

# New Insights into the Dynamic Stability of Wholesale Electricity Markets

Victor M. Zavala and Mihai Anitescu

**Abstract**— We summarize some recent results in the analysis of the dynamic stability of wholesale electricity markets. We discuss how to construct control-theoretic market analysis frameworks using concepts of market efficiency, Lyapunov stability, and predictive control. Such frameworks can be used to design, analyze, and monitor the stability and robustness properties of different market designs. In particular, we discuss how short forecast horizons, incomplete gaming, and physical ramping constraints can give rise to stability issues.

## I. INTRODUCTION

Understanding the sources of instability of electricity markets has significant economic implications. In particular, market instability leads to strong price fluctuations and to inefficient spread of social welfare among consumers and producers. Different market models have been derived in the past in trying to predict the presence of strong variations in prices resulting from dynamic forcings (e.g., weather, load, fuel prices, and wind supply), physical constraints (e.g., ramping, transmission congestion), and gaming behaviors (e.g., bidding strategies) [23]. These models range from data-based time-series models [21], [8] to mechanistic models based on agent-based systems [7], [22] and game-theoretical formulations [6], [14].

Among all these models, game-theoretical formulations enable one to understand not only how instability might arise but also how it can be prevented through better market designs. A widely used game-theoretical dynamic market model was originally proposed in [1], [2]. This model assumes that the market players (e.g., suppliers and consumers) bid recursively in time in the direction that minimizes their marginal cost. Every bidding step can be interpreted as a steepest-descent step that converges to a steady-state equilibrium as time evolves. While this model is useful for analyzing stability properties of the market equilibrium, it is based on mathematical rather than mechanistic assumptions and thus has limited applicability. Recently, a dynamic market model based on predictive control concepts was proposed in [13], [12]. Here, supply functions and receding horizon concepts are incorporated in the model. This provides a more natural representation of actual bidding procedures where the market players use forecast information in day-ahead and real-time markets. This model has been used to analyze the effect of dynamic disturbances such as wind on prices under high penetration levels. A limitation of this framework,

however, is that the dynamic model of the players is based on the marginal-cost descent assumption.

Mechanistic dynamic market models based on bidding and physical constraints considerations have also been proposed [9], [18]. These models can be used to explain how price fluctuations can arise from physical dynamic constraints such as generation ramping. Ramping constraints depend on multiple physical factors such as generator controller performance, thermal stresses, and wall capacitances [3], [4]. These dynamic constraints affect market performance in a similar way as transmission congestion does [11]. The difference, however, is that the effect of ramp constraints propagates forward in time and thus cannot be detected instantaneously. In [16], a game-theoretical framework was presented based on dynamic games and predictive control concepts. Using this model, the authors explained how ramping constraints propagate forward in time through the generation levels and thus affect long-term dynamic stability. The model also explained how increasing the foresight horizons can anticipate the ramping effects more effectively and decrease the price fluctuations.

A caveat of existing market analysis tools is the lack of a coherent framework that enables a systematic assessment of the stabilizing properties of different market designs. In [24], we proposed a control-theoretical framework using market efficiency, Lyapunov stability, and predictive control concepts. A market-specific Lyapunov function was derived using a summarizing state that measures the progress of the market efficiency. This Lyapunov function can be used to establish conditions under which a given market design can guarantee long-term stability. In particular, we used the framework to explain how incomplete gaming solutions (as those used in practice), short foresight horizons, and limited ramping capacity can lead to instability. These insights were used to propose a new stabilizing market design by making use of a stabilizing constraint for the market efficiency.

In this report, we summarize some of these findings. The paper is structured as follows. In Section II we present the market structure under consideration. In Section III we discuss implementation issues arising from incomplete gaming. In Section IV we derive a framework to analyze market stability properties. In Section V we present a numerical case study. In Section VI we provide concluding remarks and recommendations for future extensions.

## II. MARKET STRUCTURE

We first define the market structure under consideration and discuss the underlying modeling assumptions.

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V. M. Zavala and M. Anitescu are with the Mathematics and Computer Science Division, Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439 USA [vzavala@mcs.anl.gov](mailto:vzavala@mcs.anl.gov), [anitescu@mcs.anl.gov](mailto:anitescu@mcs.anl.gov)

## A. Suppliers

We consider a supply-function equilibrium market structure similar to those proposed in [15], [16]. Here, the supplier decisions are the parameters  $a_k^i, b_k^i$  of the affine supply function:

$$q_k^i(p_k, b_k^i, a_k^i) = b_k^i \cdot (p_k - a_k^i). \quad (1)$$

Here,  $q_k^i$  is the production quantity of supplier  $i \in \mathcal{S} := \{1..S\}$  at time  $k$ ;  $p_k \geq 0$  is the price at time  $k$ ; and  $a_k^i, b_k^i$  are the bidding coefficients at time  $k$  for supplier  $i$ . We assume that the supply function is nondecreasing in  $p_k$ . Consequently, we impose the requirement that  $b_k^i \geq 0$ . In our analysis, we assume that the generation quantities  $q_k^i$  and  $p_k$  are always non-negative. Consequently, we restrict the intercept parameter  $a_k^i$  to be non-negative as well. The supply function can also be expressed in inverse form as

