Mesh Generation

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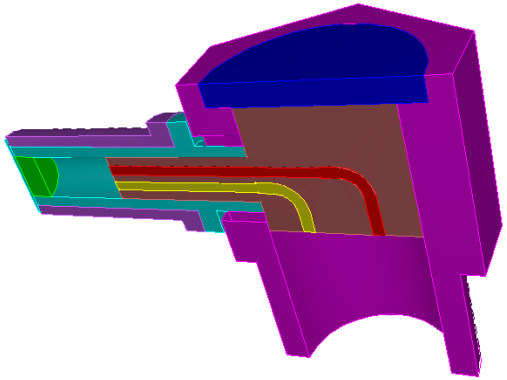
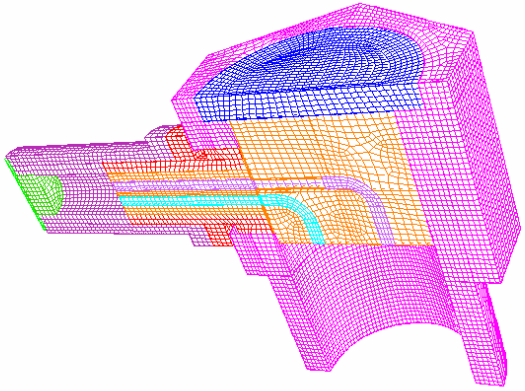
What is mesh generation?

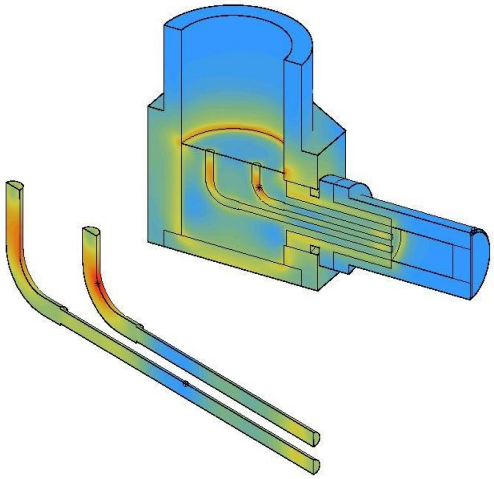
Continuous domain (Geometry)



Discretized Domain (Mesh)



Discrete Solution (FEA Results)



- **Geometry from CAD (SDRC Ideas, Pro/Engineer, etc.)**
- **Mesh type determined by analysis; tetrahedra, hexahedra most common**

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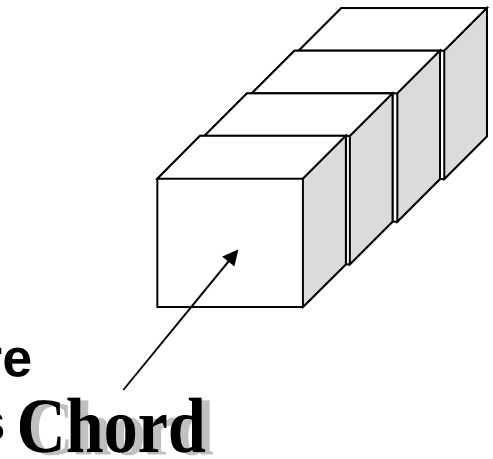
Overview
Why Is Hex Meshing Difficult?

- Hex meshing is difficult, because of chord propagation:
  - Small change in surface mesh can effect non-local changes in mesh
  - Mesh changes propagate across entire assembly because of shared surfaces



- In assemblies, shared surfaces introduce coupling
- Coupling is *reduced* for tet meshes
- Coupling introduces global components to hex meshing

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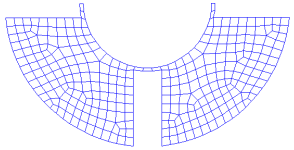
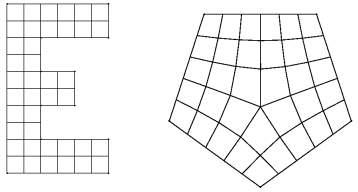
Hexahedral Mesh Generation

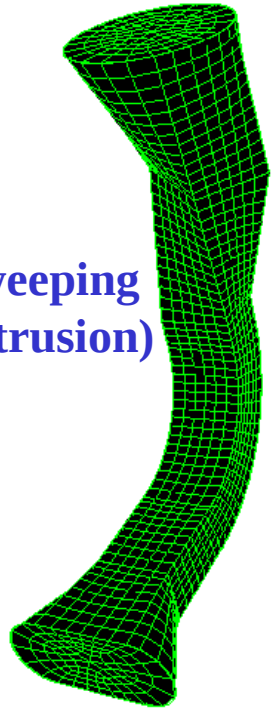
- *No "automatic" hex meshing algorithm known!*
- **Lots of semi-automated algorithms**

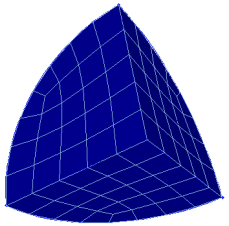
Quad (surface) algorithms





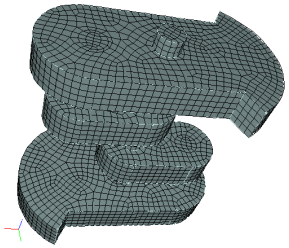
Sweeping (extrusion)





N-Side Primitives

Multisweep



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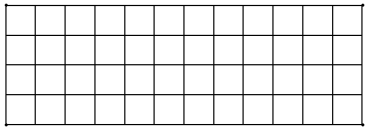
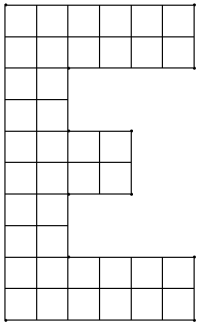
Hex Meshing
Begins With Quad Meshing

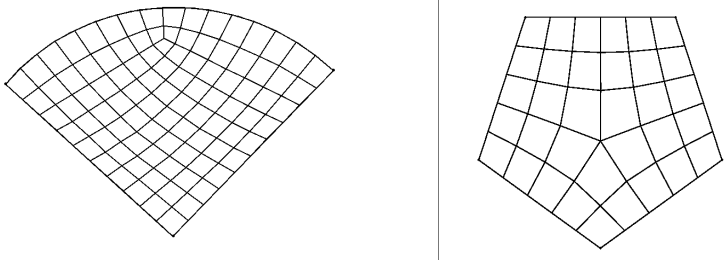
- **Structured algorithms**
  - **Mapping [COO82]**



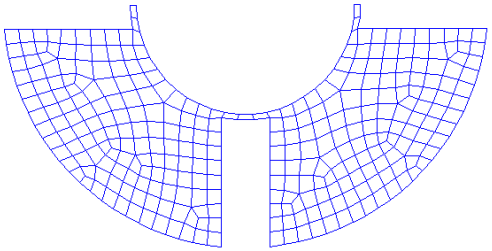


  - **Submapping [WHI96]**

- **Primitives [LI95]**



- **Advancing-front algorithms**
  - **Paving [BLA91, CAS96]**



  - **Qmorph [OWE99a]**

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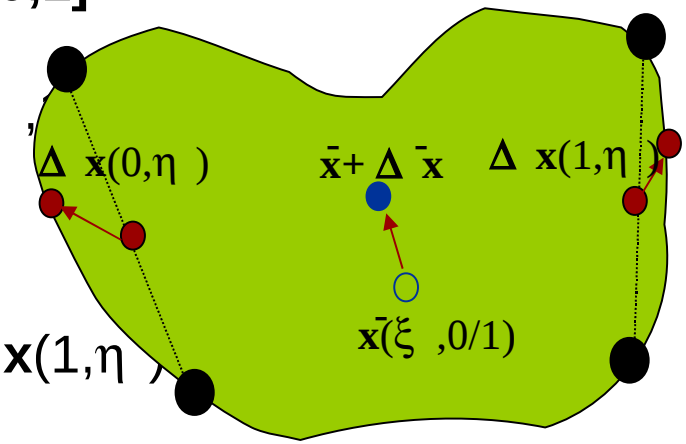
2D Mapping
a.k.a. “Trans-finite Interpolation”, TFI

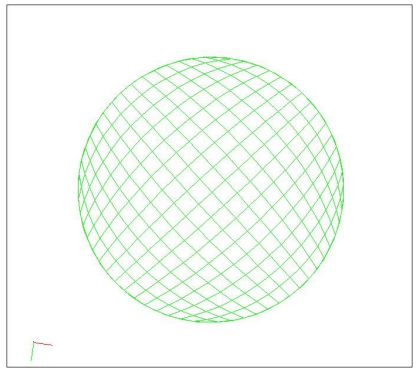
- **Parameterization of boundary into $\xi, \eta \in [0,1]$ using corners & interval settings**
- **Boundary mesh $\mathbf{x}(0,\eta)$, $\mathbf{x}(1,\eta)$, $\mathbf{x}(\xi,0)$, $\mathbf{x}(\xi,$**
- **Compute interior mesh $\mathbf{x}(\xi,\eta)$ using TFI:**

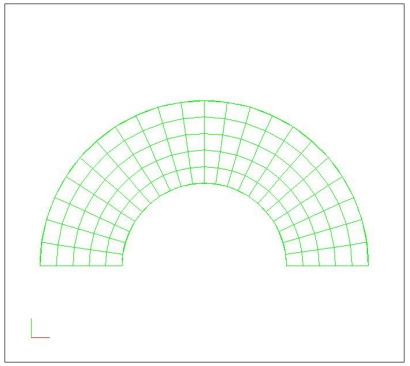


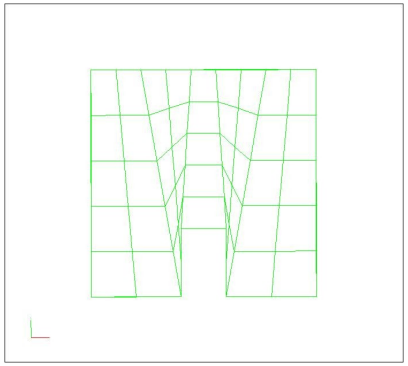
$$\mathbf{x}(\xi,\eta) = (1-\eta)\mathbf{x}(\xi,0) + \eta\,\mathbf{x}(\xi,1) + (1-\xi)\,\Delta\,\mathbf{x}(0,\eta) + \xi\,\Delta\,\mathbf{x}(1,\eta)$$

$$\Delta\,\mathbf{x}(0,\eta) = \mathbf{x}(0,\eta) - [(1-\eta)\mathbf{x}(0,0) + \eta\,\mathbf{x}(0,1)]$$

$$\Delta\,\mathbf{x}(1,\eta) = \mathbf{x}(1,\eta) - [(1-\eta)\mathbf{x}(1,0) + \eta\,\mathbf{x}(1,1)]$$







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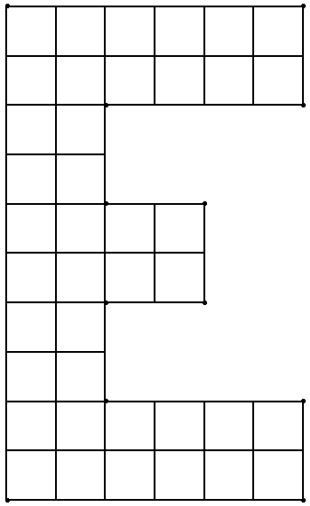
Hex Meshing Primitives
Submap

- **Parameterize boundary using vertex types *and intervals***
- **Divide domain into mapped subregions**
- **Interpolate between subregion boundaries to get lower-dimensional mesh boundaries**
- **Utilize mapping implementation to generate interior nodes from mesh boundaries**



