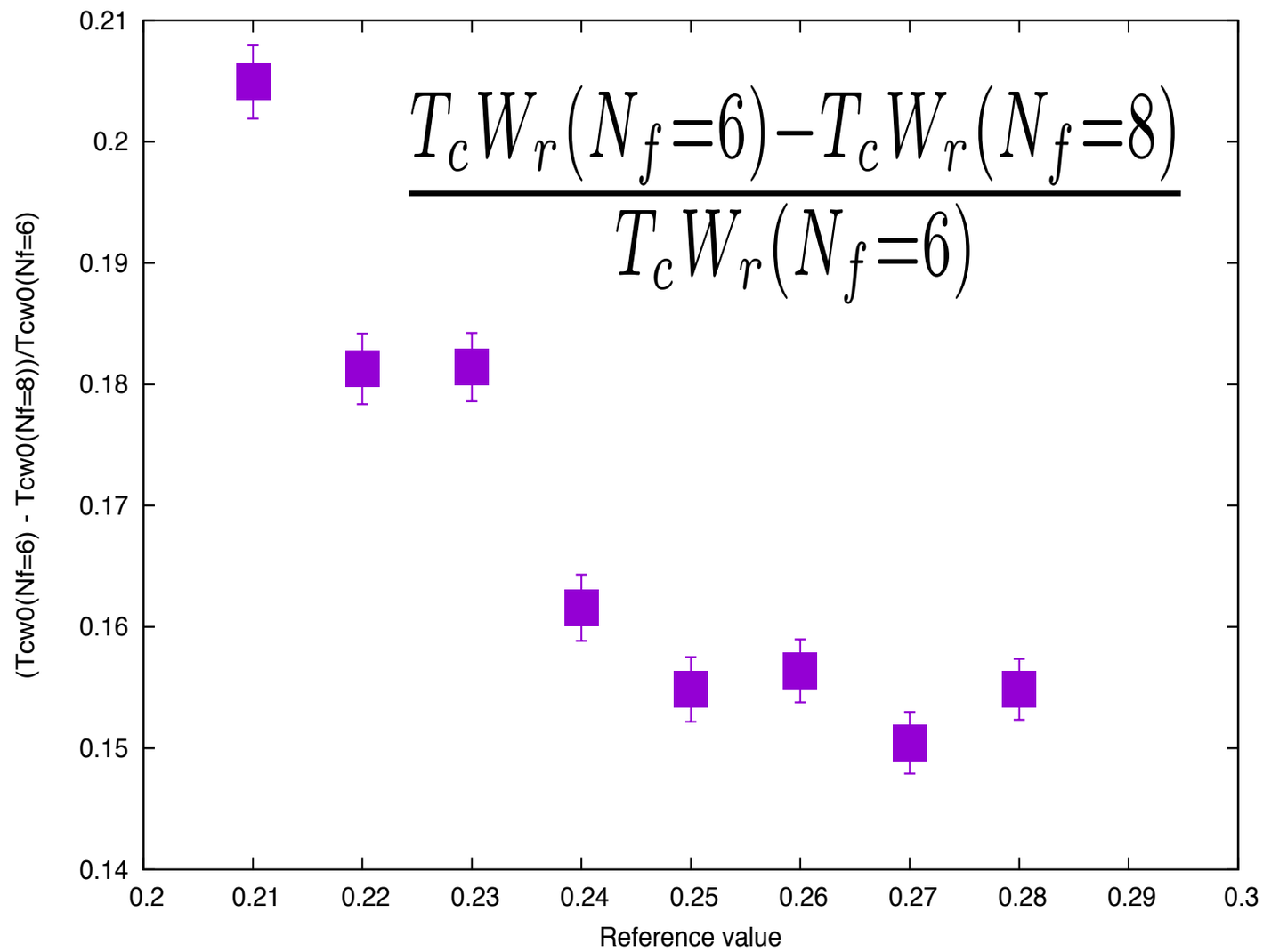


Preliminary

Moving the scale with Wilson flow



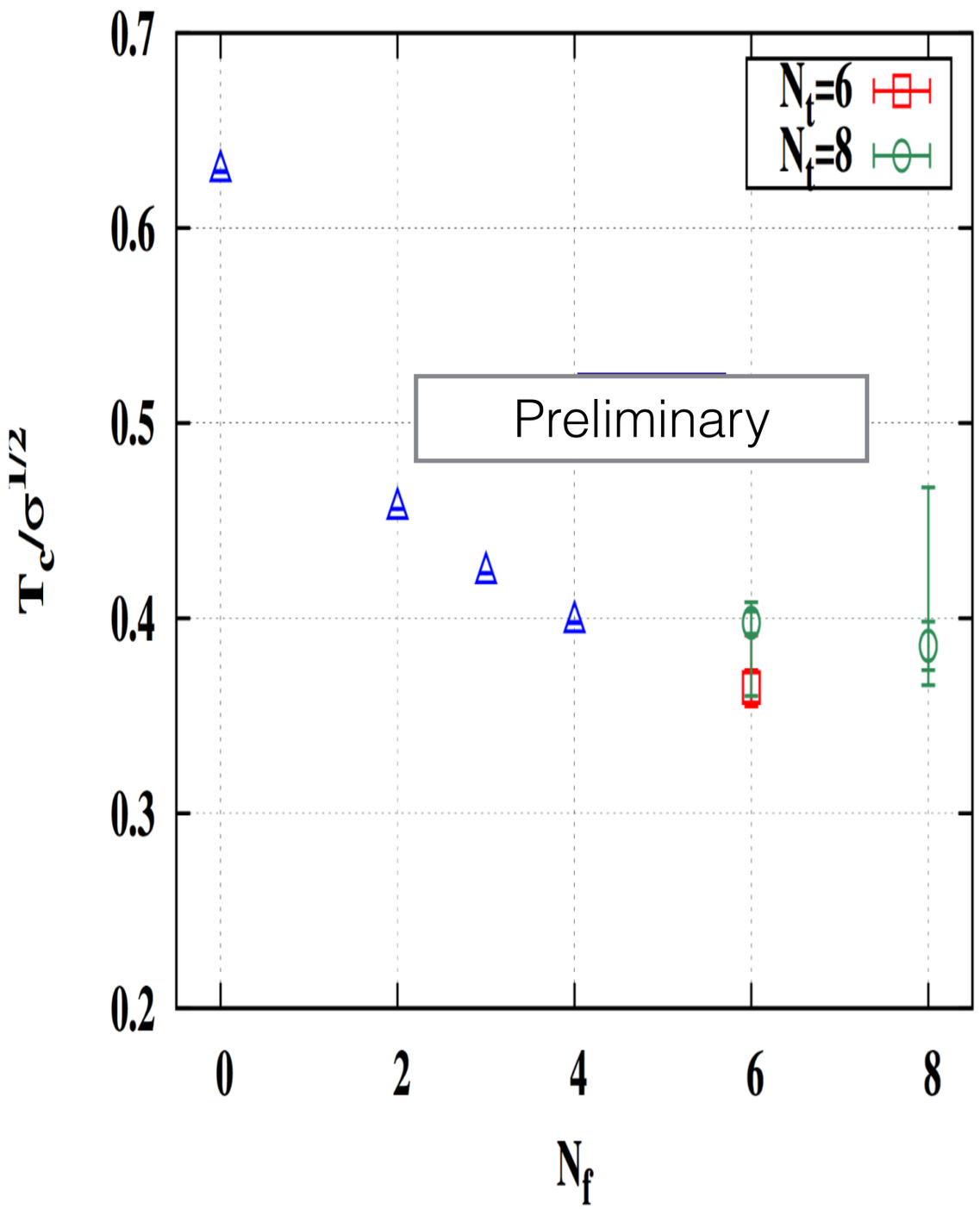
Qualitatively as expected,
limited by lattice artifacts



$$\frac{T_c W_r(N_f=6) - T_c W_r(N_f=8)}{T_c W_r(N_f=6)}$$

UV

Tc and the string tension



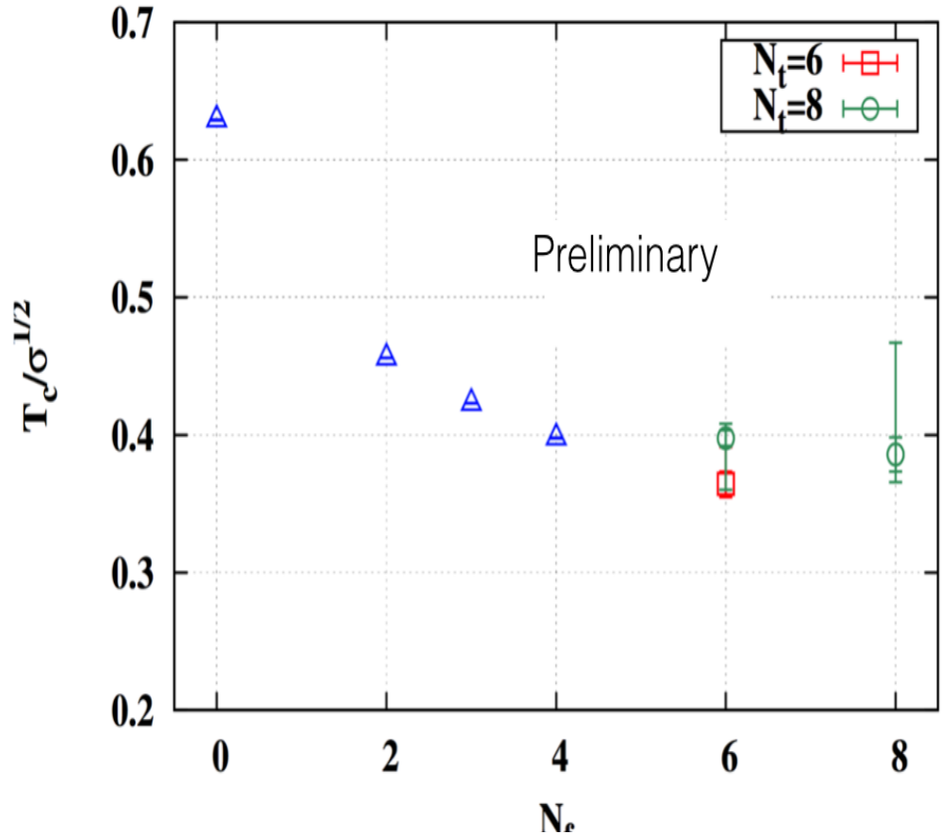
Mild decrease, possibly constant as $N_f \to N_f^c$