$$p_k(q_k^i, b_k^i, a_k^i) = \frac{1}{b_k^i} q_k^i + a_k^i. \quad (2)$$

We observe that multiple combinations of  $a_k^i, b_k^i \geq 0$  can reach the same quantities or prices. Since this ill-posedness introduces difficulties in analyzing the properties of the supplier problem, we will assume that the intercept parameters  $a_k^i$  are zero. This assumption will not affect the analysis as long as the price is assumed to be non-negative. The consumer demands will be assumed to be fixed (inelastic) and denoted by  $d_k^j$ , where  $j \in \mathcal{C} := \{1..C\}$  is the set of consumers.

The supplier problem can be posed as follows. Starting at time  $k$ , given the price signals  $p_k$  over the future horizon  $\mathcal{T}_k := \{k..k+T\}$ , where  $T$  is the horizon length and the current states  $q_k^i, b_k^i$ , find the bidding parameter trajectories  $b_t^i, t \in \mathcal{T}_k$ , that maximize the future profit (revenue minus marginal cost). We can pose the supplier problem entirely in terms of the prices  $p_k$  and the supply function parameters  $b_k^i$ . In addition, we can interpret the bidding parameters  $b_k^i$  as the suppliers states. These modifications lead to the following equivalent formulation in state-space form:

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} \phi_t^i := \sum_{t \in \mathcal{T}_k} (p_t \cdot b_t^i \cdot p_t - c_t^i(b_t^i \cdot p_t)) \quad (3a)$$

$$\text{s.t. } b_{t+1}^i = b_t^i + \Delta b_t^i, t \in \mathcal{T}_k^- \quad (3b)$$

$$\underline{q}^i \leq b_t^i \cdot p_t \leq \bar{q}^i, t \in \mathcal{T}_k \quad (3c)$$

$$b_t^i \geq 0, t \in \mathcal{T}_k \quad (3d)$$

$$b_k^i = \text{given}, \quad (3e)$$

where  $\underline{q}^i, \bar{q}^i \geq 0$  are the lower and upper generation limits, respectively. We also have  $\mathcal{T}_k^- := \mathcal{T}_k \setminus \{k+T\}$ . The bidding increments  $\Delta b_k^i$  are interpreted as the control actions of the supplier. Note that these are unconstrained, implying that the suppliers can adjust their bids infinitely fast. A direct consequence is that the feasible set of the problem is invariant to the initial states  $b_k^i$ . In addition, the feasible set is invariant to the price signals  $p_k$  since it is always possible to find  $b_k^i \geq 0$  mapping any  $p_k$  to a feasible quantity  $q_k^i$ . Consequently, we denote the feasible set of this problem as  $\Omega^i$ .

The accumulated future profit is denoted by  $\sum_{t \in \mathcal{T}_k} \phi_k^i$ . The marginal cost function is assumed to have the form

$$c_k^i(q_k^i) = h_k^i \cdot q_k^i + \frac{1}{2} g_k^i \cdot (q_k^i)^2. \quad (4)$$

We make the common assumption that  $g_k^i > 0$  so the marginal cost is convex in  $q_k^i$  [20].

*Property 2.1:* If  $p_t \geq 0$  and  $g_t^i \geq 0, t \in \mathcal{T}_k$  then, problem (3) is convex. If  $p_t > 0$ , the problem has a feasible solution for any  $\underline{q}^i, \bar{q}^i \geq 0$ . If  $p_t = 0$ , the problem admits a solution only if  $\underline{q}^i = 0$ .

## B. ISO Market Clearing

The independent system operator (ISO) receives the bidding states  $b_k^i$  and clears the market by determining the generation quantities (and implicitly the prices) that balance total supply and demand. The main objectives of the ISO are to maximize social welfare and efficiency and to ensure market stability. The interaction between the ISO and the suppliers results in a game in which each player tries to maximize its own performance metric.

In our analysis, market stability will be interpreted as the ability to keep prices bounded from a given reference in the presence of dynamic fluctuations of demands and renewable supply and physical constraints. To account for this, we propose to use the basic concept of *market efficiency* as a measure of stability. To define efficiency, we first define an *ideal unconstrained* market clearing problem. This problem can be stated as follows. Given supply function states  $b_k^i$ , solve [6]:

$$\min_{q_t^i} \sum_{t \in \mathcal{T}_k} \bar{\varphi}_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq \quad (5a)$$

s.t.