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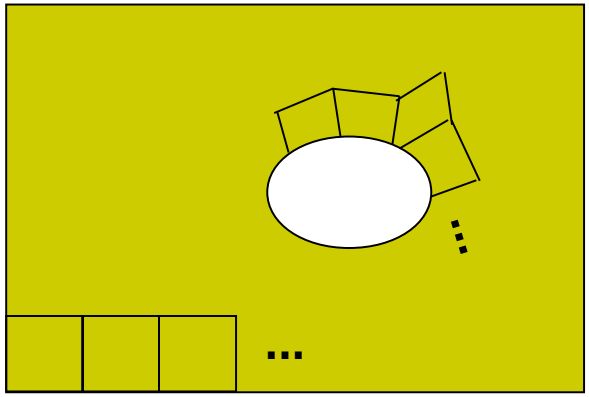
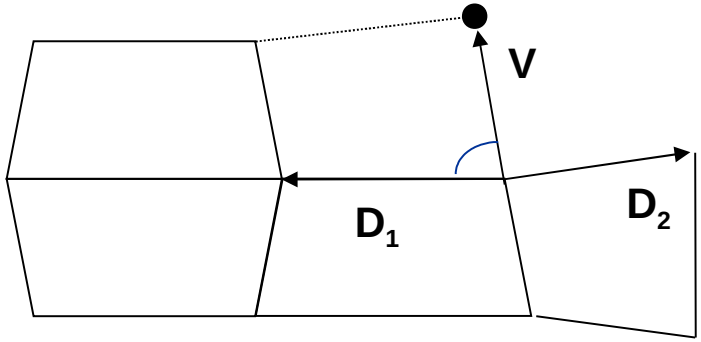
The Paving Algorithm

1) insert quadrilaterals by geometric projection into void





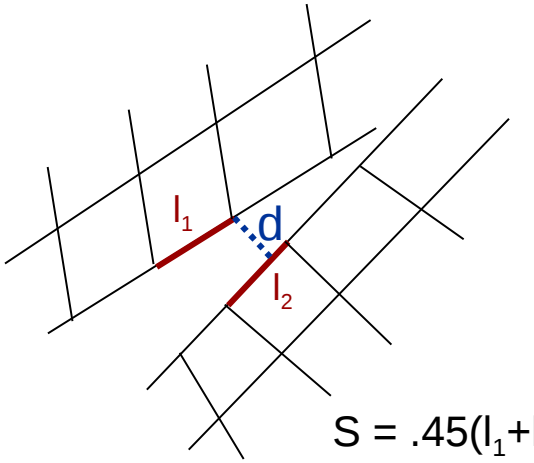
$$|\mathbf{V}| = (|\mathbf{D}_1| + |\mathbf{D}_2|) / 2\sin\alpha$$

The Paving Algorithm
Intersection Detection

2) Detect & resolve intersecting fronts [CAS96]

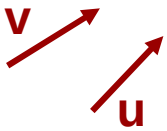
- **Intersections in 3-space difficult, because edges rarely intersect (chords of 3D surface)**
- **"Intersection" becomes decision on whether to join fronts**

Proximity:



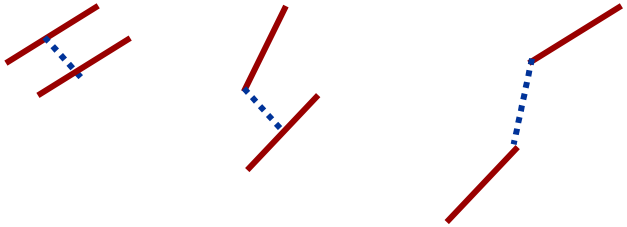
$S = .45(l_1+l_2)/2$

Angle:



Case (C):

mid-mid (0) **mid-end (1)** **end-end (1)**



$$F = d/S + .25(1-|\mathbf{u}\bullet\mathbf{v}|) + .1C$$
$$F < 1 \Rightarrow \text{join edges}$$

- **Connect by joining nodes & edges on opposing fronts**
- **Need even-node loops on both sides**

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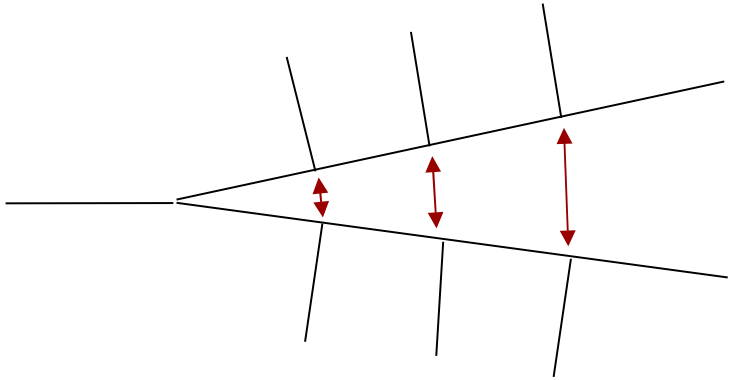
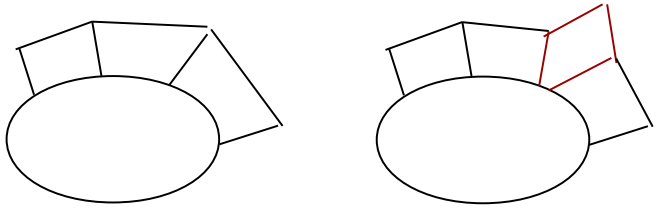
The Paving Algorithm
Smooth & Seam

3) Smooth locally
4) Seam, insert tucks/wedges

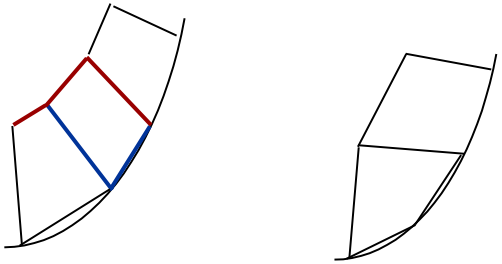
Seam:



Wedge:



Tuck:



5) Repeat 1-4 until closed

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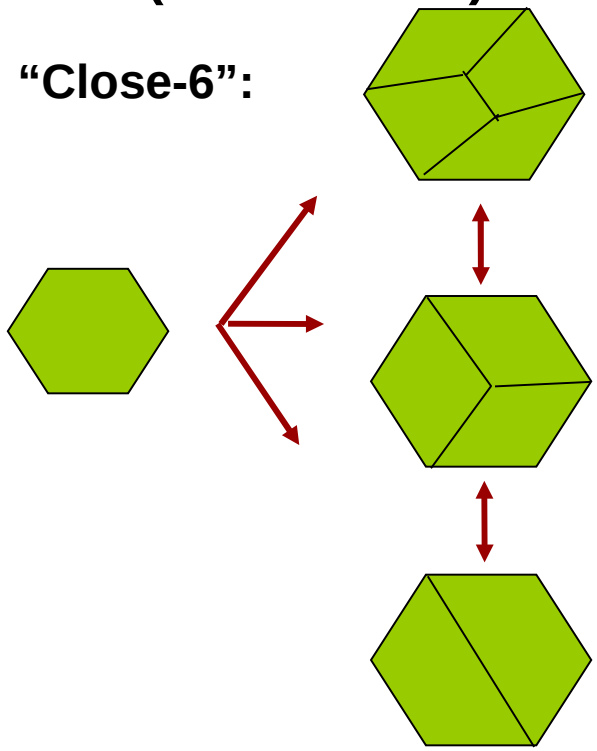
The Paving Algorithm
CleanUp

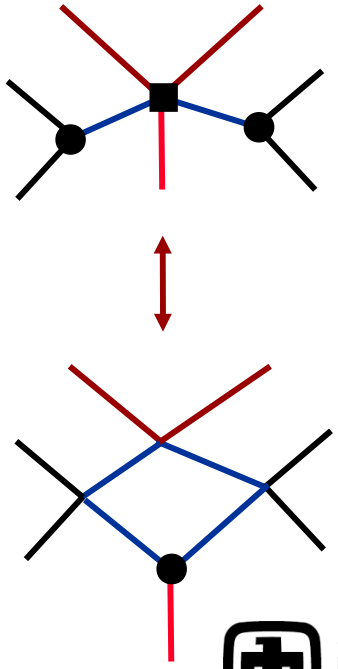
6) Clean up mesh topology

- **Large family of topological cleanup operations [KIN96]**
- **Ultimate goal: minimize number of irregular nodes (valence != 4)**

"Close-6":



"Face Open":



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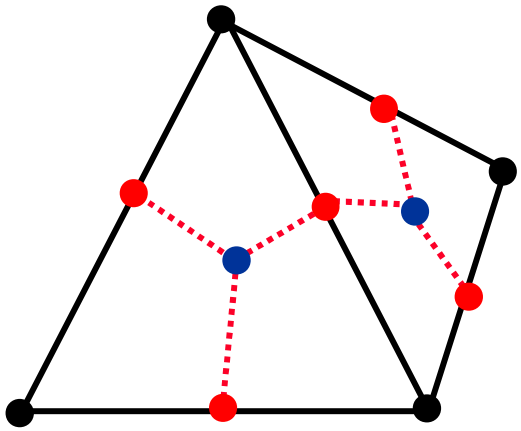
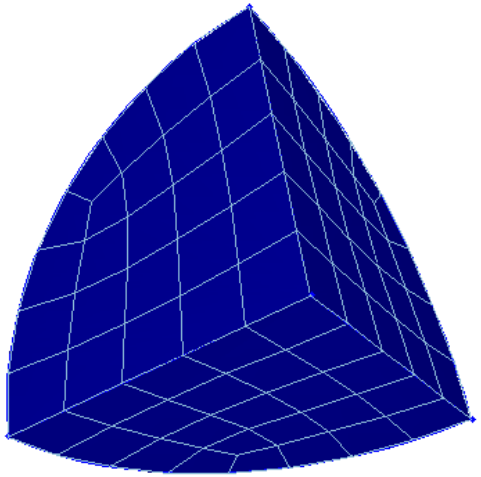
Mid-point Subdivision

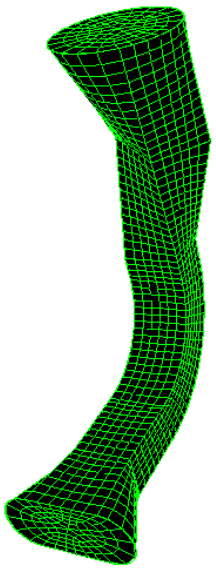
- Split each d-dimensional region with a node, connect to all bounding (d-1)–dimensional region midpoints
- Utilize mapping to map subregions
- In 3d, requires 3-valent vertices/nodes at “corners”
- Automatic detection / scheme assignment challenging for non-trivial topology

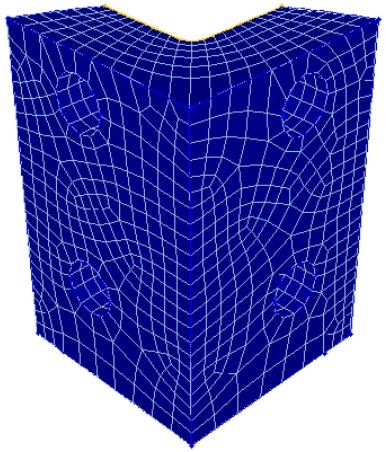




Hex Meshing Primitives
Sweeping

- Fundamental assumption: match all source faces, nodes with counterparts on target (may be implied mesh on target)
- Fundamental problem: placement of nodes inside boundary loops on target & intermediate layers
- Fidelity to:
  - Side surface variations
  - Source/target surface variation
- Implementations:
  - Morph-based (CUBIT)
  - BMSweep (Ansys)
  - Cooper