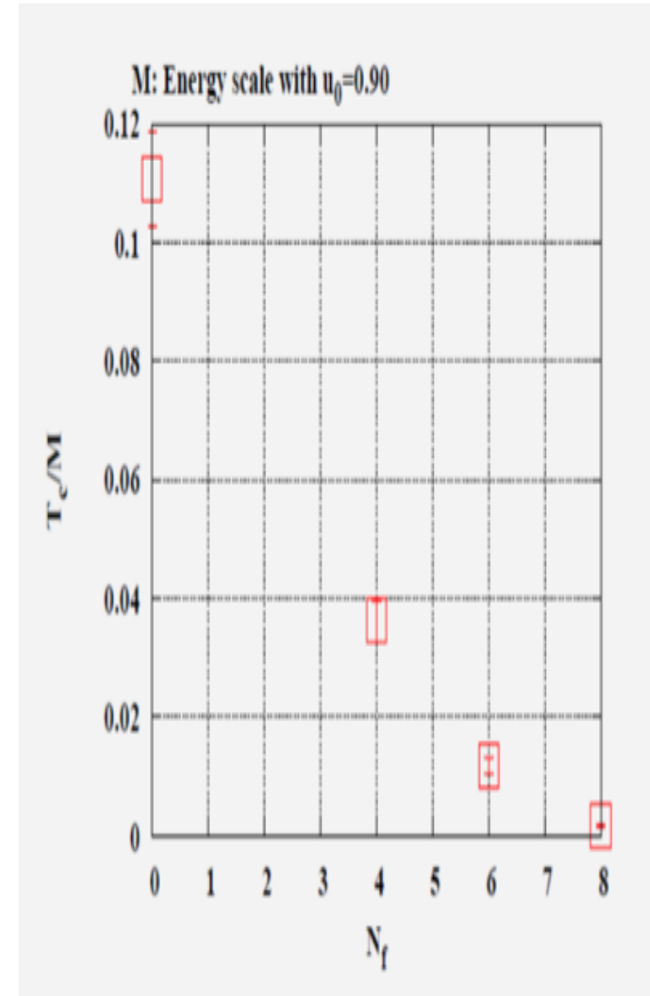
Again similar to the prediction of the WSS model:

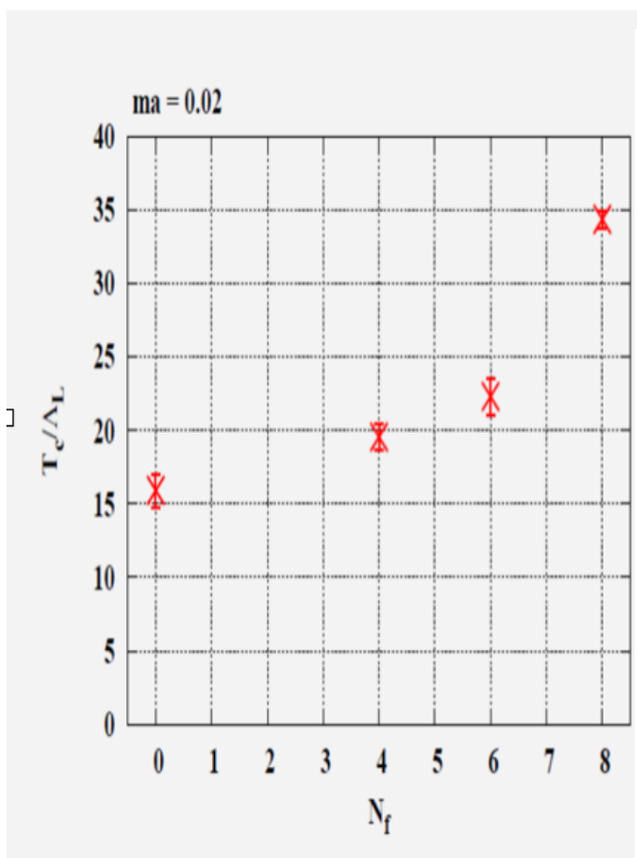
$$\frac{T_c}{\sqrt{\sigma}} \propto (1 - \epsilon N_f / N_c)$$

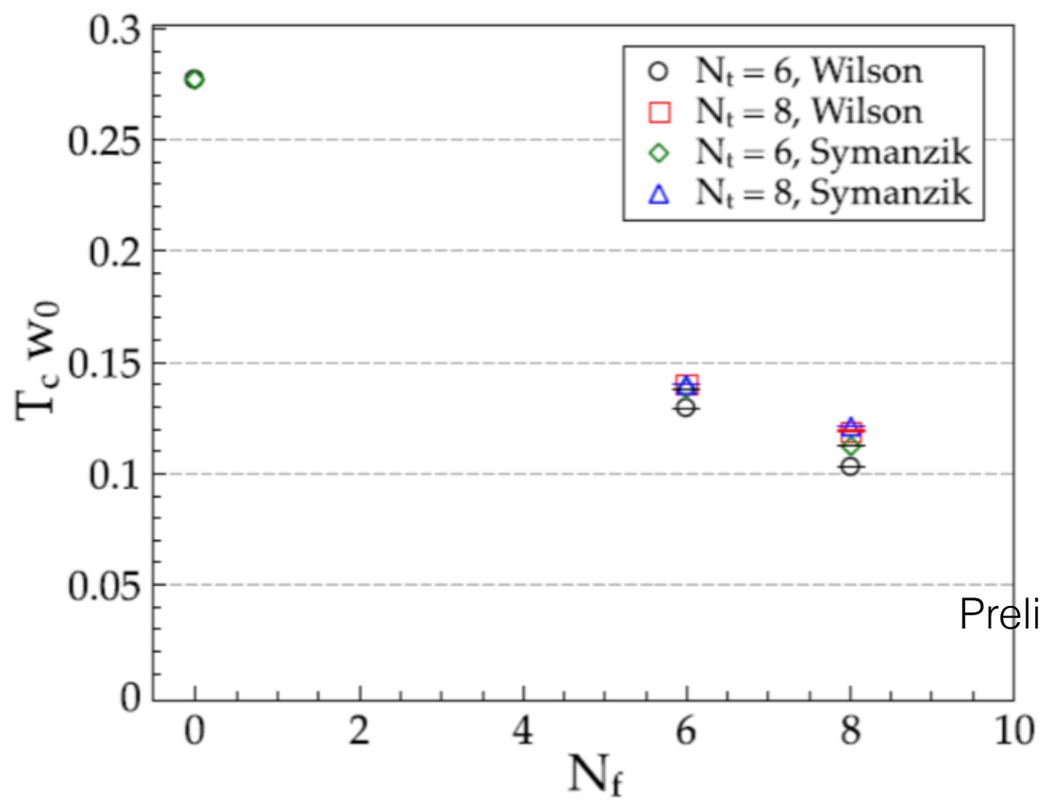
communicated by F. Bigazzi

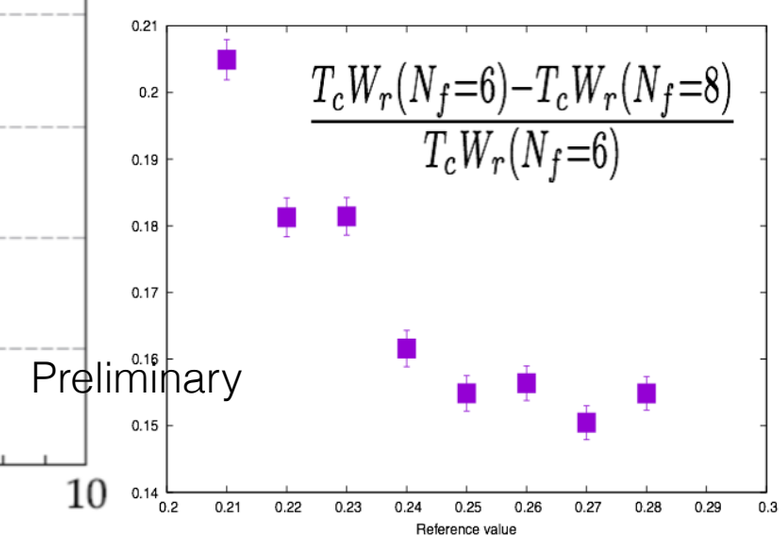
Scale separation in the preconformal region of QCD