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_t^j, t \in \mathcal{T}_k \quad (5b)$$

$$\underline{q}^i \leq q_t^i \leq \bar{q}^i, i \in \mathcal{S}, t \in \mathcal{T}_k, \quad (5c)$$

where

$$\int_0^{q_k^i} p_k(q, b_k^i) dq = \frac{1}{2} b_k^i (q_k^i)^2. \quad (6)$$

The objective function is the *negative social welfare*, denoted as  $\sum_{t \in \mathcal{T}_k} \bar{\varphi}_t$ . Since we have assumed that the consumers do not bid into the market, this reduces to the aggregated income of the suppliers. We have that  $\bar{\varphi}_t \geq 0$  since  $q_t^i, b_t^i \geq 0, t \in \mathcal{T}_k$ . The multipliers for the constraint (5b) are the prices  $\bar{p}_t \geq 0$ . Note that the feasible set of this problem is not affected by the bidding parameters, since they enter only in the objective function. In addition, in this unconstrained formulation, we assume that the generators can move infinitely fast between production levels (no ramp constraints). This assumption decouples the problem in time (i.e., each time step can be solved separately). Hence, the feasible set of this problem is invariant to the current state of the generators  $q_k^i$ . Nevertheless, the feasible set of this problem does depend on the demands. Accordingly, the

feasible set of this problem will be denoted as  $\Omega_{UNCO}^{ISO}(d_{\mathcal{T}_k}^j)$ , where  $d_{\mathcal{T}_k}^j := \{d_k^j, \dots, d_{k+T}^j\}$ .

*Property 2.2:* If  $b_t^i \geq 0, t \in \mathcal{T}_k$ , problem (5) is convex. The problem has a feasible solution if  $\sum_{i \in \mathcal{S}} q_t^i \leq \sum_{j \in \mathcal{C}} d_j^t \leq \sum_{i \in \mathcal{S}} \bar{q}^i$  holds. If  $b_t^i > 0$ , feasibility holds for any  $q_t^i, \bar{q}^i \geq 0$ . If  $b_t^i = 0$ , the problem admits a solution only if  $\underline{q}^i = 0$ .

For our analysis, we note that having infinitely fast dynamics in the generators is equivalent to assuming that their ramp capacities are equal to the distance between the maximum and minimum generation capacities  $\bar{q}^i - \underline{q}^i$ . Thus, we can pose (5) in the following equivalent state-space form:

$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \bar{\varphi}_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq \quad (7a)$$

s.t.

$$q_{t+1}^i = q_t^i + \Delta q_t^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \quad (7b)$$

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_j^t, \quad t \in \mathcal{T}_k \quad (7c)$$

$$-(\bar{q}^i - \underline{q}^i) \leq \Delta q_t^i \leq (\bar{q}^i - \underline{q}^i), \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \quad (7d)$$

$$\underline{q}^i \leq q_t^i \leq \bar{q}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k \quad (7e)$$

$$q_k^i = \text{given}, \quad i \in \mathcal{S}. \quad (7f)$$

The variables  $\Delta q_t^i$  are the generation ramp increments that are bounded by  $\pm(\bar{q}^i - \underline{q}^i)$ , the maximum generation ramp that is physically possible. Since problems (7) and (5) are equivalent, their feasible sets are the same. The multipliers of the constraints (8c) are the prices  $\bar{p}_t$ .

The solution of the unconstrained market clearing problem represents the *ideal* performance for the market (in the absence of ramping constraints). We now consider the dynamically *constrained* market clearing problem:

$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq \quad (8a)$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \quad (8b)$$

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_j^t, \quad t \in \mathcal{T}_k \quad (8c)$$

$$-\underline{r}^i \leq \Delta q_t^i \leq \bar{r}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \quad (8d)$$

$$\underline{q}^i \leq q_t^i \leq \bar{q}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k \quad (8e)$$

$$q_k^i = \text{given}, \quad i \in \mathcal{S}. \quad (8f)$$

The multipliers for the constraint (8c) are the prices  $p_t \geq 0$ . In this formulation, the ramps are bounded by  $\underline{r}^i, \bar{r}^i \leq (\bar{q}^i - \underline{q}^i)$ , respectively. This constrains the dynamic response of the generators. As before, we note that the bidding parameters  $b_t^i$  enter only the cost function and thus do not affect the feasible set. In this case, however, the dynamic constraints introduce time coupling because the ramp constraints might become active. Consequently, the feasible set does depend on the current state  $q_k^i$ . Accordingly, we denote the feasible set of this problem as  $\Omega_{UNCO}^{ISO}(q_k^S, d_{\mathcal{T}_k})$ , where  $q_k^S = \{q_k^1, \dots, q_k^S\}$ .

The constrained social welfare is denoted as  $\sum_{t \in \mathcal{T}_k} \varphi_t$  with  $\varphi_t \geq 0$  since  $b_t^i, q_t^i \geq 0, t \in \mathcal{T}_k$ . It is straightforward to prove that  $\sum_{t \in \mathcal{T}_k} \varphi_t \geq \sum_{t \in \mathcal{T}_k} \bar{\varphi}_t$  since  $\Omega_{UNCO}^{ISO}(q_k^S, d_{\mathcal{T}_k}) \subseteq \Omega_{UNCO}^{ISO}(d_{\mathcal{T}_k})$ . In other words, the performance of the constrained clearing problem is bounded by that of the unconstrained counterpart. We also have the following property, proven in [24].