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Morph-Based Sweep

- **Given: source surface mesh $\mathbf{x}^S$, boundary mesh Ω^T, compute interior mesh $\mathbf{x}^T$ by solving:**

$$D_n\mathbf{x}^T = P_n|\mathbf{u}_n^T|^{\gamma-1}\,\mathbf{u}_n^T + Q_n|\mathbf{v}_n^T|^{\gamma-1}\,\mathbf{v}_n^T$$

where

$$P_n = \{(\mathbf{v}_n^S\bullet\mathbf{v}_n^S)(\mathbf{u}_n^S\bullet D_n\mathbf{x}^S) - (\mathbf{u}_n^S\bullet\mathbf{v}_n^S)(\mathbf{v}_n^S\bullet D\mathbf{x}^S)\}/|\mathbf{u}_n^S|^{\gamma-1}|\mathbf{u}_n^S\times\mathbf{v}_n^S|^2$$

$$Q_n = \{(\mathbf{u}_n^S\bullet\mathbf{u}_n^S)(\mathbf{v}_n^S\bullet D_n\mathbf{x}^S) - (\mathbf{u}_n^S\bullet\mathbf{v}_n^S)(\mathbf{u}_n^S\bullet D\mathbf{x}^S)\}/|\mathbf{v}_n^S|^{\gamma-1}|\mathbf{u}_n^S\times\mathbf{v}_n^S|^2$$

$$D_n\mathbf{x} = G_{22n}D_{\xi\xi}\,\mathbf{x}_n - 2G_{12n}D_{\xi\eta}\,\mathbf{x}_n + G_{11n}D_{\eta\eta}\,\mathbf{x}_n$$

$G..._n$, $D..._n$, $u_n, v_n = f(x_n, x_{nm})$

- **Compute each layer mesh based on source & bounding meshes**
- **Work by Pat Knupp, Sandia National Labs [KNU99a]**

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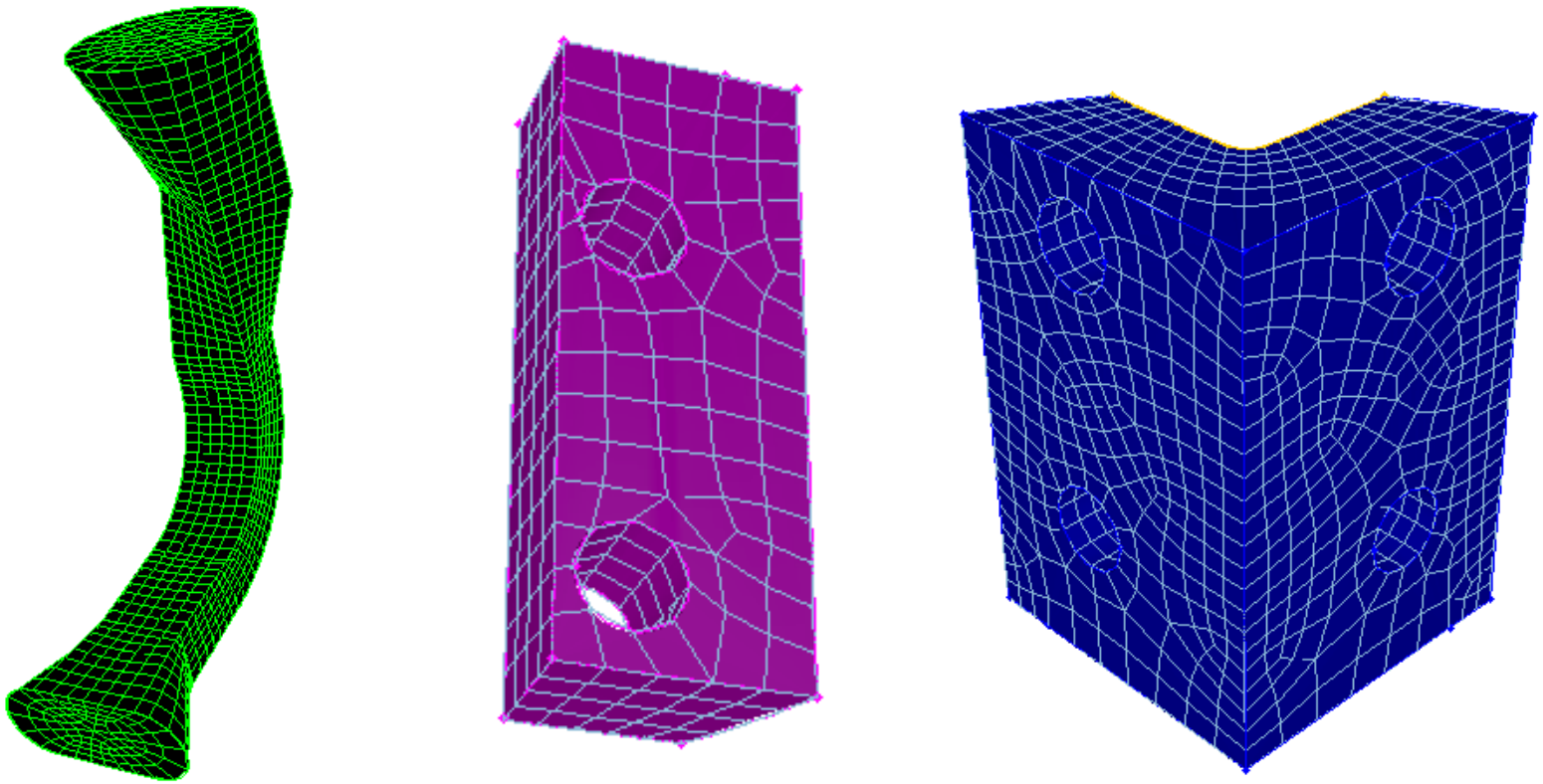
Morph-Based Sweep Examples



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BMSweep Algorithm [STA98]

- **Compute sweepable mesh on source surface**
- **Compute background triangular mesh for source surface, using only boundary nodes**
- **Compute barycentric coordinates and offset distance of nodes with respect to the containing background triangle on source and target surfaces**
- **At each layer, place interior nodes according to interpolated barycentric coordinates & offsets wrt "elevated" background triangles**

<u>On source/target:</u>

$$\mathbf{x}_i'^{S} = a_i^S \mathbf{x}_{1i}^S + b_i^S \mathbf{x}_{2i}^S + c_i^S \mathbf{x}_{3i}^S$$

$$di^S = |\mathbf{x}_i^S - \mathbf{x}_i'^{S}|$$

<u>On layer u:</u>

$$\mathbf{x}_i'^{u} = a_i^{ST} \mathbf{x}_{1i}^u + b_i^{ST} \mathbf{x}_{2i}^u + c_i^{ST} \mathbf{x}_{3i}^u$$

$$\mathbf{x}_i^u = \mathbf{x}_i'^u + d_i^t \, \mathbf{v}(\mathbf{x}_{1i}^u, \mathbf{x}_{2i}^u, \mathbf{x}_{3i}^u)$$

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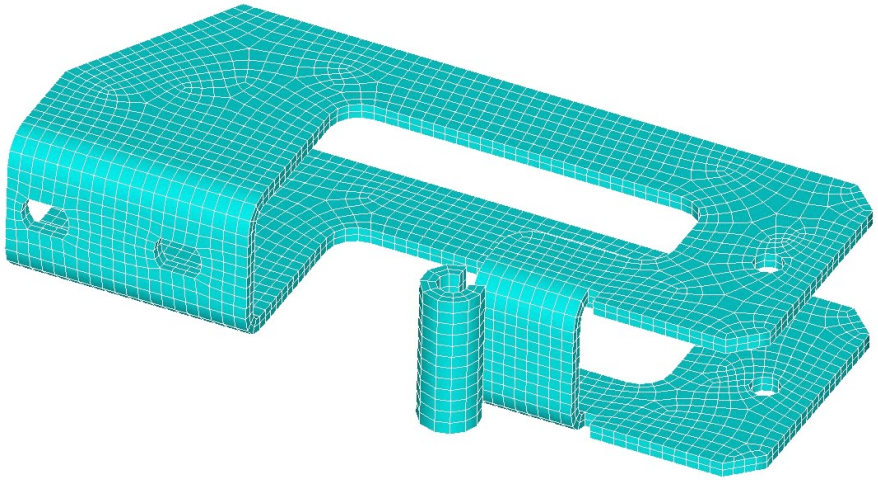
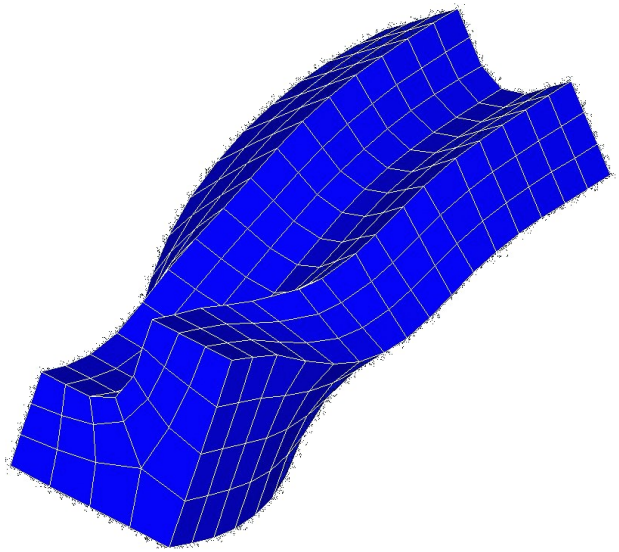
BMSweep Algorithm Examples





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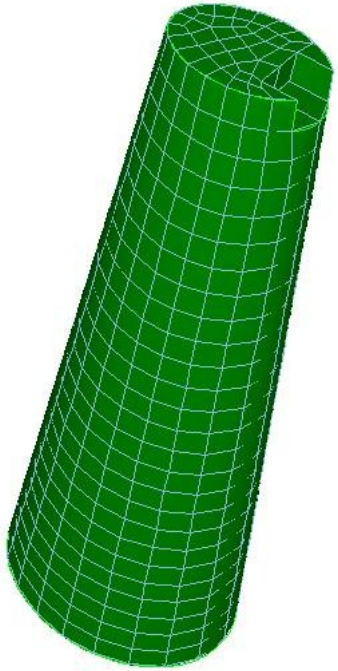
(simple 3d example, manual setting)

- Frustum height 5 maj rad 1 min rad .8 top .5
- Brick x 0.4
- Body 2 move z 2.5 x .4
- Subtr 2 from 1
- Vol 1 scheme sweep source 14 12 targ 2
- Vol 1 size .2
- Surf 14 12 2 scheme pave
- Surf 15 scheme submap
- Mesh vol 1
- Quality vol 1 allmet
- Export acis ‘frust.sat’



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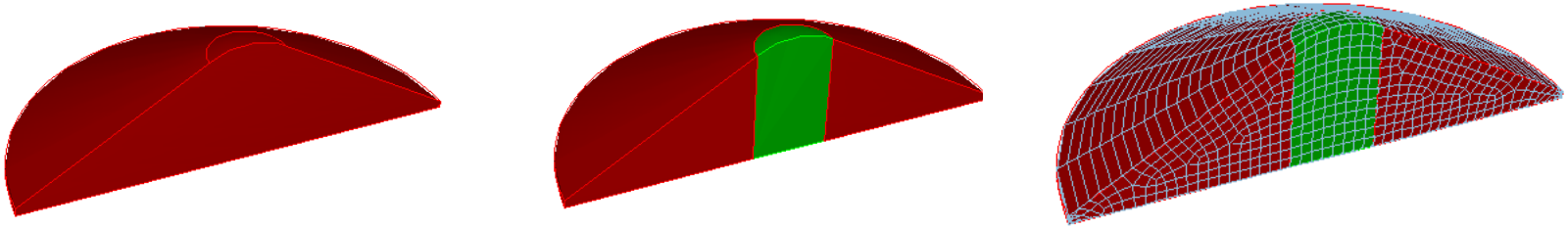
Decomposition-Based Hex Meshing Technique

- Use decomposition to reduce the problem to simpler pieces



- Requires:
  - Primitive algorithms, to mesh pieces
  - Tools, to choose algorithms and setup input
  - Infrastructure, for common data (mesh, geometry, etc.) and support (visualization, etc.)

