

# Fast Poisson Solvers and the Fokker-Planck Collision Operator

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A variety of collisional kinetic theories require the computation of the Coulomb collision operator. This can be expressed succinctly in terms of Rosenbluth potentials as

$$C_{ab} = \partial_{\mathbf{v}} \cdot \left[ \partial_{\mathbf{v}} \cdot (f_a \partial_{\mathbf{v}} \partial_{\mathbf{v}} \Phi_b) - 2 \left( 1 + \frac{m_a}{m_b} \right) f_a \partial_{\mathbf{v}} \Psi_b \right],$$

where the Rosenbluth potentials  $\Phi_b, \Psi_b$  are given by

$$\Phi_b(\mathbf{v}) = \int f_b(\mathbf{v}') \|\mathbf{v} - \mathbf{v}'\| d\mathbf{v}', \quad \Psi_b(\mathbf{v}) = \int \frac{f_b(\mathbf{v}')}{\|\mathbf{v} - \mathbf{v}'\|} d\mathbf{v}',$$

$\mathbf{v}$  is the vector of velocity coordinates and  $f$  is a given distribution function. While  $\Phi$  and  $\Psi$  are simply biharmonic and Coulomb potentials, there are three areas of difficulty in such computations. First,  $\Phi$  and  $\Psi$  are often defined by an axisymmetric density  $f(\mathbf{v})$ , requiring a fast solver in cylindrical coordinates. Second,  $\Phi$  and  $\Psi$  are defined in all of space and appropriate boundary conditions need to be imposed. Finally, the construction of the collision operator involves four derivatives of  $\Phi$  and two derivatives of  $\Psi$ .

We have developed a method for computing the Rosenbluth potentials that overcomes these difficulties, using separation of variables and Fourier analysis. We show that a weak singularity arises in the Fourier transform that can be handled very effectively with special purpose quadratures and that spectral accuracy can be achieved in all derivatives without loss of precision.

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