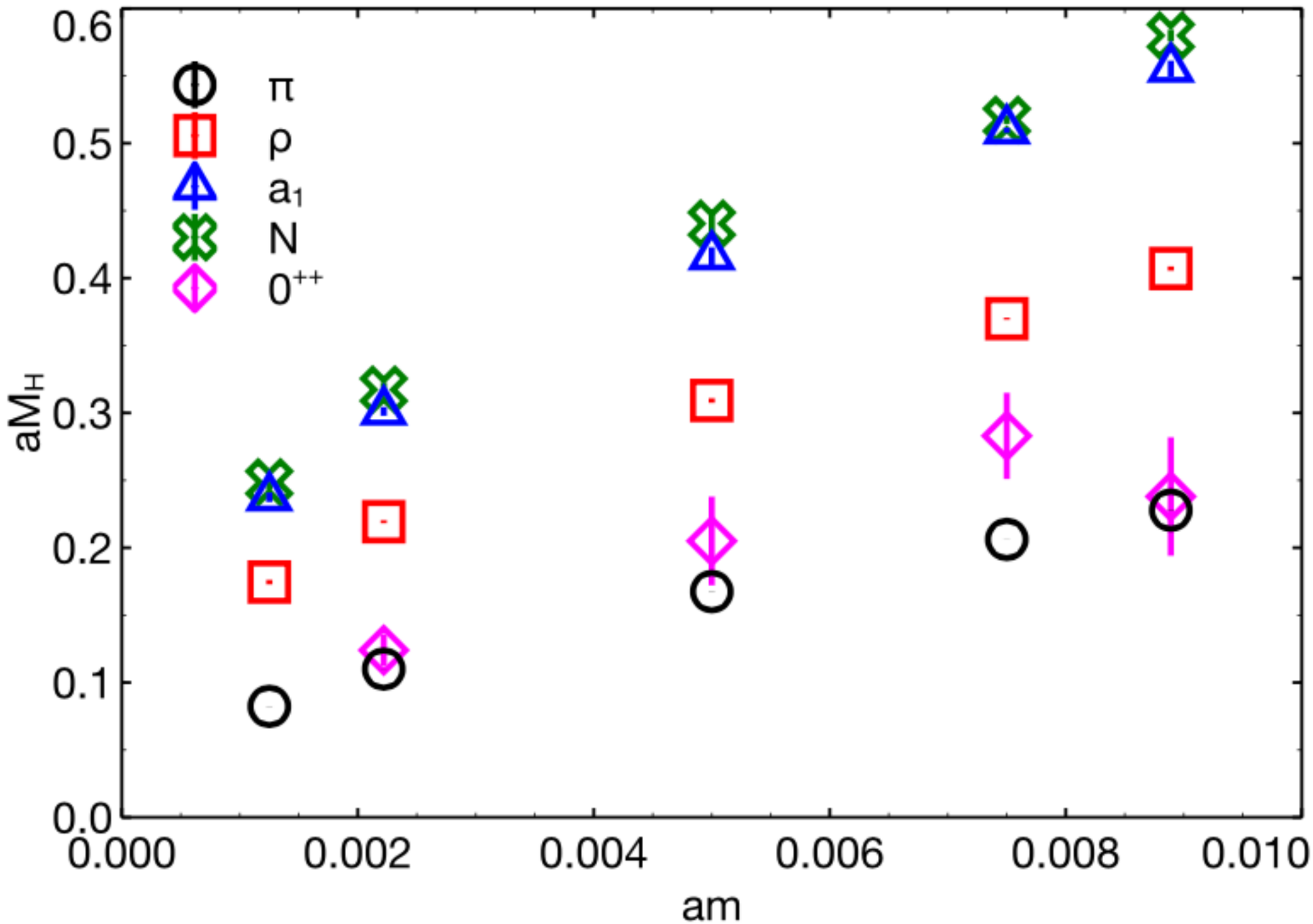




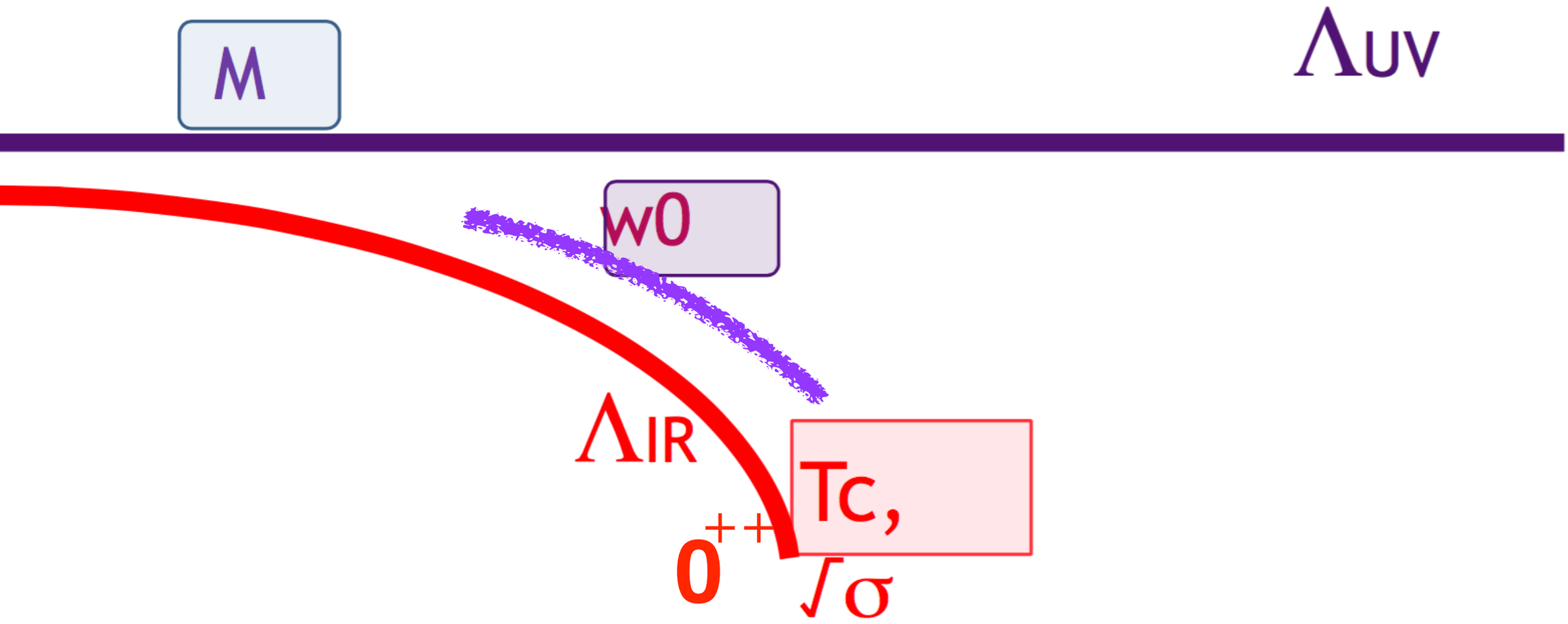
UV

Strongly interacting dynamics and the search for new physics at the LHC

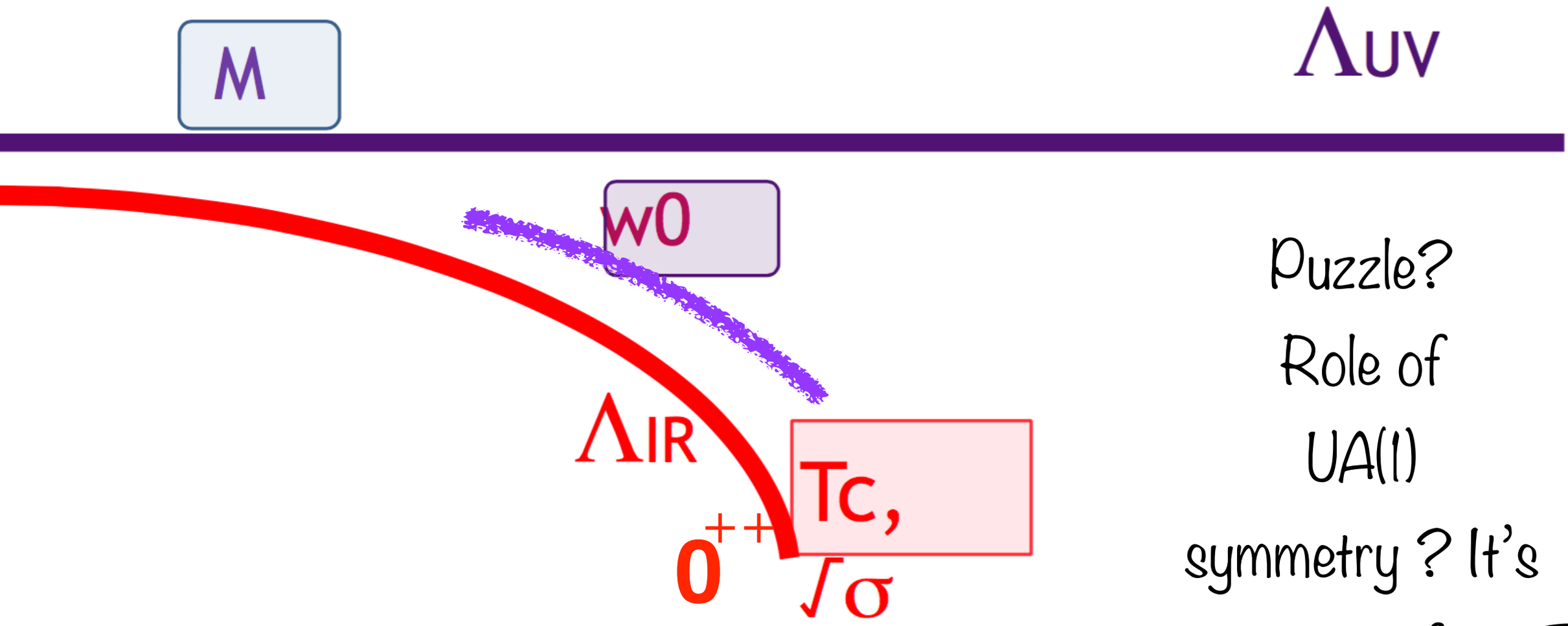
T. Appelquist,[1] R. C. Brower,[2,3] G. T. Fleming,[1,3] A. Hasenfratz,[4,3] X. Y. Jin,[5] J. Kiskis,[6] E. T. Neil,[4,7,3] J. C. Osborn,[5,3] C. Rebbi,[2] E. Rinaldi,[8,3] D. Schaich,[9,3,10] P. Vranas,[8] E. Weinberg,[11] and O. Witzel[12,3]
(Lattice Strong Dynamics (LSD) Collaboration)