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(assembly meshing example)

- Sphere rad 1 xpos ypos zpos
- Web bod 1 cylinder rad .2
- Merge all
- Vol 1 scheme sweep source 13 targ 15
- Vol 3 sch sweep
- Vol all siz .05
- Mesh vol all

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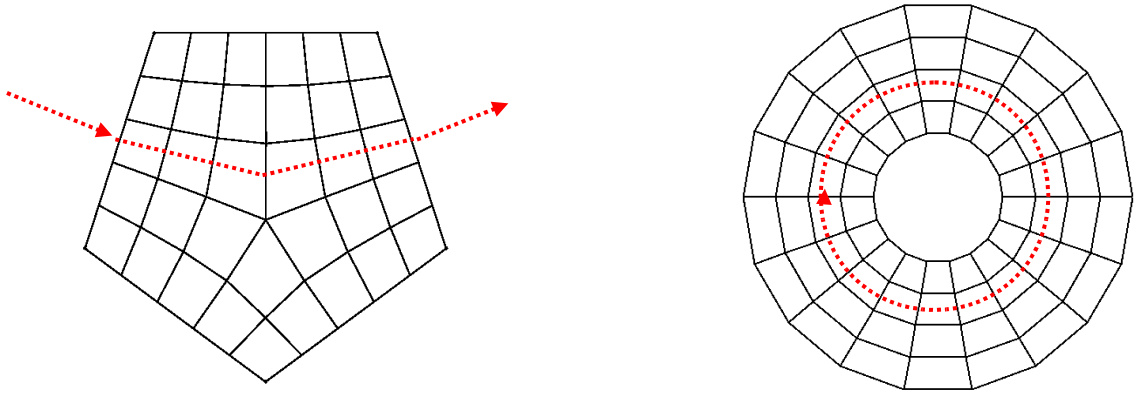
Other Useful Tools for Hex Mesh Generation

- Interval matching
- Automatic mesh scheme selection
- Geometry decomposition tools
- Multisweep

Hex Meshing Tools
Interval Assignment

- **Task: quad-mesh a collection of surfaces which share edges**
- **Challenge: quad meshes constrain the number of intervals on boundary of a surface to be even**

Surface Chords:



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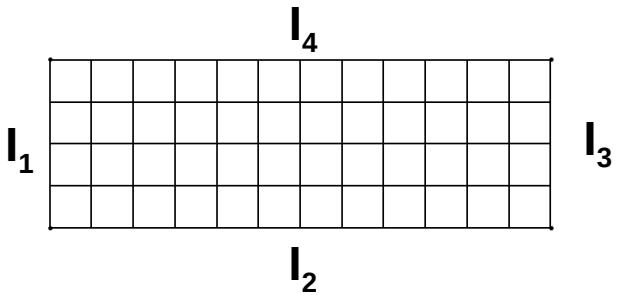
Interval Assignment Constraints

- **Some quad algorithms further constrain intervals**

Map:

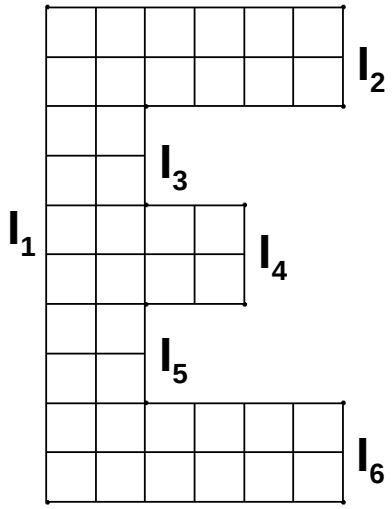
$I_1 = I_3$

$I_2 = I_4$



Submap:

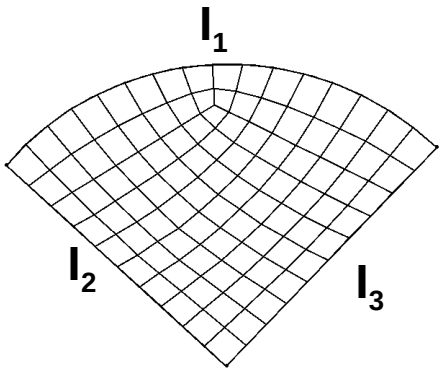
$I_1 = I_2 + I_3 + I_4 + I_5 + I_6$



Triangle Primitive:

$I_2 + I_3 > 2I_1$

...



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Interval Assignment Optimization Problem

- Cast interval constraints as an optimization problem:
- "Over a collection of surfaces, minimize deviation from initial intervals (x) subject to surface algorithm constraints (Ax=b)
  - I, i = Desired, computed interval settings
  - Independent variable: $x := |I-i|$
  - Constraints: $Ax = b$
  - Minimize: $c^T x$
- Different forms of c have been tried
  - Minimize summed deviation from goal (Li95)
  - Minimize max deviation from goal (TAM93)
  - Optimize lexicographic deviation from goal (MIT97)

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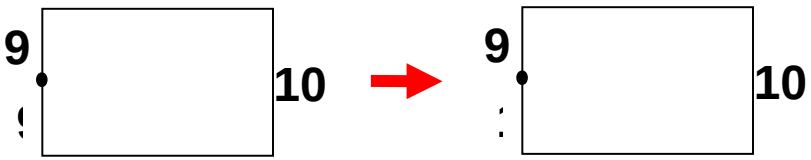
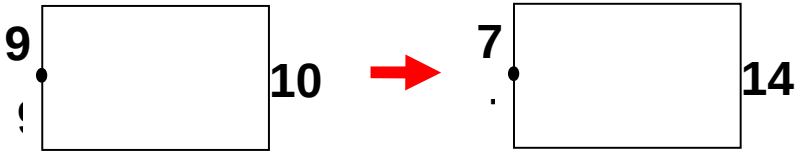
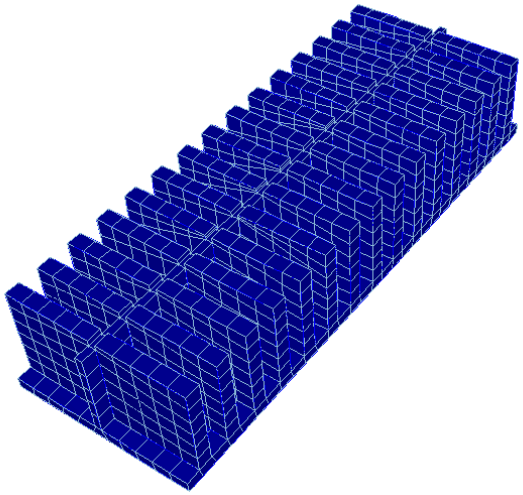
Interval Assignment Examples

Minimize sum of deviations:



Minimize lexicographic vector of deviations:





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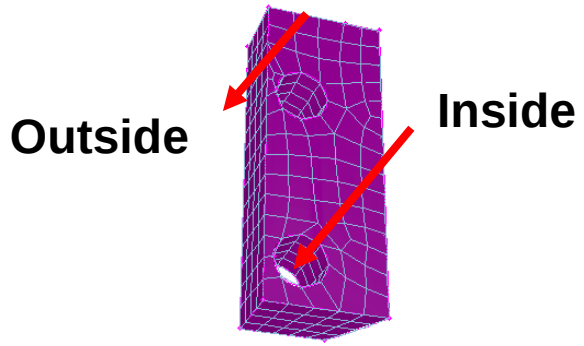
Interval Assignment Issues

- Affects both robustness & quality
- New constraints being discovered and added
  - Volume sweeping constraints [SHEP99] :



Constraint:
Intervals inside = Intervals outside

  - Skew reduction constraints [KER99]
  - Hex meshes intrinsically constrain surface mesh (not really interval constraints, but close; see [MIT96])
- Automation important because it reduces manual effort in quad-hex mesh generation

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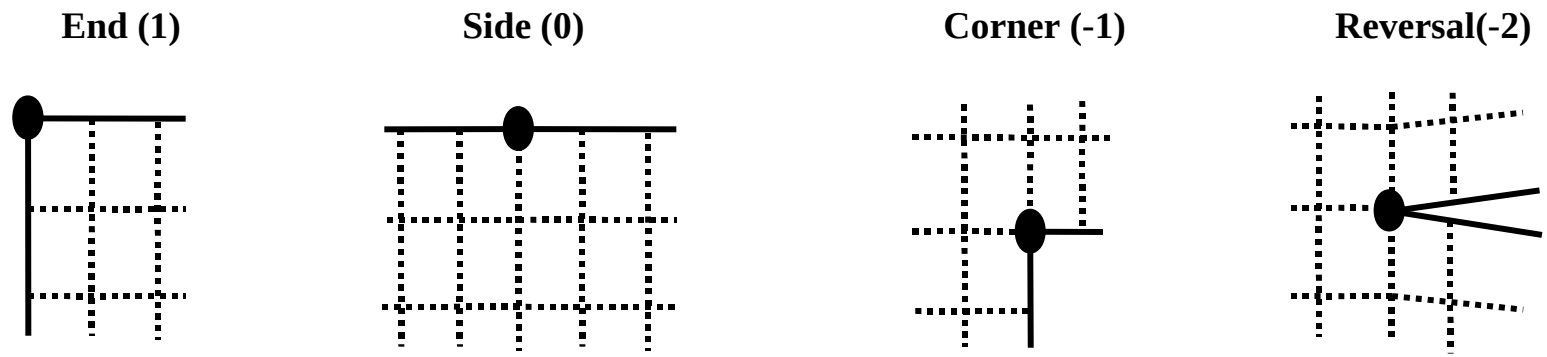
Hex Meshing Tools
Auto Algorithm Selection

- Meshing toolkit implies many tools
- Need to automatically choose tools:
  - Eliminates need to “touch” entire model
  - Relieves user from knowing about all the tools
- Three classes of meshing algorithms:
  - Map, submap, primitive
  - Sweep, Multisweep
  - Free-meshing

Auto Algorithm Selection
Vertex, Edge Types

- Vertex types designate the type of mesh around a vertex:



- Angle of curves meeting at a vertex can be in a range about some ideal value
- First step in automatic algorithm selection is assignment of vertex types
- Edge types can be classified in same manner

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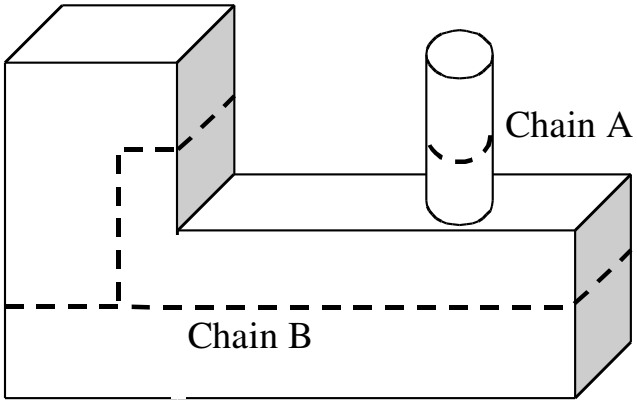
Auto Algorithm Selection
Map, Submap

- A surface can be mapped or submapped if the sum of vertex types is four [WHI96]
- A surface is mapped if all vertices are either Side or End, submapped otherwise
- A volume is mapped or submapped if all its surfaces can be mapped or submapped; submappable volumes have vertices with valence > 4

