Meshing the Universe: In Situ Voronoi and Delaunay Tessellation

Overview
SDAV technologies aim to help cosmologists unravel the mysterious nature of dark matter and energy by transforming raw data into meaningful representations. For example, mesh tessellations help analyze point data because they transform sparse discrete samples into dense continuous functions. Similarly, large-scale structures such as halos and voids are extracted, tracked, and summarized in high-level models. The goal of SDAV’s partnership with computational cosmology is to bring such methods to extreme scale.

Voronoi tessellation of cosmological simulations reveals regions of irregular low-density voids amid clusters of high-density halos.

Algorithm
1. Compute local Voronoi tessellation.
2. Examine all circumspheres and identify those that extend past region boundaries.
3. Exchange points that support the circumspheres + points on the convex hull.
4. Update the local diagram with newly received points.
5. Exchange all points on the convex hull that are not finalized.
6. Finalize the Voronoi tessellation.

Points on the convex hull require special attention. Three solutions:
1. Send them everywhere.
2. Use supported halfspaces (see figure below): the halfspaces are the limits of circumspheres.
3. Use a heuristic: send to nearest neighbors, followed by a second phase, where still active points get sent everywhere.

Circumspheres: the green empty circumsphere is contained entirely inside the local region and, therefore, cannot change (the dual triangle is guaranteed to be in the Delaunay triangulation); the red circumspheres extend past the local region, their dual triangles are not finalized.

Key idea to reduce communication: we are affected by a point in a neighbor if and only if they are affected by our point. Therefore, we do not need to request points; all the destinations can be determined locally.

Cosmology Applications
Left: Tessellation of halo 6606326352 shows substructures inside the halo.
Right: Delaunay tessellation of 128^3 dark matter tracer particles.

Above: Density contrast distribution of evolving Voronoi cells at three time steps statistically are consistent with the formation of large-scale cosmological structures.

Above: Cosmology tools plugin in ParaView promotes interactive feature exploration.

Points on the convex hull, the external halfspaces they support (green), and their nearest neighbor blocks.

Problem with the old approach: fixed radius ghost region do not guarantee correctness for too small radius; create large overhead for too large radius.

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Three representations of the same halo. From left to right: original raw particle data, Voronoi tessellation, and regular grid density sampling.