*Property 2.3:* For fixed  $b_t^i \geq 0$ , the point social welfare  $\varphi_t$  evaluated at a solution of problem (8) and  $\bar{\varphi}_t$  evaluated at a solution of (5) satisfy  $\varphi_t \geq \bar{\varphi}_t, t \in \mathcal{T}_k$ .

We now formally define the *market efficiency* as

$$\eta_k := \frac{\bar{\varphi}_k}{\varphi_k}, \quad \forall k. \quad (9)$$

By definition and from Proposition 2.3, we have that  $\eta_k \in [0, 1]$ . The case where  $\eta_k = 1$  is achieved if  $\varphi_k = \bar{\varphi}_k$ . This case implies that the prices  $p_k$  are close to those of the unconstrained market clearing problem  $\bar{p}_k$ , which represents the ideal market performance. The case where  $\eta_k = 0$  occurs if the constrained social welfare diverges to infinity. This case occurs when the future demands cannot be met given the current states the generators and the ramping constraints. This implies that the prices  $p_k$  diverge from  $\bar{p}_k$  (i.e., a small change in demand leads to large changes in price). It is possible to show that the efficiency and price difference between the constrained and unconstrained games can be bounded by the magnitude of the ramp limits. This can be done using the following Lipschitz property (see [24]).

*Property 2.4:* If at a solution of the game (3) and (8), each of the optimization problems satisfy LICQ and the prices  $p_t, t \in \mathcal{T}$  and the production  $q_t^i, t \in \mathcal{T}, i \in \mathcal{S}$  values are large enough, then the solution is locally stable and the solution is a Lipschitz continuous function of the game data.

### III. IMPLEMENTATION ISSUES

To represent the game given by (3) and (8) in abstract form, we define the market states  $x_k$  as the set of quantities  $q_k^i$  and prices  $p_k$  and define the aggregated vector over the set  $\mathcal{T}_k$  as  $x_{\mathcal{T}_k} := \{x_k, \dots, x_{k+T}\}$ . The controls  $u_k$  are defined as the set of ramps for all suppliers  $\Delta q_k^i, i \in \mathcal{S}$  with  $u_{\mathcal{T}_k} = \{u_k, \dots, u_{k+T-1}\}$ . The bidding increments  $\Delta b_k^i$  are interpreted as the supplier controls and are denoted as  $w_k^i$ , and we define  $w_k := \{w_k^1, \dots, w_k^S\}$ . We define the disaggregated supplier vectors  $w_{\mathcal{T}_k}^i, i \in \mathcal{S}$  and the total aggregated vector  $w_{\mathcal{T}_k}$ . The bidding states  $b_k^i$  are interpreted as the supplier states  $z_k$  with aggregated vector  $z_{\mathcal{T}_k}$ . We include the problem data over the horizon (e.g., the demands) in the aggregated vector  $m_{\mathcal{T}_k}$ . We define the abstract dynamic system as

$$(x_{k+1}, z_{k+1}) = \phi_k(x_k, z_k, u_k, w_k), \quad \forall k \geq 0. \quad (10)$$

We can eliminate the states  $x_k, z_k$  by forward propagation of (10). With this, we can express the supplier and market clearing problem entirely in terms of the controls and initial

state conditions. We thus have the supplier problem,

$$\min_{w_{\mathcal{T}_k}} \sum_{t \in \mathcal{T}_k} \phi_t^i(w_t^i, u_t) \quad (11a)$$

$$\text{s.t. } w_{\mathcal{T}_k}^i \in \Omega^i, \quad (11b)$$

for  $i \in \mathcal{S}$  and the constrained market clearing problem,

$$\min_{u_{\mathcal{T}_k}} \sum_{t \in \mathcal{T}_k} \varphi_t(u_t, w_t) \quad (12a)$$

$$\text{s.t. } u_{\mathcal{T}_k} \in \Omega^{ISO}(x_k, m_{\mathcal{T}_k}). \quad (12b)$$

Since the decisions of the players do not affect each others feasible sets, the resulting game is a pure Nash equilibrium problem (see [10]).

For implementation, the game given by (11) and (12) can be solved over a receding horizon. This can be done as follows. At time  $k$  we use the forecast data  $m_{\mathcal{T}_k}$  (e.g., demands  $d_t^j$ ,  $t \in \mathcal{T}_k = \{k..k+T\}$ ) and the current states  $x_k, z_k$ . We solve the game (11) and (12) over the horizon  $\mathcal{T}_k$  to obtain  $u_{\mathcal{T}_k}^*, w_{\mathcal{T}_k}^*$ . From these sequences, we extract only the first actions  $u_k \leftarrow u_k^*, w_k \leftarrow w_k^*$ . The system will evolve from its current state  $x_k, z_k$  into the states  $x_{k+1}, z_{k+1}$  according to the model (10). In the nominal case (no forecast errors in the data  $m_{\mathcal{T}_k}$ ), the state will evolve as predicted. At the next step  $k+1$ , we introduce feedback in the market by shifting the horizon of the game to obtain  $\mathcal{T}_{k+1} \leftarrow \{k+1..k+T+1\}$  and use the new state  $x_{k+1}, z_{k+1}$  as initial conditions. The new data  $m_{\mathcal{T}_{k+1}}$  is forecast and the game problem is solved to obtain the new decisions  $u_{k+1}, w_{k+1}$ . This approach generates the feedback law  $(u_k, w_k) = h(x_k, z_k, m_{\mathcal{T}_k})$ .