Auto Algorithm Selection
Auto Sweep Detect [WHI99]

- Swept volumes must be bounded by non-intersecting chains of mapped/submapped surfaces:



- Can parameterize chain surfaces in a consistent global parameter system
  - Consistent means C0-continuous parameters where surfaces meet
  - Chains will have periodic boundary where parameter "jumps"

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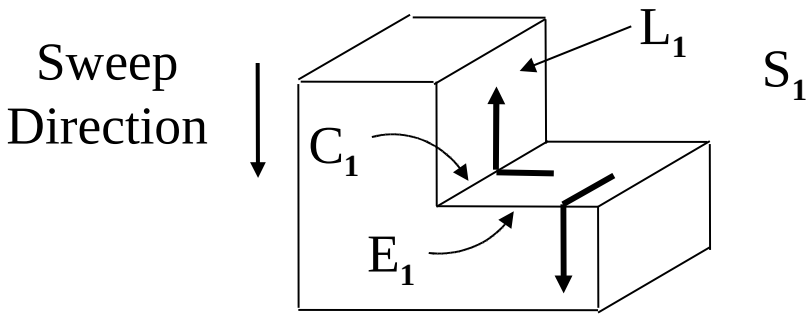
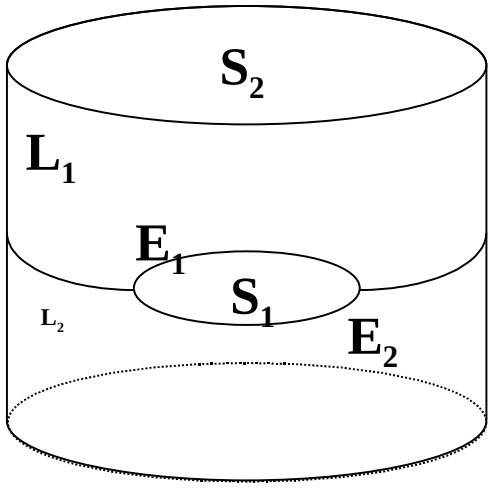
Auto Algorithm Selection
Auto Sweep Detect (cont)

- **Two additional constraints: traversing from source/target to linking surface over two edges heads in:**
  - **Same global parametric direction if edges of same edge type (End or Corner)**
  - **Opposite global parametric direction if different edge type**





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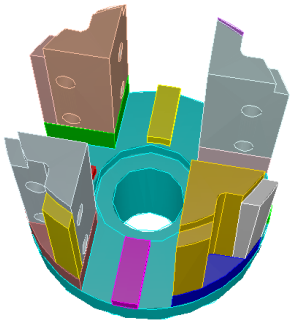
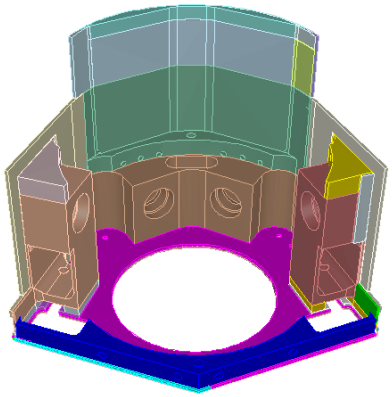
Hex Meshing Tools
Geometry Decomposition

- Manual decomposition:
  - Similar to typical CAD operations
  - Some tuning necessary for application to meshing
    - Cut with existing surfaces/bodies
    - Cut by specifying cutting surface data instead of geometry entity
    - Keep both pieces (decomposing, not removing material)
- Automatic decomposition
  - EVG
  - UW automatic decomposition work:





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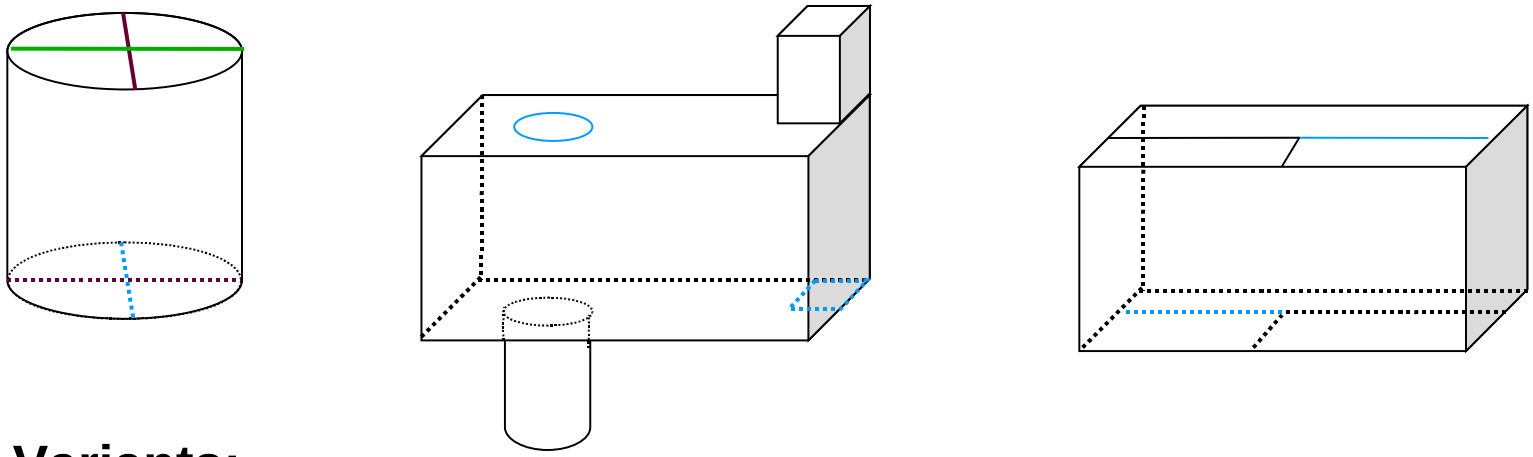
Hex Meshing Algorithms
Multi-sweep Algorithm

- Primarily a topological variant of sweeping
- Problem: Find/compute/effect correspondence between mesh nodes & faces on all sources and targets



- Variants:
  - CooperTool [BLA96]
  - CUBIT Multisweep [LAI99]

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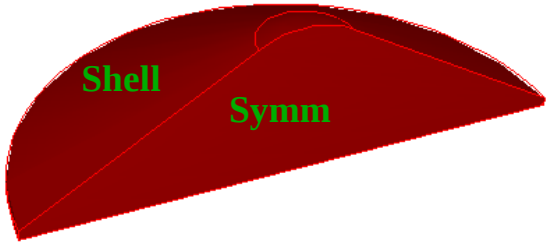
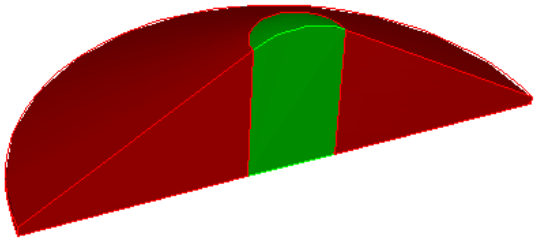
Assembly Hex Mesh Generation *Toolkit* Approach

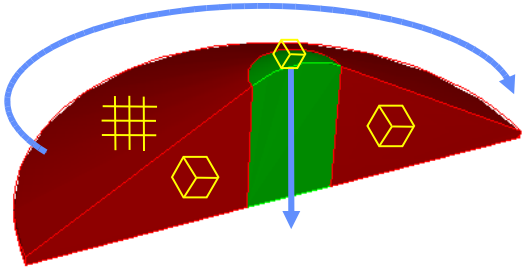
Geometry Validity, BC Assignment

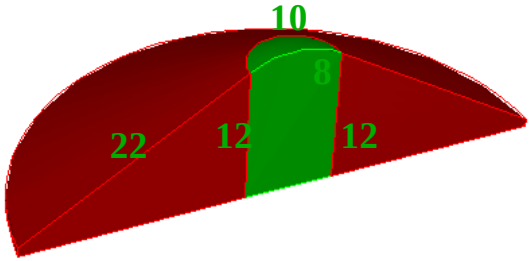


Geometry Decomposition

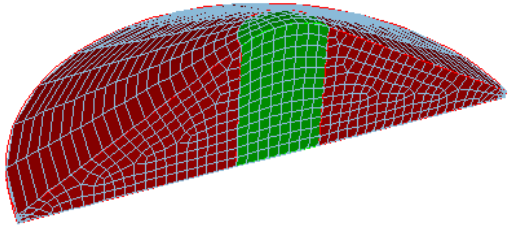


Mesh Scheme Selection

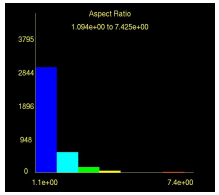


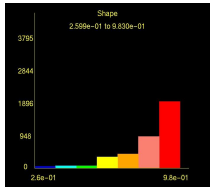


Interval Assignment



Mesh Generation





Mesh Quality Analysis

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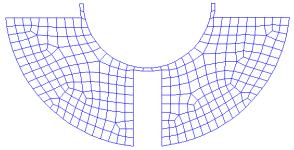
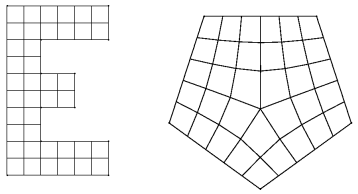
Hexahedral Mesh Generation

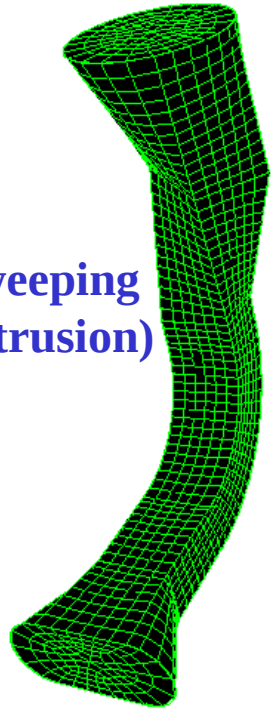
- *No "automatic" hex meshing algorithm known!*
- **Lots of semi-automated algorithms**

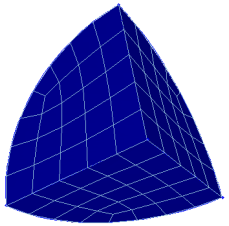
Quad (surface) algorithms





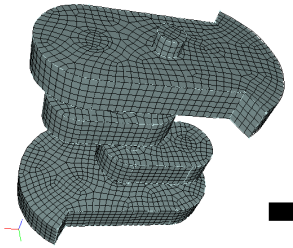
Sweeping (extrusion)