Scale separation

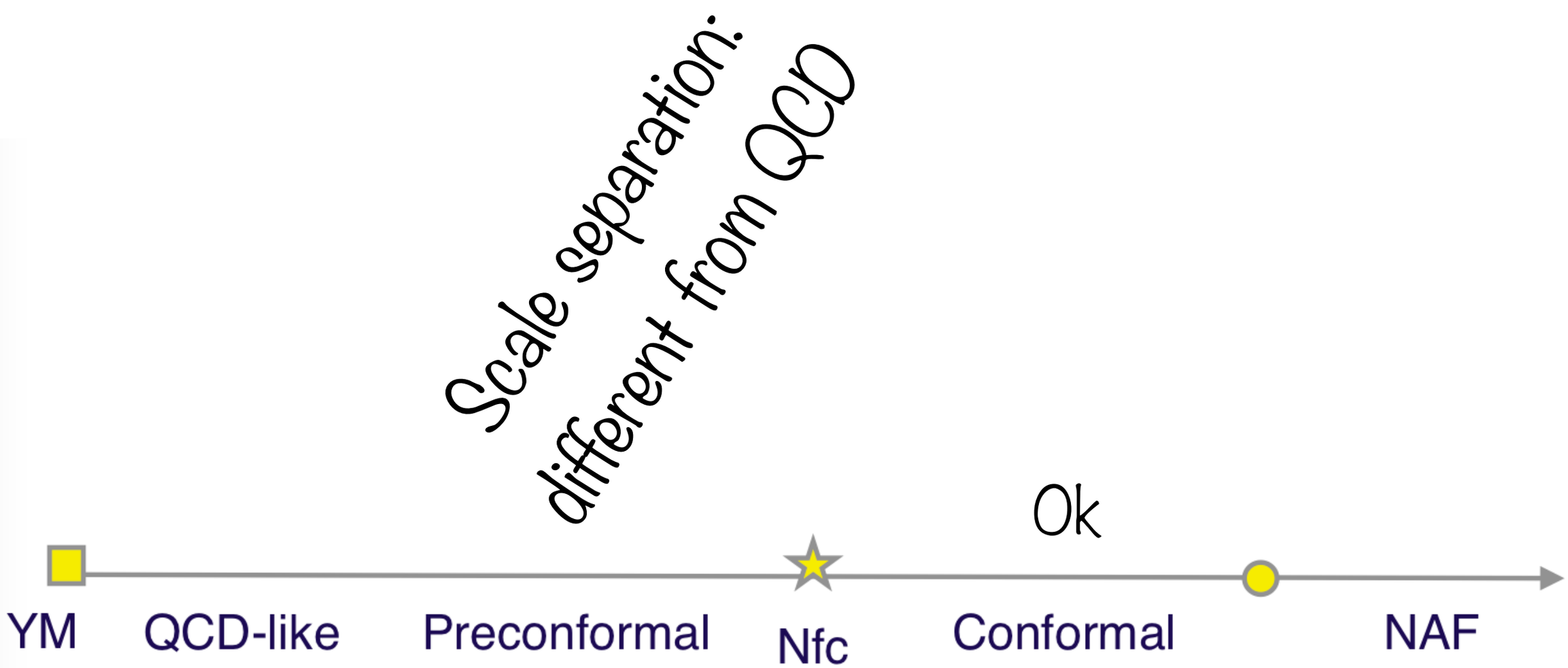


Scale separation



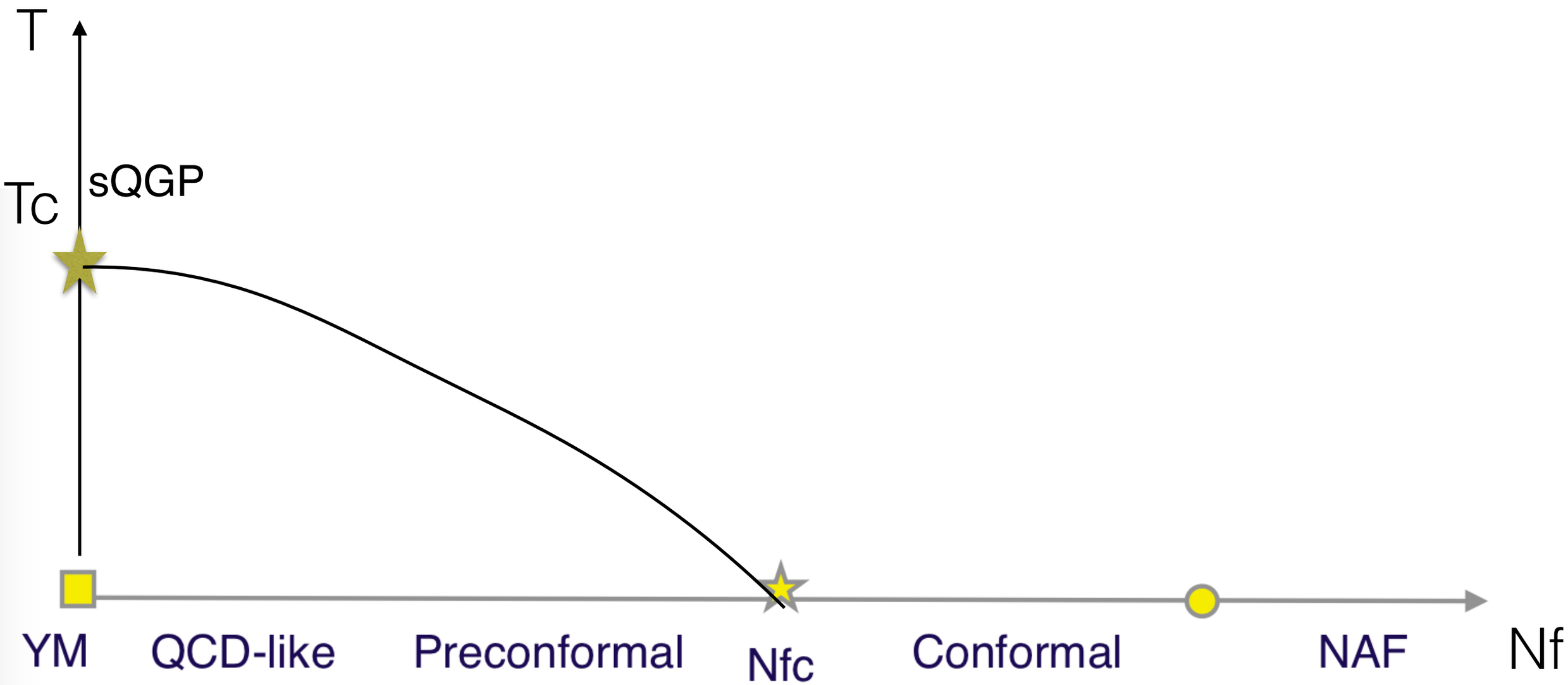
Puzzle?

Role of UA(1) symmetry ? It's important at finite T..


Scale separation:
different from QCD
Ok
YM
QCD-like
Preconformal
Nfc
Conformal
NAF

II. High temperature:

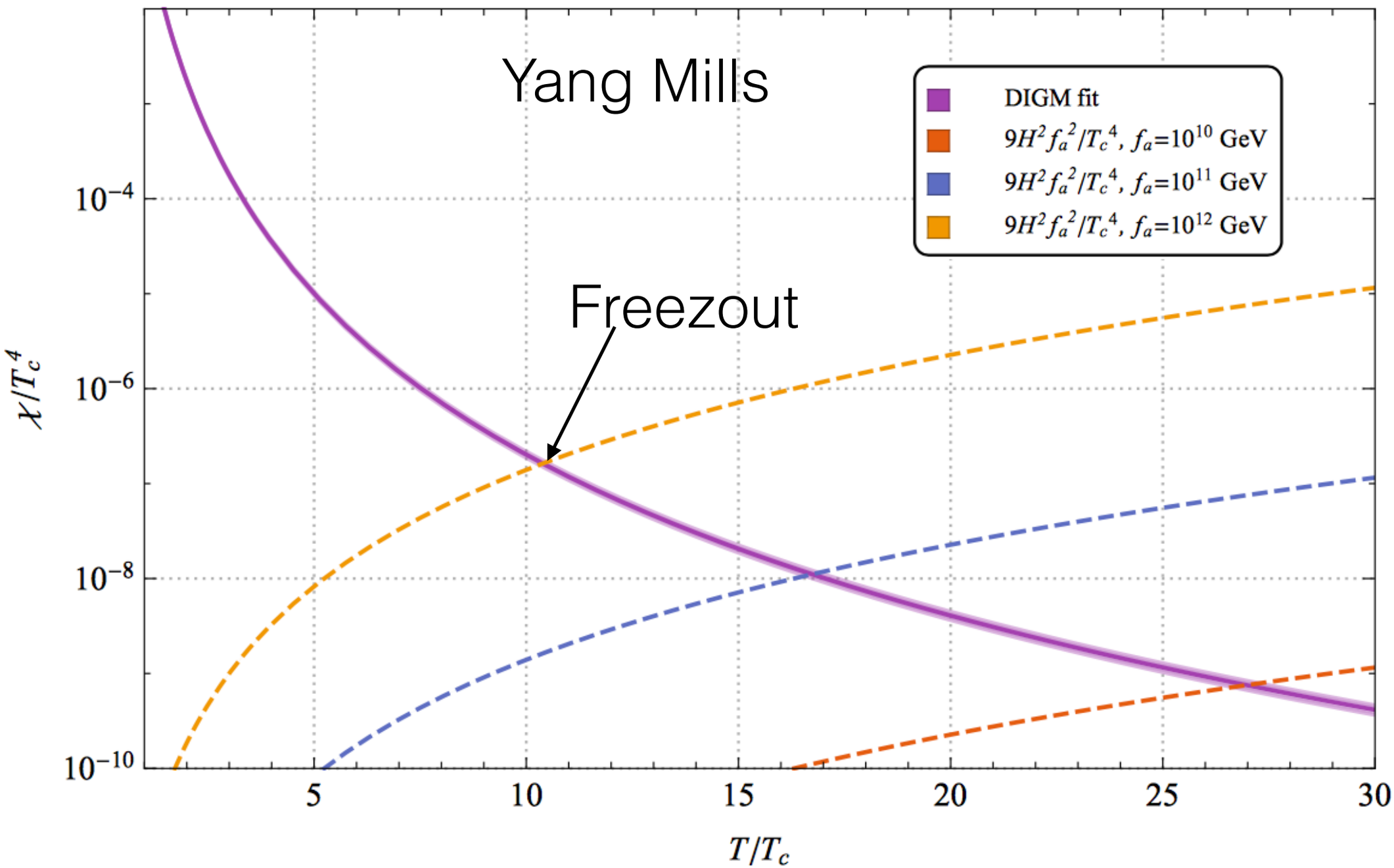
Topological susceptibility


T
Tc
sQGP
YM
QCD-like
Preconformal
Nfc
Conformal
NAF
Nf

Axion freezout : $3H(T) = m_a(T)$

$$m_a(T_1) = \frac{\sqrt{\chi}}{f_a}$$

Berkowitz Buchoff Rinaldi 2015



Axion density at freezout controls axion density today

Axions 'must' be there: solution to the strong CP problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

Postulate axions, coupled to Q:

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)$$

$$m_a^2(T)f_a^2 = \left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T),$$