Note that, even in the nominal case, feedback is required because the horizon  $T$  is usually finite (i.e., at time  $k$  it is not possible to foresee demands beyond time  $k+T$ ). This implementation strategy that solves the game over a receding horizon is intuitive but it is not used in practice. This might be because of constraints in information exchange and in decision times.

The current strategy used in practice iterates *once* between the suppliers and the ISO in a distributed manner (see [19], [5]). Here, each supplier guesses the ISO states (e.g. prices) or, implicitly, its decisions. This can be done, for instance, by using price forecasting. This guess is denoted by  $u_{\mathcal{T}_k}^\ell$ , where  $\ell$  is an iteration counter. The suppliers compute bidding parameters  $w_{\mathcal{T}_k}^\ell$  by solving (11). These are sent to the ISO to solve the market clearing problem (12) to update the decisions  $u_{\mathcal{T}_k}^{\ell+1}$ . This strategy can be interpreted as a single Jacobi-like iteration (see [10]).

The Jacobi iterate  $u_{\mathcal{T}_k}^{\ell+1}, w_{\mathcal{T}_k}^{\ell+1}$  is feasible but not optimal for the game. Feasibility follows since the suppliers decisions  $w_{\mathcal{T}_k}$  do not enter the feasible set  $\Omega^{ISO}(\cdot, \cdot)$  and since the supplier problems always have a feasible solution for any feasible decision of the ISO  $u_{\mathcal{T}_k}$ . This suboptimal strategy is an *incomplete gaming* strategy between the suppliers and the ISO. A key observation is that *the resulting incomplete gaming error generated at each step* results in suboptimal control actions  $(u_k, w_k)$  that are propagated forward

in time through the dynamic system (10). This introduces additional error dynamics into the market that can lead to instability. For instance, the suboptimal gaming solution  $u_k, w_k$  obtained at time  $k$  might place the generators at a future state  $x_{k+1}, w_{k+1}$  from which the future demands  $\{d_{k+1}^j, \dots, d_{k+1+T}^j\}$  can only be in a suboptimal manner (e.g., using expensive generators) or not reached at all, leading to load shedding.

#### IV. DYNAMIC STABILITY ISSUES

Stability, in the context of wholesale electricity markets, reflects strong fluctuations and divergence of prices. Traditional control-theoretic stability analysis tools are not directly applicable in this context because the market is inherently dynamic and does not exhibit a natural equilibrium for the states. While it is possible to design market clearing procedures (these can be viewed as *market controllers*) that artificially introduce equilibria (i.e., by enforcing periodicity in some form), this strategy can constrain and degrade market performance. New stability analysis tools are thus needed to enable a systematic design, analysis, and implementation of *robust* and stabilizing market clearing procedures that can sustain market manipulation and strong dynamic variations of demands and renewable supply. In this section, we take a first step toward this goal by making use of a market-specific Lyapunov stability framework.

We can express the market efficiency as an implicit function of the states of the form,  $\eta_k(x_k, z_k)$  or  $\eta_k$  for short-hand notation. Here, we use the following definition of market stability.

*Definition 4.1:* The market system defined by the game (11) and (12) is said to be *stable* if, given  $\eta_0 \in \Omega^\eta(\epsilon) := \{\eta \mid \eta \geq \epsilon\}$  with  $\epsilon \in [0, 1]$ , there exist sequences  $u_k, w_k$  such that  $\eta_k \in \Omega^\eta(\epsilon)$ ,  $k = 0..∞$ .

Here,  $\epsilon$  is an *efficiency threshold value*. We note that efficiency is a state derived from the system physical states. From (9) and (8a), we can see that the states of ISO and of the suppliers  $x_k, z_k$  can be detected through the efficiency  $\eta_k$ .

The market efficiency implicitly sets a measure of stability for the prices. We propose to measure price stability as the distance between the prices of the constrained and unconstrained market clearing problems  $|p_t - \bar{p}_t|$ . Having such a relative measure is important since high efficiencies do not necessarily imply large prices and vice-versa. We now define the *summarizing market state*:

$$\delta_{k+1} := (1 - (\eta_{k+1} - \epsilon)) \cdot \delta_k, \quad k = 0..∞, \quad (13)$$

with initial conditions  $\delta_0 := (1 - (\eta_0(x_0, z_0) - \epsilon)) \geq \alpha > 0$ . We can also use  $\delta_0 := (1 - (\eta_0 - \epsilon)) \cdot \mu$  with  $\mu > 0$  as long as  $\eta_0 \geq \epsilon$ . If  $\eta_k \geq \epsilon$ ,  $k = 0..∞$ , then for any  $\alpha > 0$  such that  $\delta_0 \geq \alpha$ , there exists  $\kappa \geq 0$  such that  $\delta_k \rightarrow \kappa$  for all  $k = 0..∞$ . In other words, the summarizing market state has a stable origin. Stability of this origin implies market stability in the sense of Definition 4.1. On the other hand, if at any step we have  $\eta_k < \epsilon$ , the summarizing market state will increase. Subsequent violations of the efficiency threshold

will make the summarizing state diverge from the origin. We note that the states  $x_k, z_k$  can be detected through the efficiency  $\eta_k$  and that the efficiency can in turn be detected through the summarizing state  $\delta_k$ .