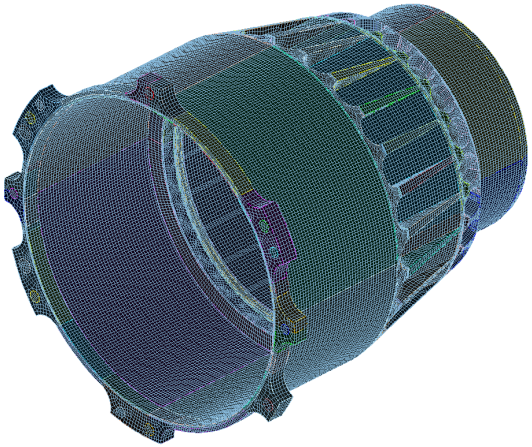


N-Side Primitives

Multisweep

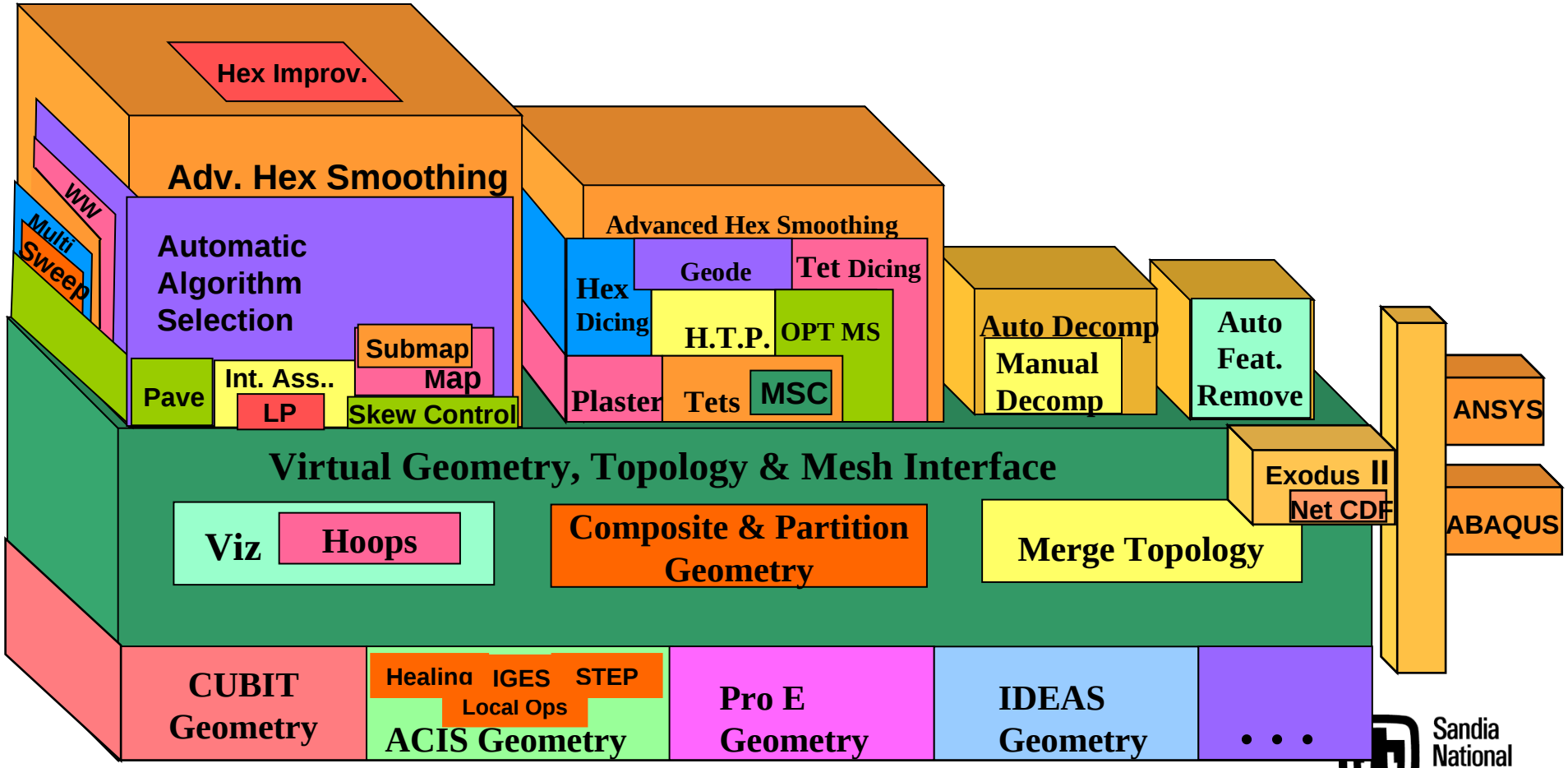


Combined Use in Large Assemblies



B61 Antenna Support
Structural Analysis
~258K Hex elements

The CUBIT Mesh Toolkit



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Day 2 Outline

- Geometry decomposition
- Example: assembly hex mesh generation
- Boundary conditions
- Exporting/translating mesh
- Importing to Ansys, running analysis
- Advanced topics in meshing
- Assignment

Geometry Decomposition

- General goal: split one volume into 2 using a cutting surface
- Variants: by how cutting surface is defined
  - Explicit vs implicit
  - Geometric shape
- ‘Webcut’ command used for most decomposition in CUBIT
- Geometric variants:
  - With plane; defined by:
    - Existing model surface
    - Three model vertices
    - Coordinate plane, possibly offset
  - With cylinder
  - With extended surface
  - With “tool” body

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Example, assembly mesh generation with decomposition

- Import acis ‘frust.sat’
- Cyl rad .05 hei .2
- Bod 2 rot 90 ab y
- Bod 2 mov x .3 y .1 z 2.38
- Bod 2 copy mov y -.2
- Unite all
- Section bod all yplane
- Webcut body all plan zplan offset 2.3
- Web bod all except 4 plan yplan off .2
- Imprint all
- Merge all
- Volume all size .1 scheme auto
- Mesh vol all

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Boundary conditions

- Three primary types of boundary conditions in meshing:
  - *Material:* not really a BC; groups elements for purposes of material type identification
    - *In CUBIT: element block*
  - *Dirichlet:* applied (usually) at nodes; assigns a dependent variable (e.g. Temperature) for a set of independent variables (e.g. spatial/node position)
    - *In CUBIT: nodeset*
  - *Neumann:* applied over lengths/areas/volumes; assigns derivative of dependent variable (e.g. dF/dA = pressure) for a set of independent variables (e.g. spatial positions over a mesh face)
    - *In CUBIT: sideset*

Exporting mesh

- **CUBIT commands:**
  - `[block | nodeset | sideset] <id> <geometry_entity_list>`
  - `Export genesis ‘<filename>’ [block <block_ids>]`
- **Examples:**
  - `Block 100 volume 1`
  - `Nodeset 2001 surface 1 2 3 curve 100 volume 3`
  - `Sideset 300 surface 2`
  - `Export genesis ‘test.g’`
- **Notes/gotchas:**
  - **Volume blocks created automatically if no blocks explicitly created; surface blocks not (can change default behavior with setting)**
  - **For Ansys, use nodesets for neumann BC’s (will explain later)**

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Exodus to Ansys

- ExodusII is a mesh format used as a standard throughout Sandia
- ExodusII translated to Ansys using a translator outside cubit:
  - `Exoans <exo_file> <ansys_BASE_name>`
- Produces 5 files:
  - *<base_name>* : Overall command file, which imports others
  - *<base_name>.node*: node definitions
  - *<base_name>.elem*: element definitions
  - *<base_name>.eset*: component *EB<block_id>* definitions, one component per element block
  - *<base_name>.nset*: component *NS<nodeset_id>* definitions, one component per nodeset
  - *<base_name>.sset*: component definitions for sidesets, not used right now
- To use in Ansys, read *<base_name>* in as commands from a file

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Exodus to Ansys

- In Ansys:
  - Elements defined with default material, or can assign using EB components
  - Dirichlet BC's can be assigned directly to components
  - Neumann BC's:
    - Could assign to element + "side"/load step, but that's not easy in translator
    - Can assign to all faces connected to specified group of nodes, using NS components
      - Must make sure that each of those faces included in higher dimension element (e.g. hex) only once
      - If that's not the case, use > 1 nodeset/NS component
    - To assign surface load:
      - Define nodeset on desired surface (CUBIT)
      - Select nodeset component, i.e. NSxxx (Ansys)
      - Specify NS component when specifying surface load

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Putting it all together: Ansys example

1. CUBIT:

(start with previous mesh)

- **Block 100 volume all**
- **Nodeset 100 surface 27 32**
- **Nodeset 200 surface 16**
- **Export genesis 'frust.g'**

2. Sun:

- **exoans frust.g frust**

3. Ansys:

- **Enter preprocessor (\prep7)**
- **Select default material**
- **File->Read Input from…**
  - **(select 'frust')**
- **Select->Comp/Assembly->Select Comp/Assembly… (select NS100)**
- **Loads->define loads->apply->structural->pressure->on nodes (select "Pick All")**
  - **Enter load value**
- **Select->Comp/Assembly->Select Comp/Assembly… (select NS200)**
- **Loads->define loads->apply->structural->displacement->on nodes (select "Pick All")**
  - **Select "all DOF", enter 0.0 for value**
- **Proceed with analysis (remember to select all elements before doing analysis)**

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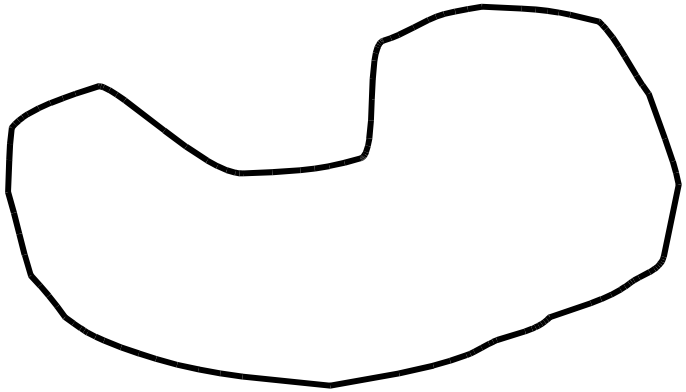
Advanced topics

Tetrahedral Mesh Generation
Delaunay Triangulation



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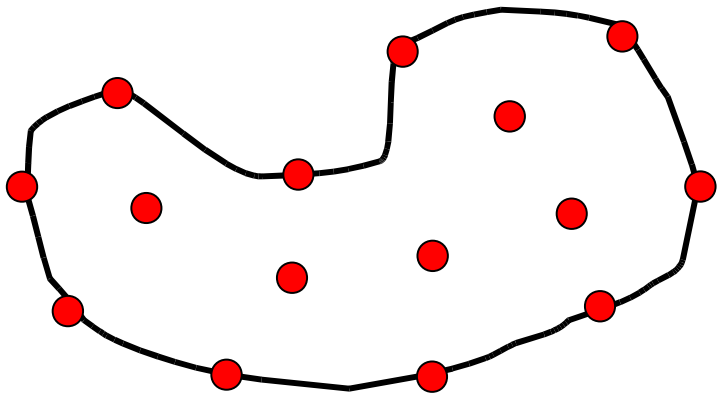
Tetrahedral Mesh Generation
Delaunay Triangulation

- **Point distribution, *P***



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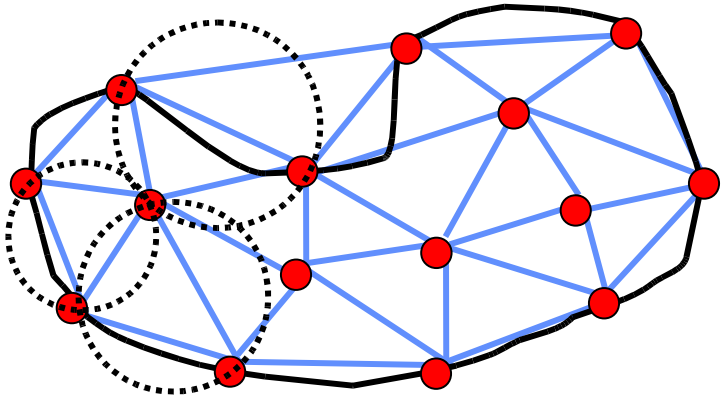
Tetrahedral Mesh Generation
Delaunay Triangulation