Tc

≈200MeV

340 –380 MeV
RHIC AuAu
200 GeV

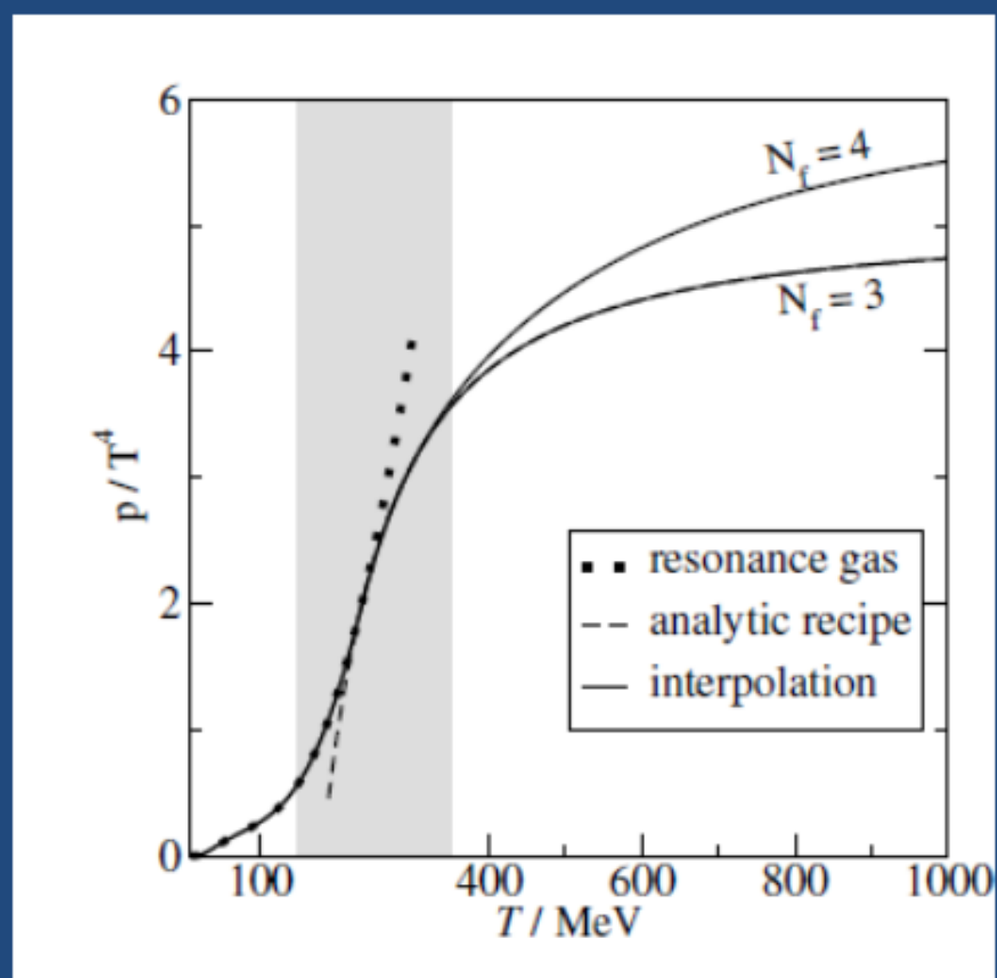
420-480 MeV
LHC
2.76 TeV

500- 600MeV
LHC hot spots
2.76 TeV

1 GeV
LHC
7 TeV

[Expected]

Quark Gluon Plasma @ Colliders



How many flavors?

Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006

Tc

≈200MeV

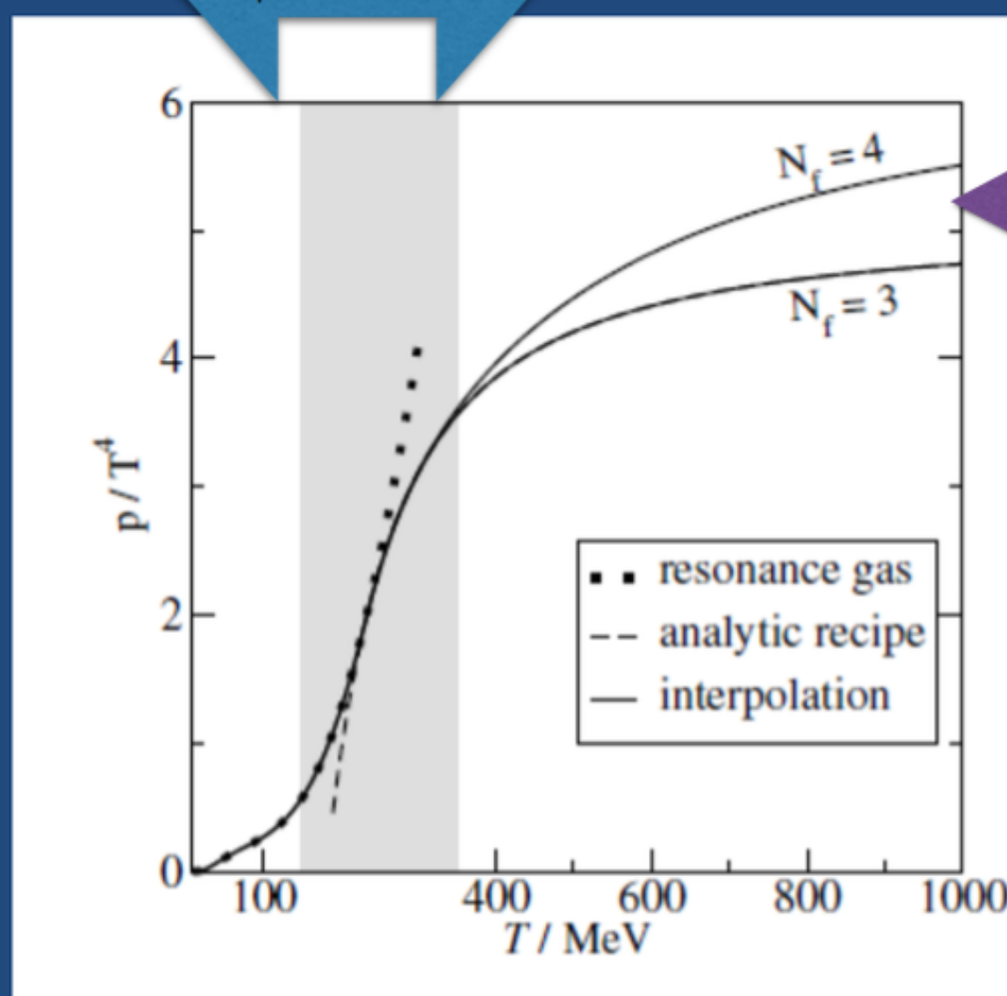
340 –380 MeV
RHIC AuAu
200 GeV

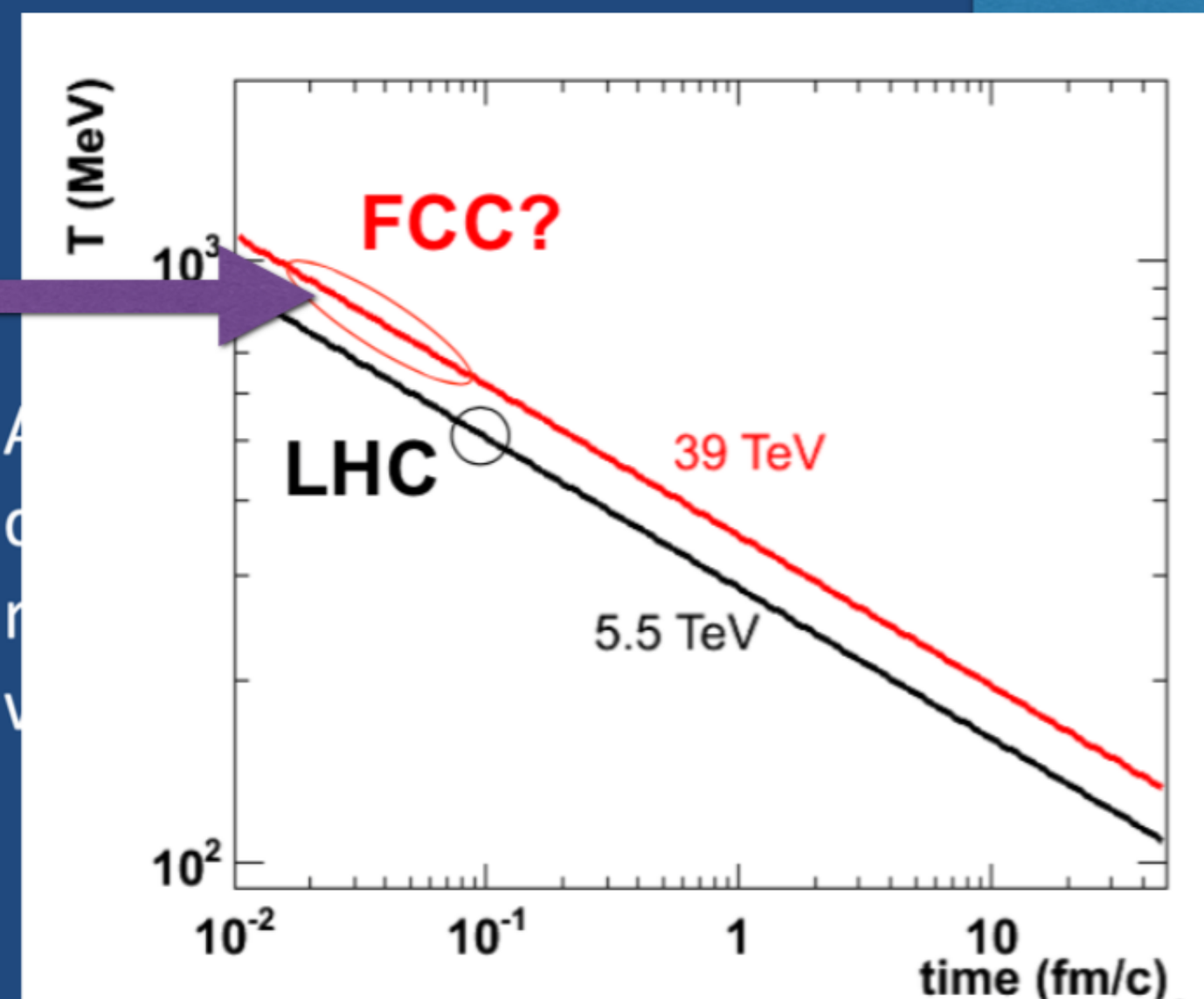
420-480 MeV
LHC
2.76 TeV

500- 600MeV
LHC hot spots
2.76 TeV

FCC

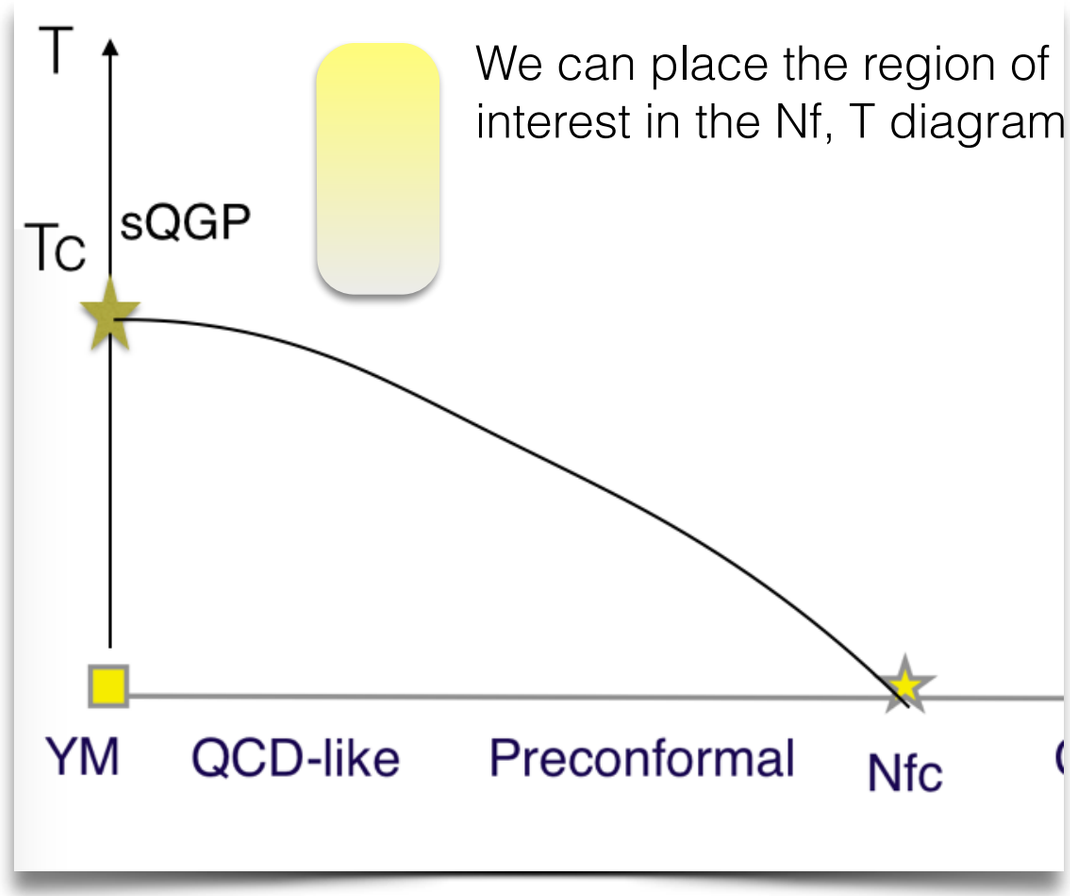
Explored so far

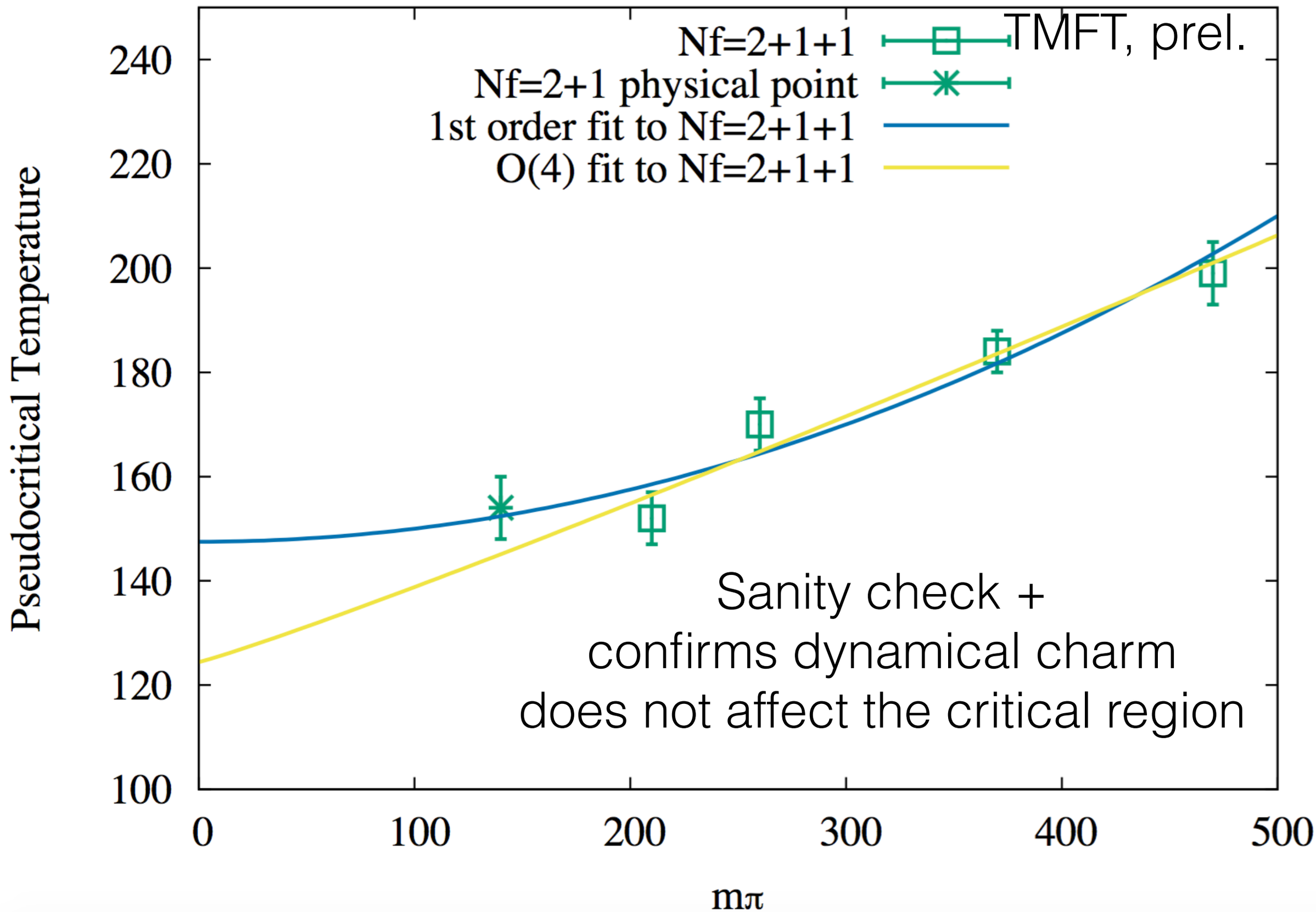




In the region of interest T > 500 MeV

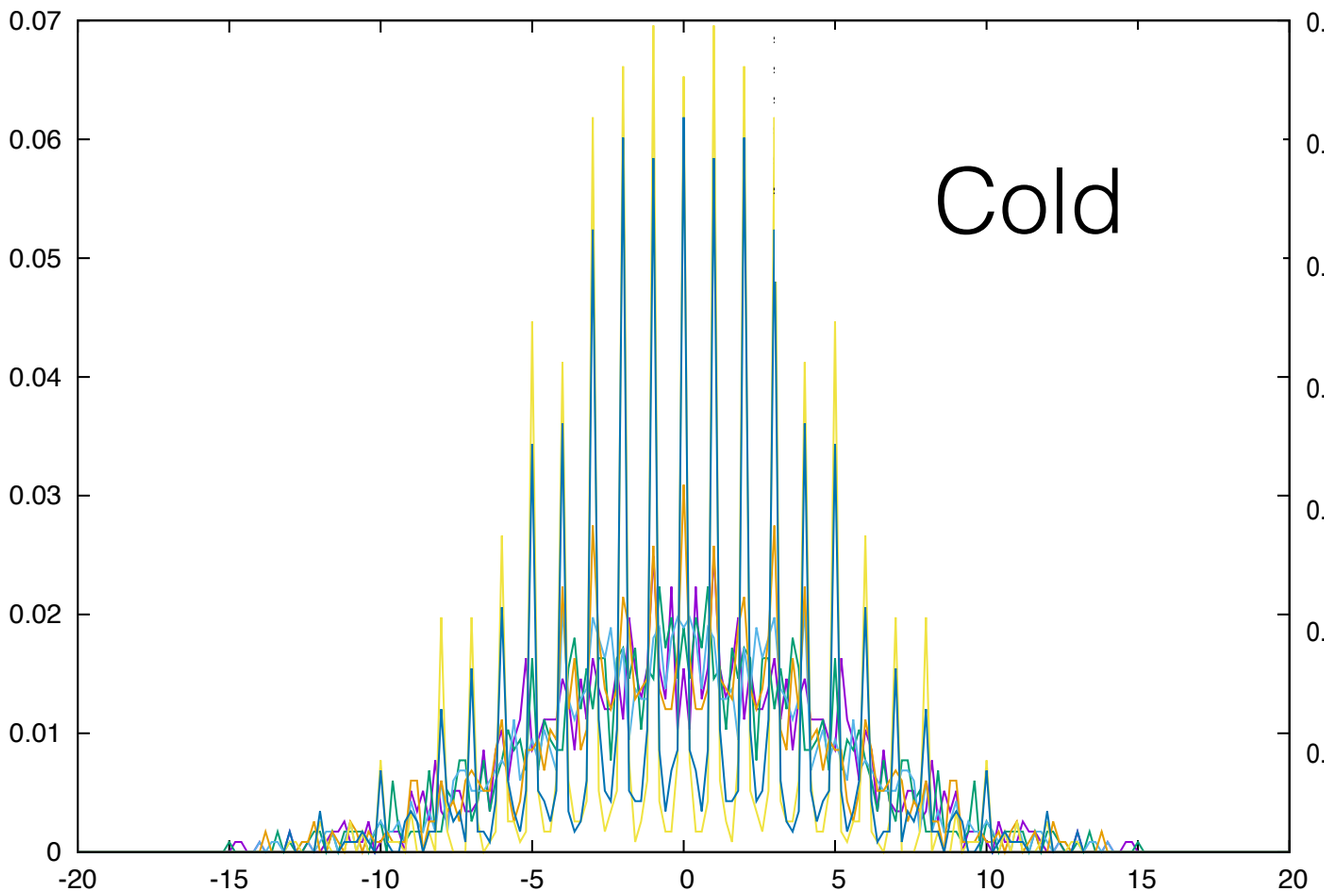
1) We need 2+1+1
2) 2+1+1 = 4 (approximatively)