For clarity, we summarize the sequence of dependencies as follows.

- Knowing states  $x_k, z_k$  and the data  $m_{\mathcal{T}_k}$ , defines  $\eta_k(x_k, z_k)$  and  $\delta_k$ .
- The control actions can be computed to give the  $(u_k, w_k) = h(x_k, z_k, m_{\mathcal{T}_k}) := \tilde{h}(\delta_k)$ .
- The states evolve as (10) or

$$\begin{aligned} (x_{k+1}, z_{k+1}) &= \phi(x_k, z_k, h(x_k, z_k, m_{\mathcal{T}_k})) \\ &= \tilde{\phi}(x_k, z_k, m_{\mathcal{T}_k}). \end{aligned}$$

This defines  $\eta_{k+1}(\tilde{\phi}(x_k, z_k, m_{\mathcal{T}_k}))$ .

- The summarizing state evolves as

$$\begin{aligned} \delta_{k+1} &= \left(1 - \left(\eta_{k+1}(\tilde{\phi}(x_k, z_k, m_{\mathcal{T}_k})) - \epsilon\right)\right) \cdot \delta_k \\ &:= f(\delta_k, \tilde{h}(\delta_k)) := \tilde{f}(\delta_k). \end{aligned}$$

Using this basic set of definitions, we now illustrate how to establish sufficient stability conditions for a given market design. In addition, we demonstrate that the current market design given by the incomplete solution of the game (11) and (12) is not stabilizing.

We propose to extend the market clearing problem (12) by using the definition of the summarizing state as follows.

$$\min_{u_{\mathcal{T}_k}^-} \sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) \quad (14a)$$

$$\text{s.t. } u_{\mathcal{T}_k} \in \Omega^{ISO}(x_k, m_{\mathcal{T}_k}) \quad (14b)$$

$$\delta_{t+1} = (1 - (\eta_{t+1} - \epsilon)) \cdot \delta_t, \quad t \in \mathcal{T}_k^- \quad (14c)$$

$$\eta_t \geq \epsilon, \quad t \in \mathcal{T}_k \quad (14d)$$

$$\delta_k = \text{given}. \quad (14e)$$

The objective function of this market clearing problem will be used as a *summarizing market function*, which we define formally as

$$V_T(\delta_k) := - \sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) = (\delta_k - \delta_{k+T}). \quad (15)$$

A crucial observation is that the summarizing market function can be used as a Lyapunov function that we can use to establish stability of the origin for the summarizing state  $\delta_k$ . To prove this, we first make the following definition.

**Definition 4.2:** A function  $V_T(\delta_k)$  is a Lyapunov function for system  $\delta_{k+1} = \tilde{f}(\delta_k)$  if (1) it is positive definite: in a region  $\Omega$  containing the origin if for  $\delta_k \in \Omega$  we have  $V_T(\delta_k) \geq 0$  for  $\delta_k \geq 0$  for all  $k$ , and (2) it is nonincreasing:  $\Delta V_T(\delta_k) \leq 0$ , for all  $k$ .

We now establish stability following the traditional approach of using the cost function of the controller (in this case market clearing problem) as a Lyapunov function [17].

**Theorem 4.3:** If the game given by (11) and (14) with  $T = \infty$  has a feasible solution  $\forall k$ , then the market is stable.

**Proof:** From feasibility of (14d) we have that  $-(\delta_{t+1} - \delta_t) \geq 0$ ,  $t \in \mathcal{T}_k^-$  so  $V_T(\delta_k) = \sum_{t \in \mathcal{T}_k^-} -(\delta_{t+1} - \delta_t) \geq 0$ .

Consequently, positive definiteness follows. To prove that the function is nonincreasing, we consider the cost function of two consecutive problems generating two trajectories  $\delta_t^k$ ,  $t \in \{k..k+T\}$  and  $\delta_t^{k+1}$ ,  $t \in \{k+1..k+1+T\}$  with  $T = \infty$ ,  $\delta_k^k = \delta_k$  and  $\delta_{k+1}^{k+1} = \delta_{k+1}$ . We then have

$$\begin{aligned} \Delta V_T(\delta_k) &= V_\infty(\delta_{k+1}) - V_\infty(\delta_k) \\ &= \sum_{t=k+1}^{\infty} (\delta_{t+1}^{k+1} - \delta_t^{k+1}) - \sum_{t=k}^{\infty} (\delta_{t+1}^k - \delta_t^k) \\ &= (\delta_{k+1} - \delta_\infty^{k+1}) - (\delta_k - \delta_\infty^k) \\ &= (\delta_{k+1} - \delta_k) \\ &= (1 - (\eta_{k+1} - \epsilon)) \cdot \delta_k - \delta_k \\ &= -(\eta_{k+1} - \epsilon) \cdot \delta_k \\ &\leq 0. \end{aligned}$$