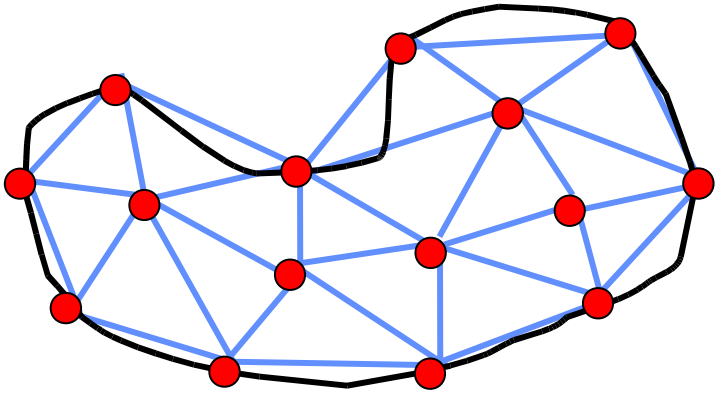
- Point distribution, *P*
- Triangulate points using "Delaunay criterion", *D(P)*



  - Delaunay criterion: circumcircle (2D) or circumsphere (3D) cannot contain any other point in P
  - Triangulation methods:
    - Point/triangle insertion: insert new point, remove elements whose circumcircles contain point, retriangulate region
    - Arbitrary starting point, refine until Delaunay criterion met
  - Transformations (diagonal flip) to improve mesh quality

Tetrahedral Mesh Generation
Delaunay Triangulation

- **Point distribution, *P***
- **Triangulate points using "Delaunay criterion", *D(P)***
- **Recover boundary**



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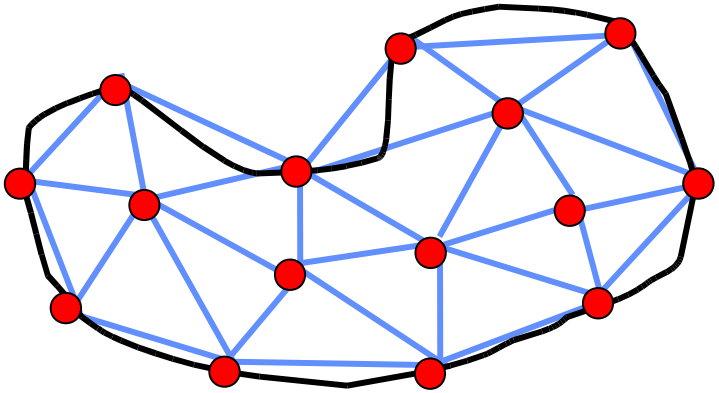
Tetrahedral Mesh Generation
Delaunay Triangulation

- **Point distribution, *P***
- **Triangulate points using "Delaunay criterion", *D(P)***
- **Recover boundary**
- **Why Delaunay? Quality guarantee:**



$$\forall T(P): \quad \alpha(D) > \alpha_{\min,Delaunay}$$

- **However, in 3D, Delaunay criterion allows "slivers"**
- **Almost everyone uses (modified) Delaunay:**
  - **Automatic**
  - **Anisotropic mesh from point distribution**
  - **Automatic**

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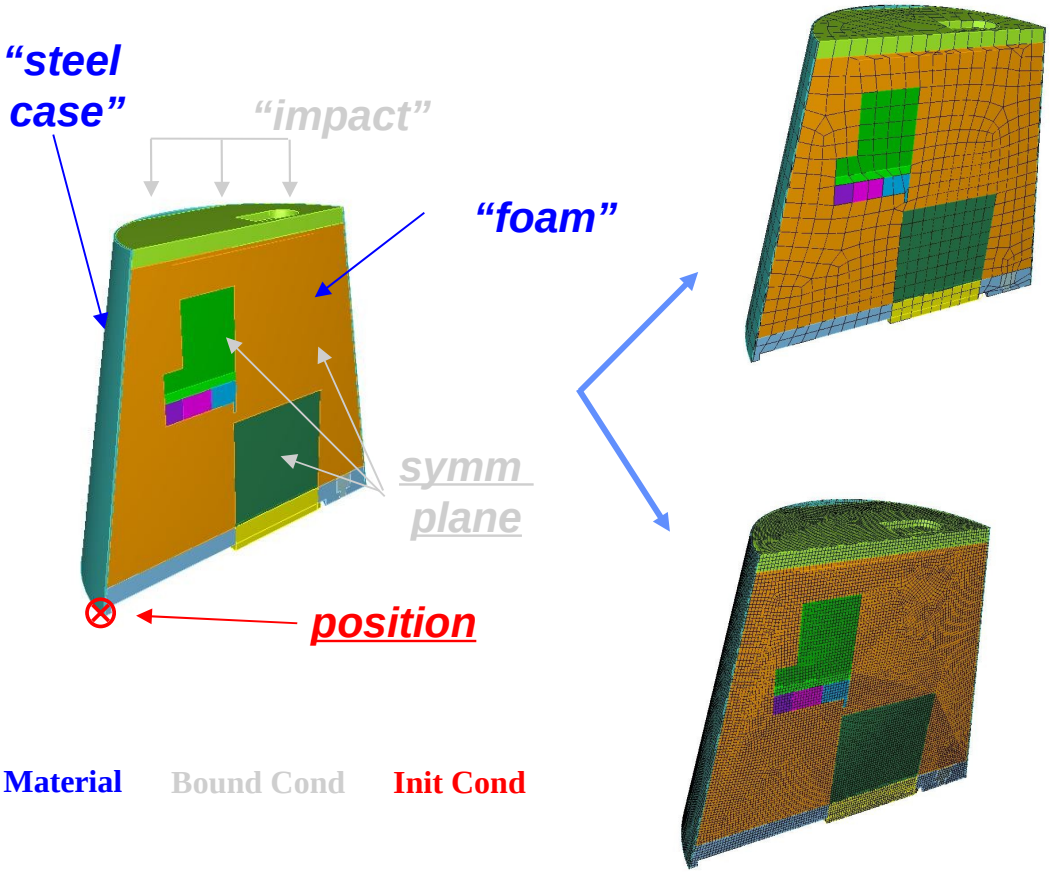
Key Technology:
A Geometry-Centric Process



- Why:

⇓interactive detail

Fine geom ≈ Coarse geom

Basis: initial CAD model

- Model generation:
  1. Start with well-defined *design model*
  2. Use simplification strategies to obtain *analysis model*
  3. Generate mesh

  …

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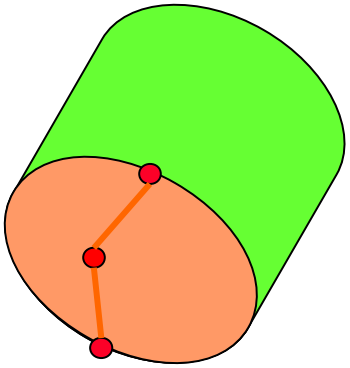
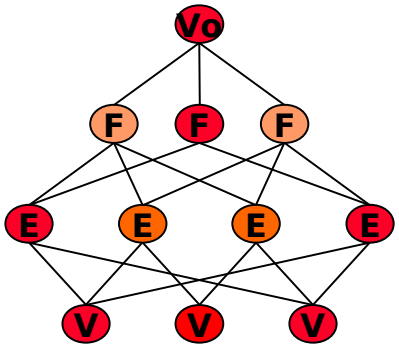
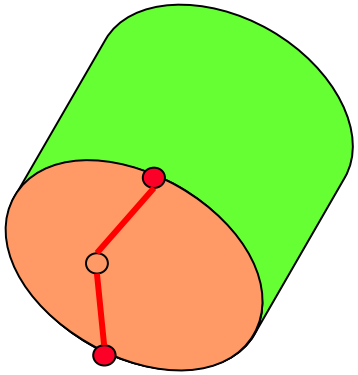
Virtual Topology
Composite Curve, Surface

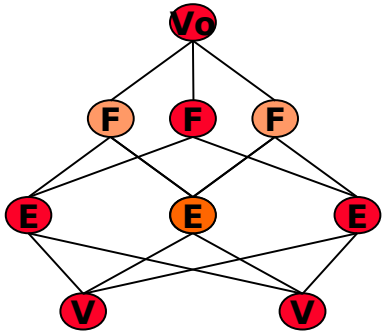
Original Geometry



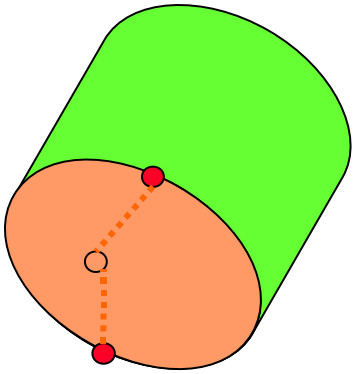


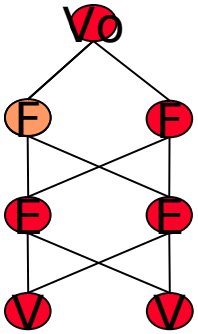
Composite Curve



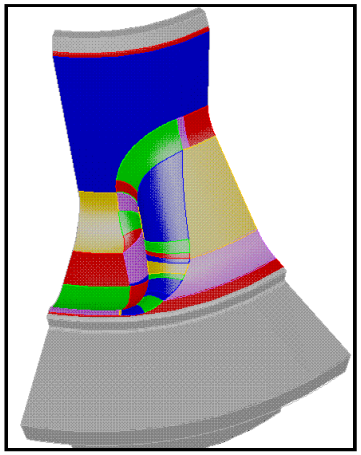


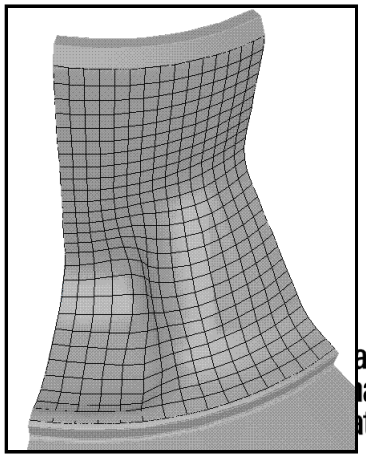
Composite Surface



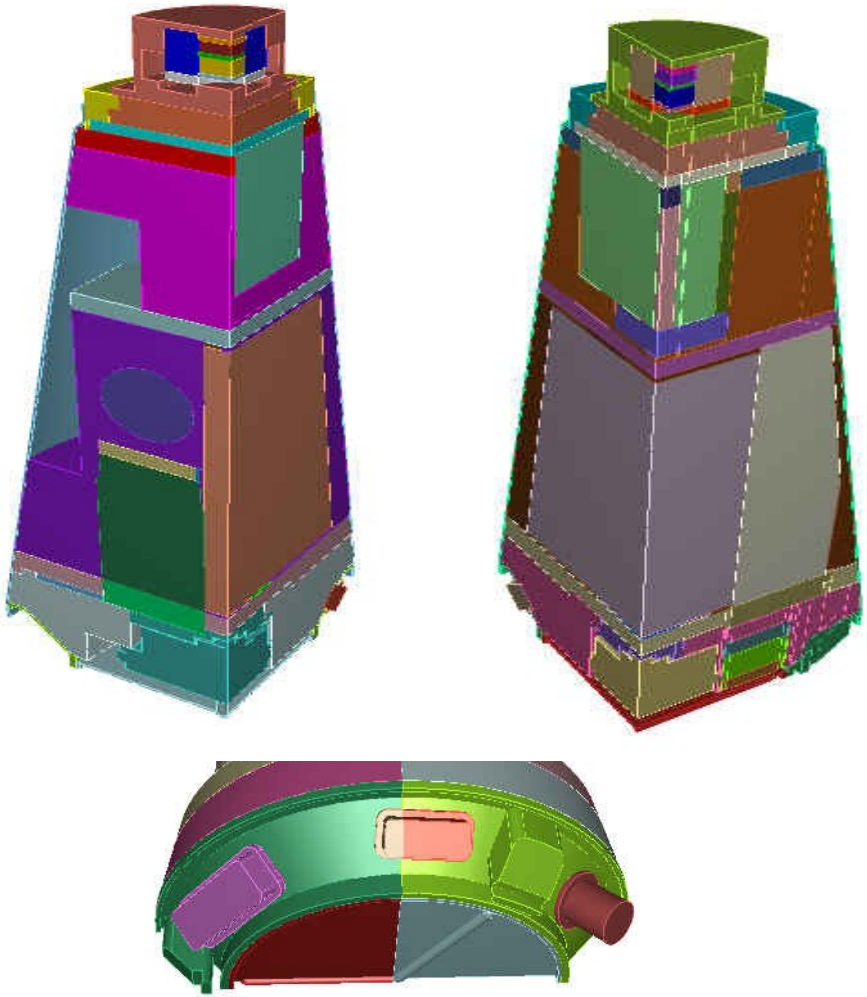


Caterpillar "wheel"