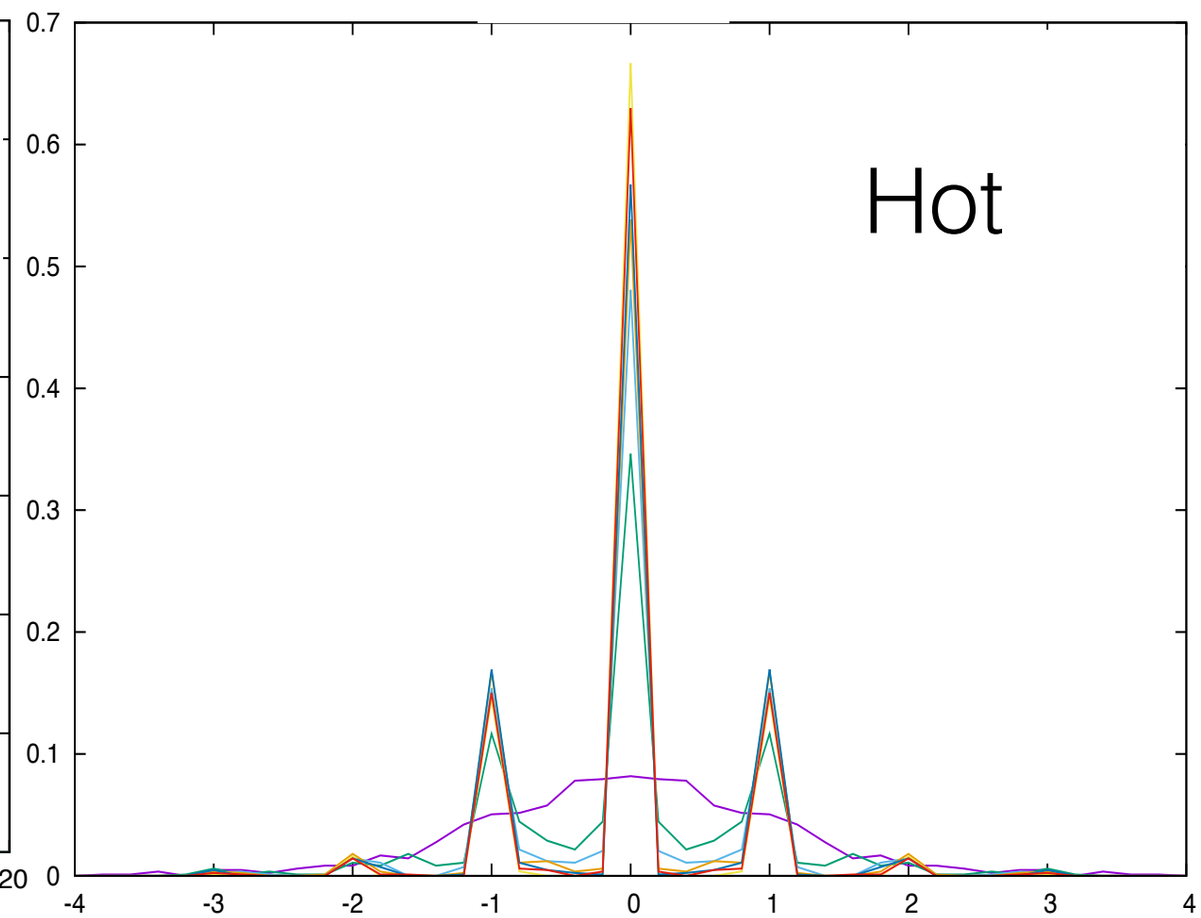

TMFT, prel.
Nf=2+1+1
Nf=2+1 physical point
1st order fit to Nf=2+1+1
O(4) fit to Nf=2+1+1
Pseudocritical Temperature
240
220
200
180
160
140
120
100
0
100
200
300
400
500
mπ
Sanity check +
confirms dynamical charm
does not affect the critical region

Shape of distributions of topological charge: different flow time

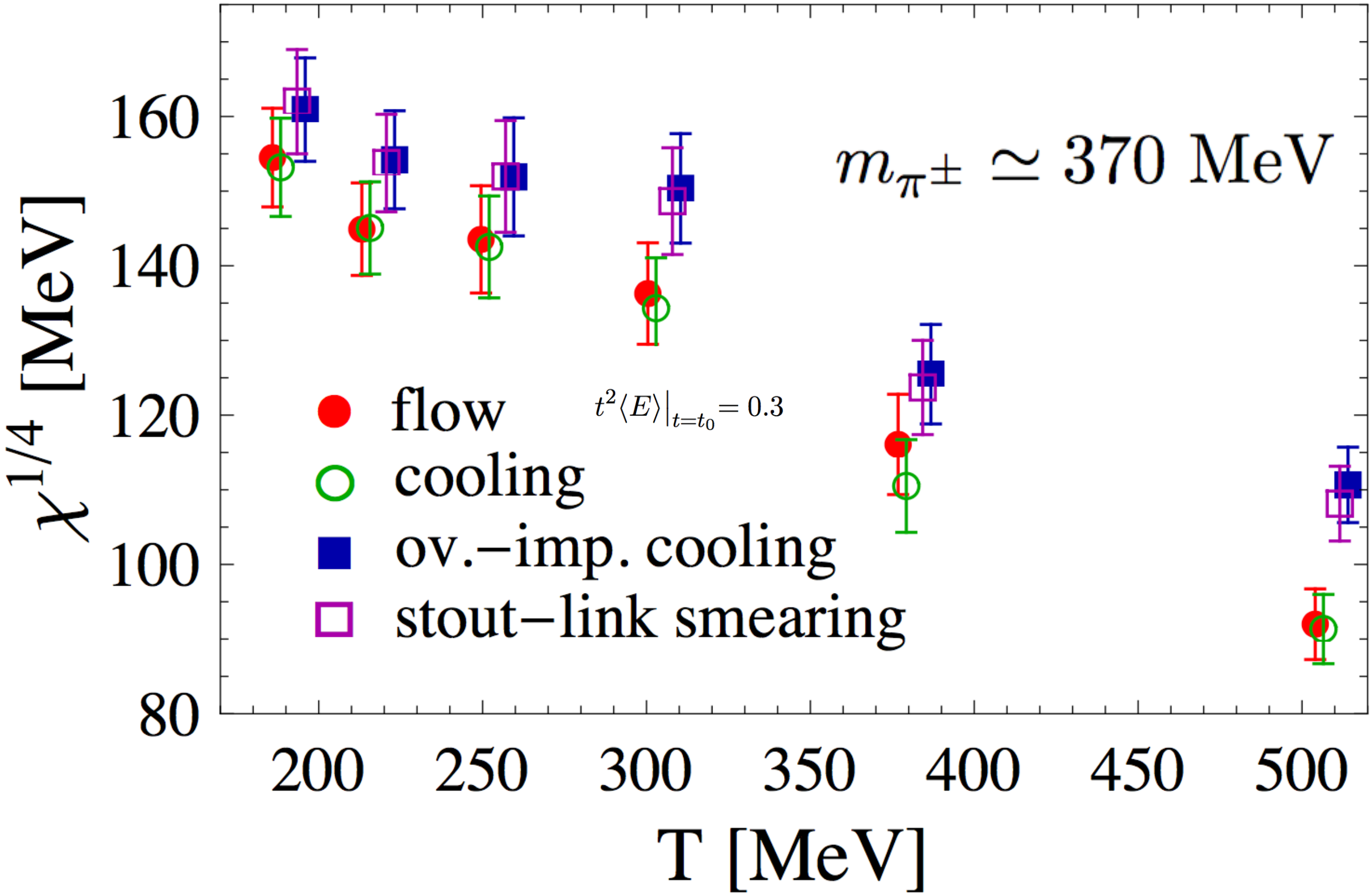
$$t^2\langle E\rangle\big|_{t=t_0} = \quad (0.1, 0.15, 0.2, 0.3, 0.4, 0.45, 0.66)$$

