

Nonconvex Compressive Sensing

Getting the most information from the fewest data (and the other way around)

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This talk will feature recent developments that move us closer to the paradigm of processing information instead of data. The young field of compressive sensing arose from the discovery that the seemingly intractable “needle in a haystack” problem of finding the sparsest solution of a severely underdetermined linear system of equations can, in fact, typically be solved via convex optimization. This leads immediately to new image or signal reconstruction methods that are successful with surprisingly few measurements, which in turn leads to data acquisition methods that effect compression as part of the measurement process (hence “compressive sensing”). Our contribution is to show that the number of measurements required to be able to reconstruct an image or signal is even lower if one replaces the convex problem that has been used with a nonconvex one. The nonconvex problem has a huge number of local minima, yet simple, local minimization algorithms, implemented properly, converge in practice to the global minimum.

The recent ASCR workshop on Mathematics for the Analysis of Petascale Data highlighted the challenges posed by the vast scale of scientific data sets. The approach made possible by compressive sensing is to only collect or generate the minimum amount of data necessary to capture the knowledge they contain. Applications include replacing high-dose CT scans with just a handful of X-rays without losing diagnostic power, the ability to reduce data sizes in remote sensing or astronomical imagery by simply measuring less in the first place, the ability to compress streaming data without having to compute with the data in hand, and the ability to infer the state of a large sensor network from just a few measurements on the periphery. Meeting the challenges of putting the methods into realistic practice will require substantial applied mathematics research, incorporating high-dimensional geometry and analysis, optimization theory, and science from application fields.

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