The bound come from the fact that  $\delta_\infty^{k+1} = \delta_\infty^k$ . The final inequality comes from feasibility. The cost function is a Lyapunov function. This implies that the summarizing state has a stable origin and the market is stable.  $\square$

With this, we have established that the decay of the summarizing function is a sufficient condition for market stability. We note that if at any point we have that  $\eta_k < \epsilon$ , then  $\delta_{k+1} > \delta_k$ , and the decay condition will not hold.

A crucial observation in our analysis is the need for the stabilizing constraint (14d). With this, the feasible set of the market clearing problem depends on the bidding states of the suppliers. A consequence is that the ISO and the suppliers might need to iterate several times (e.g., in a Jacobi manner) to be sure of obtaining a feasible solution to the game. Another consequence of this analysis is the fact that the existing market design where a single iterate is performed between the ISO and the suppliers *cannot be guaranteed to be stable* in the sense of Definition 4.1 since not every set of bidding parameters can be guaranteed to lead to a market clearing solution satisfying the stabilizing constraint. In other words, the current market design does not enable the ISO to reiterate the bidding quantities with the suppliers to stabilize the market. Hence, the market is more prone to be destabilized by the suppliers if these do not have appropriate means to anticipate the ISO decisions before bidding (e.g., by price forecasting). Finding a feasible solution to the game (11) and (14) avoids these problems. Our construct provides a mechanism to design and analyze market designs with stability guarantees.

## V. NUMERICAL CASE STUDY

In this section, we illustrate the effect of ramping constraints, foresight horizon, and incomplete gaming solutions on market stability and price dynamics. We consider a market system with three suppliers and one demand. One of the suppliers has fast dynamics (high ramping capacity) but high cost such as natural gas generators, the second one has slow dynamics but also low cost such as a coal generator, and the third one is used as a slack generator with infinite ramp limits (equal to generation capacity) and a large cost.

This last supplier acts as a slack to avoid infeasibility. The nominal parameters used are  $\underline{q} = [0, 0, 0]$ ,  $\bar{q} = [50, 70, 120]$ ,  $\underline{r} = -[5, 10, 120]$ ,  $\bar{r} = [5, 10, 120]$ ,  $h = [4, 2, 5]$ , and  $g = [2, 1, 5]$ . We used  $q_0 = [0, 40, 40]$  as initial conditions. We consider the demand profile presented in Fig. 1, which is obtained from a periodic signal perturbed with Gaussian noise. We set the market stability threshold to  $\epsilon = 0.65$ .

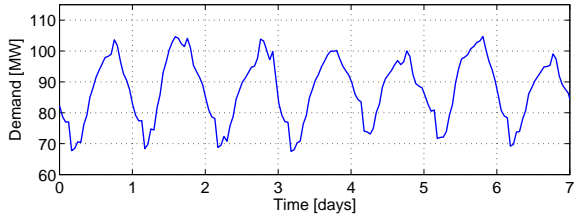


Fig. 1. Demand profile used for numerical case study.

To illustrate the main developments of the paper, we consider three market implementations. The first one uses a foresight horizon of six hours and performs a single Jacobi-like iteration at each clearing time (incomplete gaming). This implementation is labeled as ( $T = 6Jac$ ) and represents current practice. The second implementation uses the same horizon length, but the game is converged to optimality ( $T = 6Opt$ ) satisfying the stabilizing constraint. The third implementation uses an horizon of 24 hours, and the game is converged to optimality ( $T = 24Opt$ ). To compute the reference social welfare and prices, we also implemented an unconstrained market clearing procedure.

In Fig. 2 we present the profiles of the summarizing state  $\delta_k$  for the three market implementations, in Fig. 3 we present efficiency profiles  $\eta_k$ , and in Fig. 4 we present the resulting clearing price signals  $p_k$ . From Fig. 2 it is clear that the summarizing state obtained from the suboptimal implementation  $T = 6Jac$  is not strictly decreasing during days 1 and 3 and thus its market clearing cost cannot be used as a Lyapunov function. This indicates that the efficiency is crossing the threshold at certain times, as can be observed in Fig. 3. This clearly illustrates that incomplete gaming can introduce market instability. The other two control implementations remain stable, but, as expected, a longer horizon improves performance. This is observed from the faster decay of the summarizing state for  $T = 24Opt$  when compared with  $T = 6Opt$  and from the efficiency profiles. The efficiencies of  $T = 24Opt$  remain farther away from the threshold. This illustrates that the length of the foresight horizon can have important effects on market stability. The reason is that longer horizons can anticipate and manage ramping constraints more efficiently.