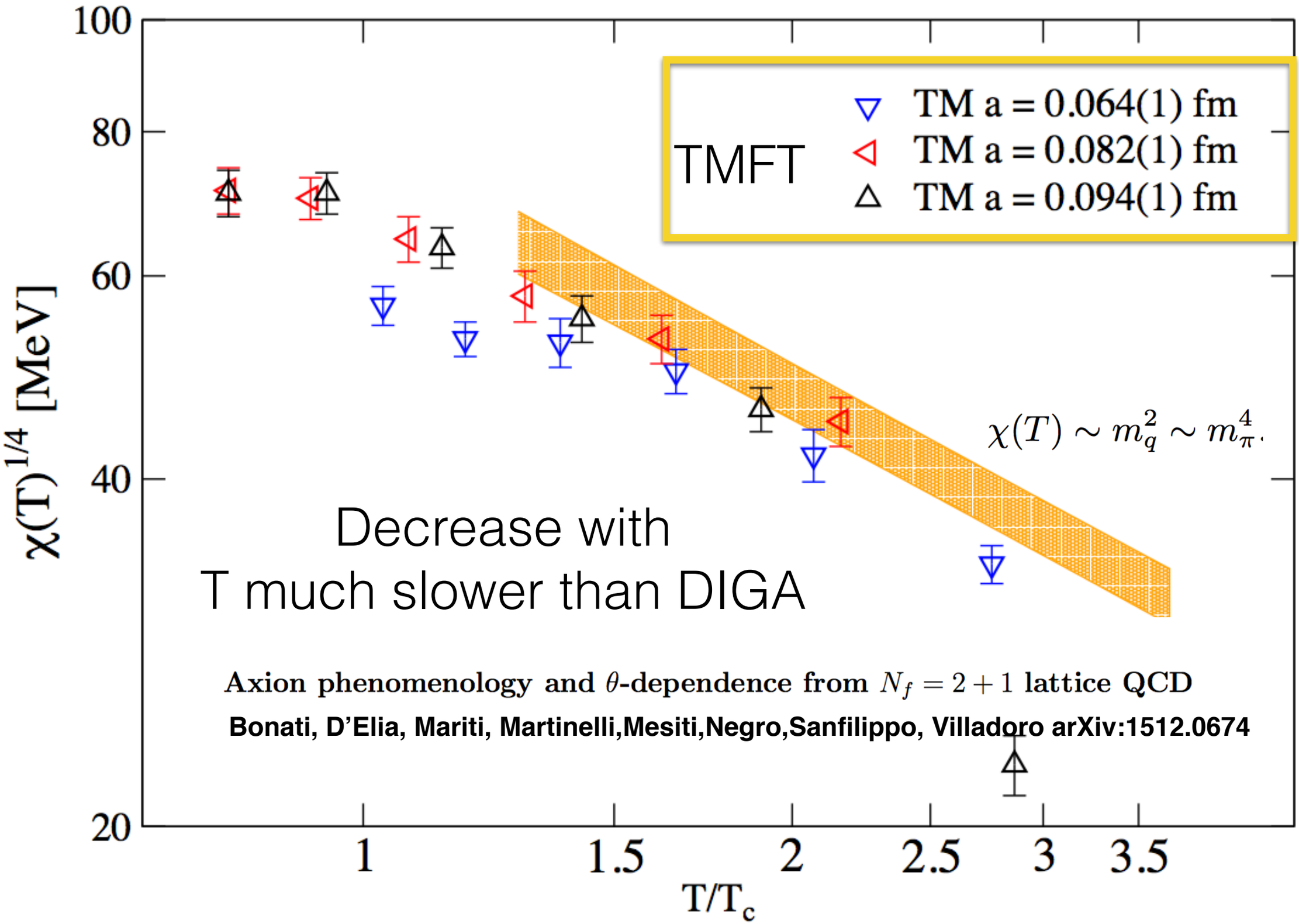


Q



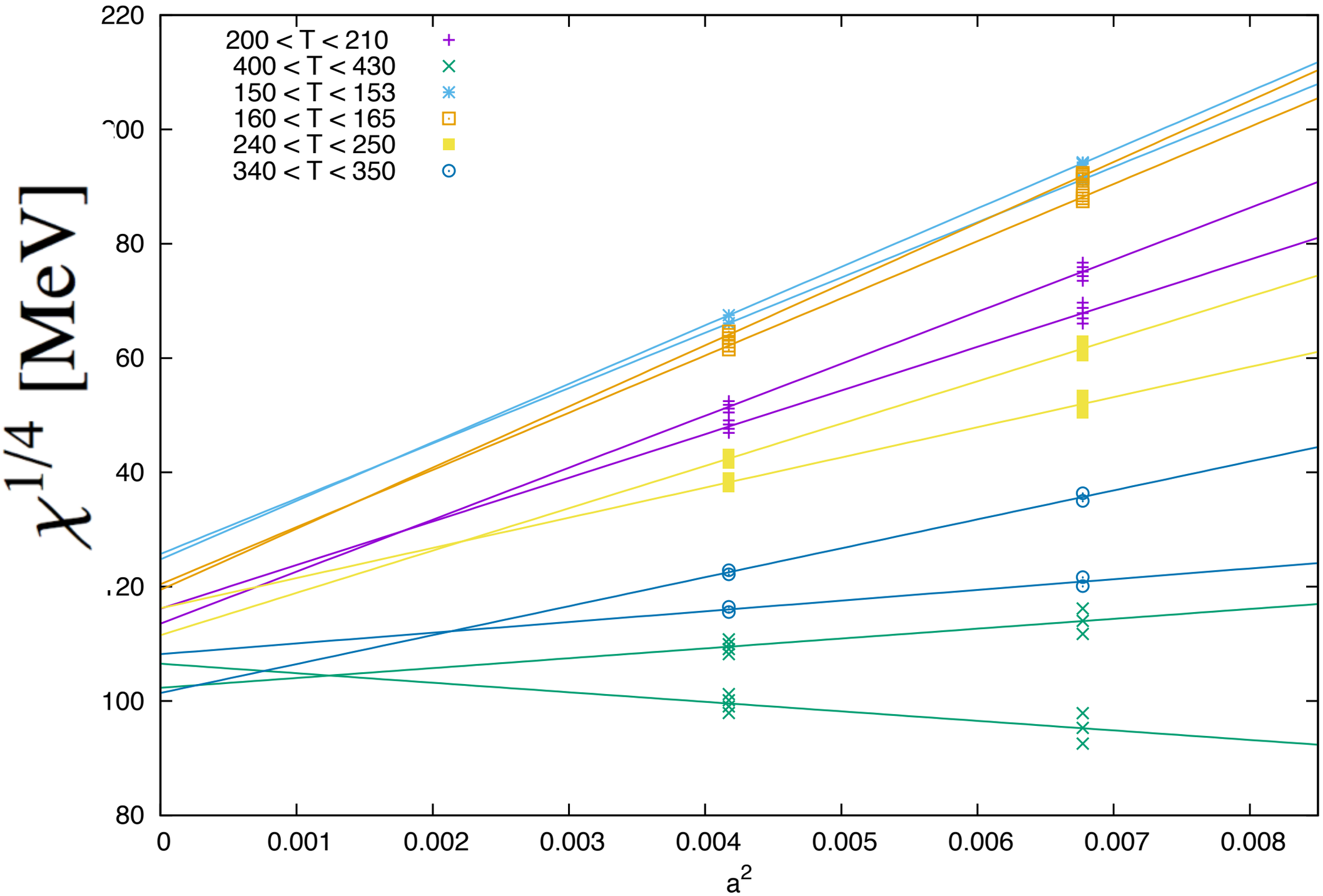
Q


$m_{\pi^\pm} \simeq 370$ MeV
$t^2\langle E\rangle|_{t=t_0} = 0.3$
flow
cooling
ov.−imp. cooling
stout−link smearing
$\chi^{1/4}$ [MeV]
T [MeV]
160
140
120
100
80
200
250
300
350
400
450
500


TMFT
TM a = 0.064(1) fm
TM a = 0.082(1) fm
TM a = 0.094(1) fm
$\chi(T) \sim m_q^2 \sim m_\pi^4$.
Decrease with
T much slower than DIGA
Axion phenomenology and θ-dependence from $N_f = 2+1$ lattice QCD
Bonati, D'Elia, Mariti, Martinelli,Mesiti,Negro,Sanfilippo, Villadoro arXiv:1512.0674
$\chi(T)^{1/4}$ [MeV]
T/T_c
100
80
60
40
20
1
1.5
2
2.5
3
3.5

Continuum limit for different scales

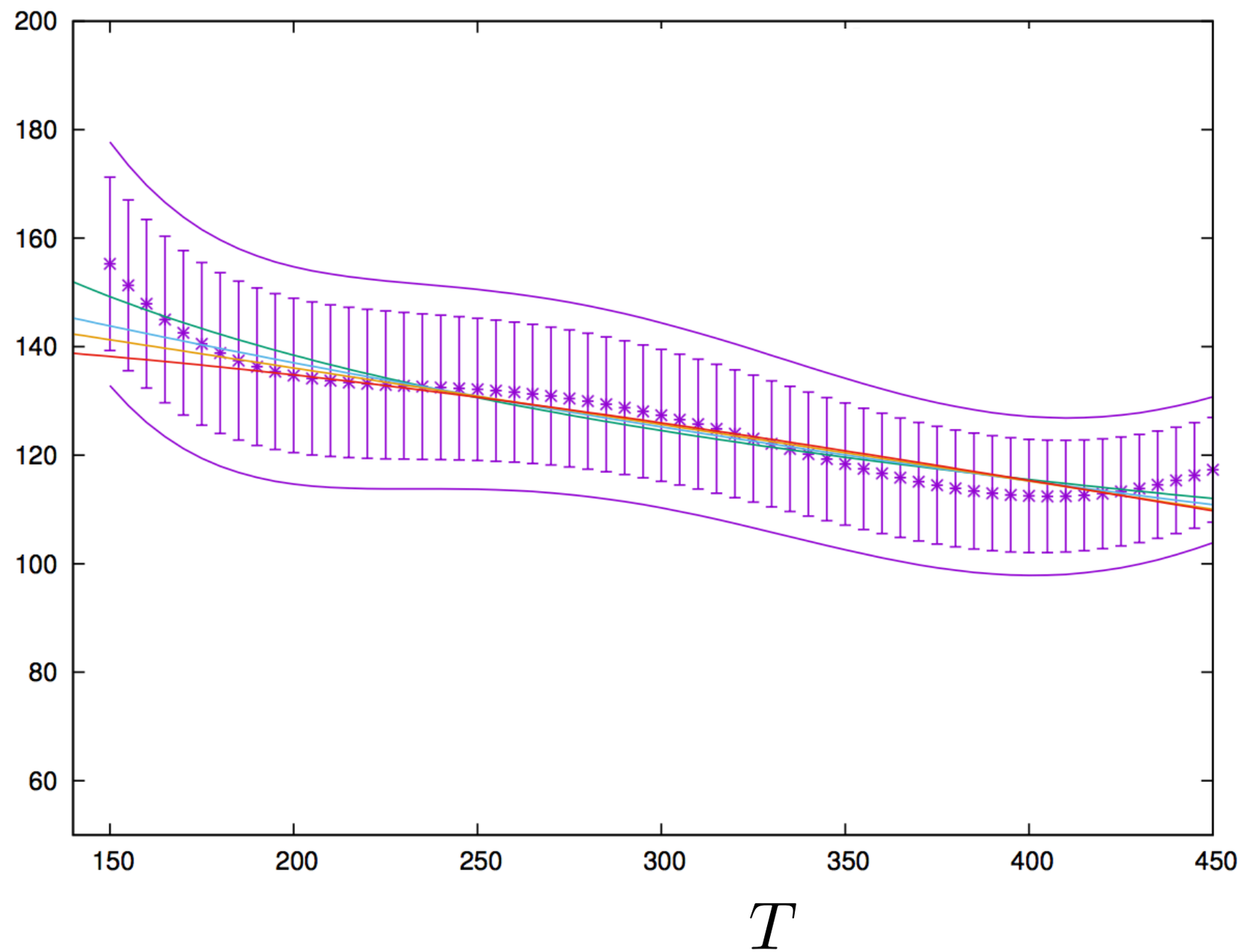
$$t^2\langle E\rangle\big|_{t=t_0} = 0.3\,,\,0.6$$



A preliminary continuum extrapolation shows an even milder decrease wrt to Nf = 2+1

strong sensitivity to Nf

$\chi(T)^{1/4}$



..to be continued

Summary

We have studied the evolution of different dimensionless observables with Nf, for a fixed quark mass.

An external mass enables communication between different phases, which are no longer qualitatively different.

The dynamics retain features of the precritical behavior, in accordance with an EoS analysis:we
have observed scale separation which indirectly supports walking of the coupling.

The theory with eight flavors is qualitatively different from QCD.

Topological susceptibility at high T, which is relevant for axion physics, seems to be particularly sensitive to the number of fermions.