You Want Me To Mesh *THAT?*



- Geometry validity
- Geometric detail removal
- Tracking BC's, other attributes
- Handling large meshes

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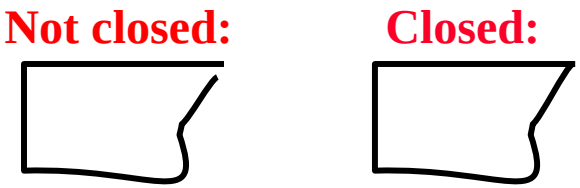
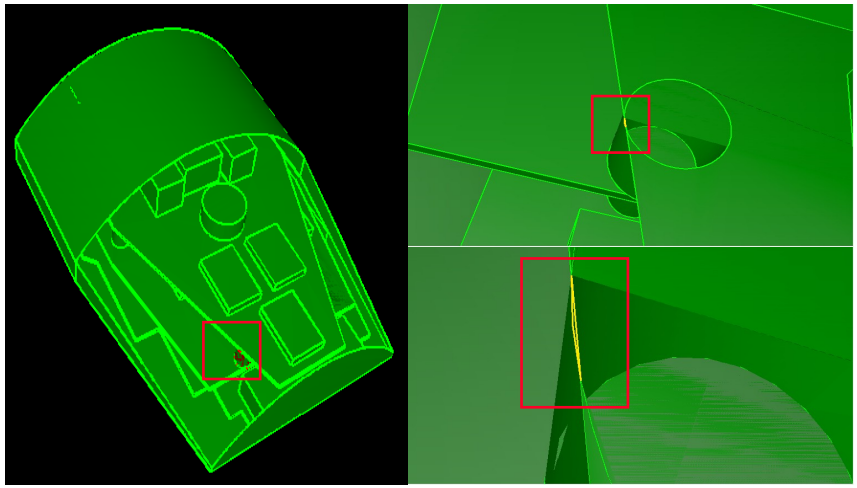
What's a "Well-Defined" Model?

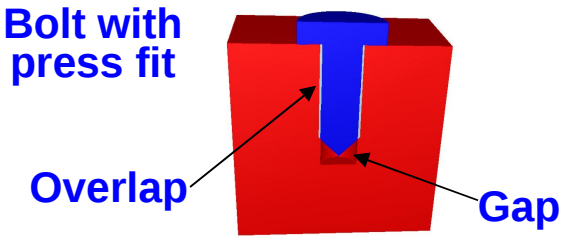
- **All volumes are closed, i.e. they "hold water"**



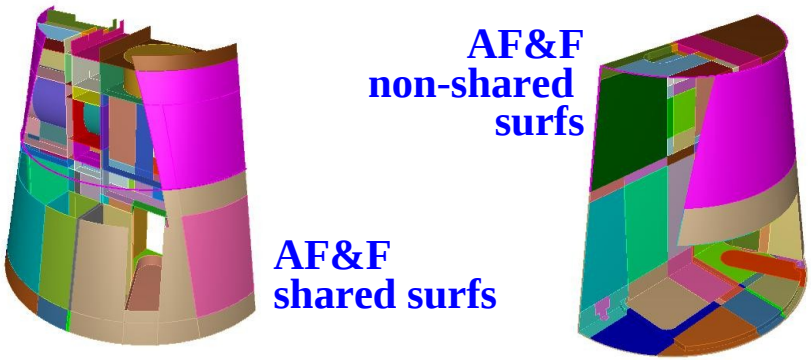
- **No "bad" geometry**



- No intra-volume gaps, overlaps

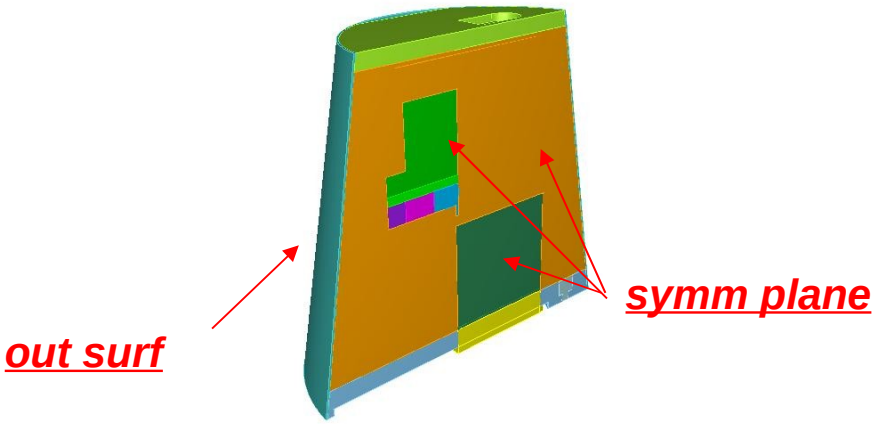


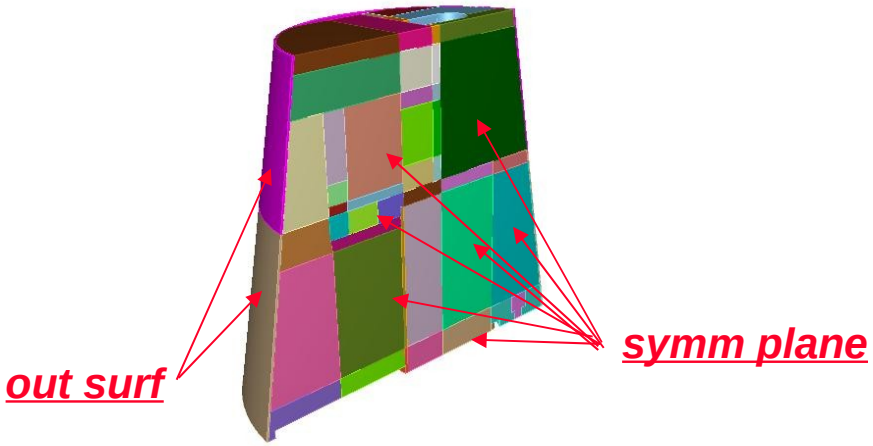
- Contiguity in multi-material models



Tracking Model through Simplification & Decomp.

- **Boundary conditions, material identifiers**





- **Strategies for accomplishing this propagation:**
  - **Names**
  - **BC groups (sidesets, nodesets, etc.)**
  - ***Requires user identification of BC handles - can we automate this?***
- **Automatic tracking makes D2A easier**

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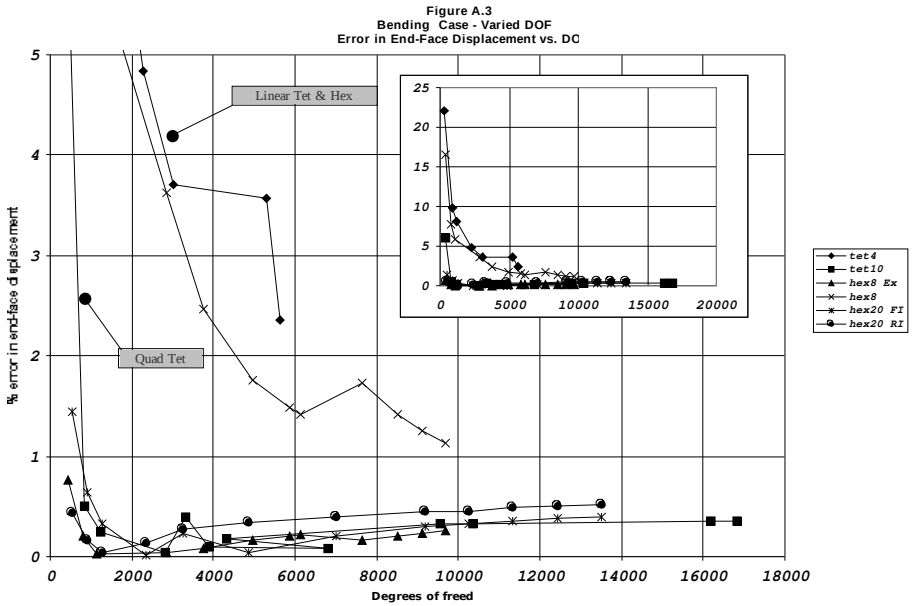
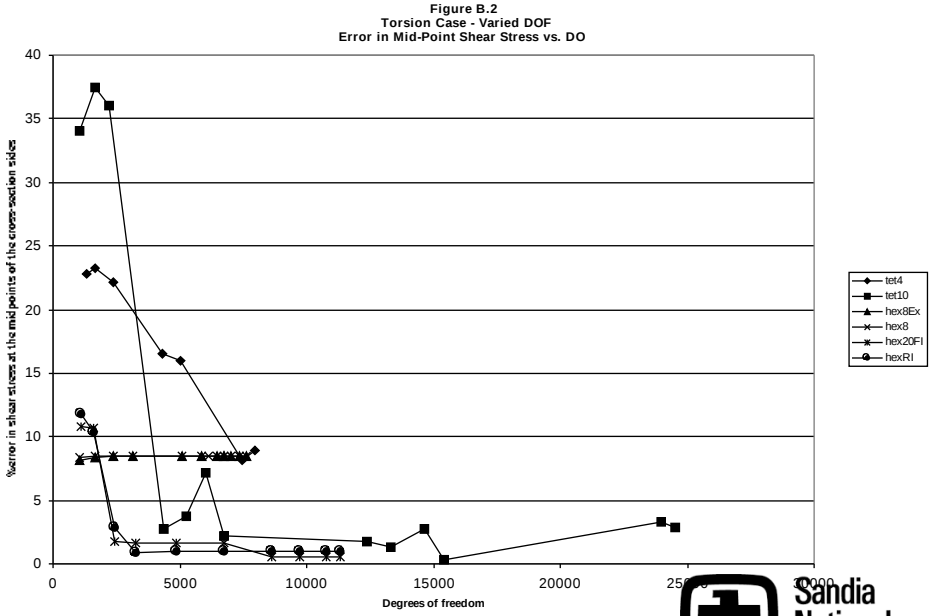
Comparative Study of Hex, Tet Element Accuracy

- **Being performed at CMU by Saigal & Owen (Ansys)**
- **Beam bending & torsion, poisson's ratio .3 - .5**
- **Comparing linear tets, hexes, quadratic tets, hexes, fully and partially (quadratic) integration**





- **Figures from Pakal et. al, "A Comparative Study of Hexahedral and Tetrahedral Elements", report to Sandia National Labs, 1999.**

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Homework 4, Problem 5

- **Run the Ansys analysis described ealier, *except:***
  - **Use a hole instead of a protrusion**
    - ***Hints:***
      - ***Can either construct protrusion cylinders with larger axial length, place properly, and use CUBIT 'subtract', OR***
      - ***Can webcut, either with implicit cylinder or by placing protrusion cylinders & using extended surface webcut, then use CUBIT 'delete body' to delete small cylinder(s)***
  - **Use material parameters given for problems 1-4 in homework 4**
  - **Find the peak stress when the shaft is subjected to an axial load of 1000 N.**

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