In Fig. 4 we observe the spikes in the prices for  $T = 6Jac$  during the first hours of the simulation and during the third day. In particular, note the strong price fluctuations when compared with the optimal unconstrained prices. These prices were obtained from the solution of the unconstrained market clearing problem. Note that in the absence of ramping constraints, the prices remain stable and nearly periodic. On the other hand, when the ramp constraints are active,

strong price variations are observed. In particular, during the third day, the prices for  $T = 6Jac$  reach levels of 150\$/MW. The prices of  $T = 24Opt$  stay well below 100\$/MW and much closer to the optimal unconstrained prices. These levels are a consequence of having a longer foresight horizon and converging the game to optimality to ensure that the efficiency is above the stability threshold. As a quantitative result, we computed the sum of squared errors  $SSE = \sum_t |p_t - \bar{p}_t|^2$  over the entire simulation horizon of 7 days. Here,  $p_t$  are the constrained price signals, and  $\bar{p}_t$  are the unconstrained price signals. For  $T = 6Jac$  we obtained  $SSE = 2.16 \times 10^5$ , whereas for  $T = 24Opt$  we have  $SSE = 4.19 \times 10^4$ , an improvement of nearly an order of magnitude. We have also observed that performing an extra Jacobi-like iteration for  $T = 6Jac$  stabilizes the prices. In addition, we have observed that extending the horizon of  $T = 24Opt$  does not improve its performance significantly.

In Fig. 5 we present price profiles for  $T = 6Jac$  and  $T = 24Opt$  with relaxed ramp constraints. In this case, we increased the ramp limits from their nominal values to  $\underline{r} = -[10, 20, 120]$ ,  $\bar{r} = [10, 20, 120]$ . As can be seen, the price signals for both implementations are close to those of the unconstrained clearing problem. The signals of  $T = 24Opt$  get closer to the unconstrained reference faster because of a combined effect of complete gaming and forecast horizon. In particular, we observe that  $T = 6Jac$  performs well in this case. The reason is that when the ramp limits are relaxed, subsequent gaming solutions become closer to each other.

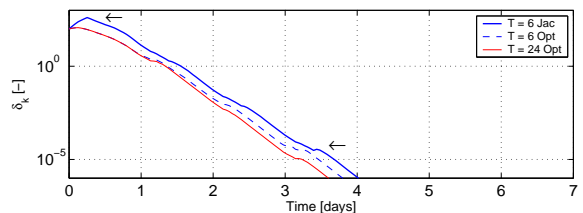


Fig. 2. Summarizing state for market implementations.

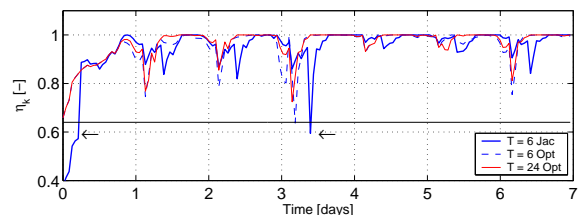


Fig. 3. Efficiencies for market implementations.

## VI. CONCLUSIONS AND CHALLENGES

In this report, we illustrate how to construct market analysis frameworks using market efficiency concepts, Lyapunov analysis, and predictive control concepts. Such frameworks can be used to explain how market stability issues arise as a result of poor market designs that allow for incomplete gaming between the ISO and the suppliers and that use

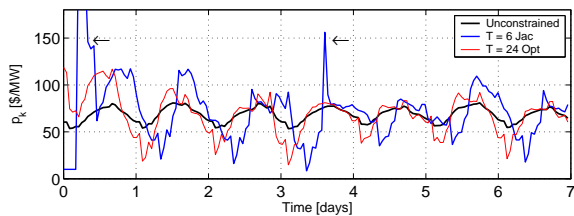


Fig. 4. Clearing prices for market implementations.

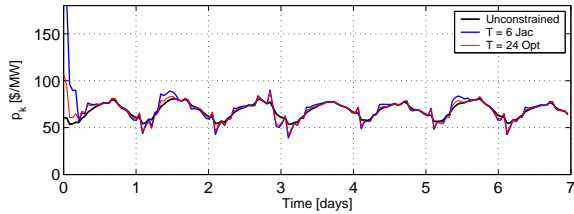


Fig. 5. Clearing prices for market implementations under relaxed ramp constraints.

short foresight horizons. The definition of stability in terms of market efficiency is general and can be extended to consider network constraints, piecewise supply functions, and Cournot games. In any of these developments, however, we believe it is critical to use a consistent framework such as that presented here to compare the stability and robustness properties of different market designs. This will enable more systematic market design procedures.

The issue of incomplete gaming opens the door to several questions regarding convergent distributed approaches to implement the bidding-clearing procedure in real time. From a stability point of view, it is necessary to understand how stability can be improved by defining markets at different time scales (e.g., forward and real-time) and through stochastic formulations. In addition, it is necessary to establish more general stability conditions under finite horizons, forecast errors, and non gaming behaviors. Finally, we note that the framework can be extended to analyze the effect of consumer elasticity on stability.

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