Predictability and Reduced Order Modeling in Stochastic Reaction Networks

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Many systems involving chemical reactions between small numbers of molecules exhibit inherent stochastic variability. Such stochastic reaction networks are at the heart of processes such as gene transcription, cell signaling or surface catalytic reactions, which are critical to bioenergy, biomedical, and electrical storage applications. The underlying molecular reactions are commonly modeled with chemical master equations (CMEs), representing jump Markov processes, or stochastic differential equations (SDEs), rather than ordinary differential equations (ODEs). As such reaction networks are often inferred from noisy experimental data, it is not uncommon to encounter large parametric uncertainties in these systems. Further, a wide range of time scales introduces the need for reduced order representations. Despite the availability of mature tools for uncertainty/sensitivity analysis and reduced order modeling in deterministic systems, there is a lack of robust algorithms for such analyses in stochastic systems. In this talk, we present advances in algorithms for predictability and reduced order representations for stochastic reaction networks and apply them to bistable systems of biochemical interest.

To study the predictability of a stochastic reaction network in the presence of both parametric uncertainty and intrinsic variability, an algorithm was developed to represent the system state with a spectral polynomial chaos (PC) expansion in the stochastic space representing parametric uncertainty and intrinsic variability. Rather than relying on a non-intrusive collocation-based Galerkin projection [1], this PC expansion is obtained using Bayesian inference, which is ideally suited to handle noisy systems through its probabilistic formulation. To accommodate state variables with multimodal distributions, an adaptive multiresolution representation is used [2]. As the PC expansion directly relates the state variables to the uncertain parameters, the formulation lends itself readily to sensitivity analysis.

Reduced order modeling in the time dimension is accomplished using a Karhunen-Loève (KL) decomposition of the stochastic process in terms of the eigenmodes of its covariance matrix. Subsequently, a Rosenblatt transformation relates the random variables in the KL decomposition to a set of independent random variables, allowing the representation of the system state with a PC expansion in those independent random variables. An adaptive clustering method is used to handle multimodal distributions efficiently, and is well suited for high-dimensional spaces.

The spectral representation of the stochastic reaction networks makes these systems more amenable to analysis, enabling a detailed understanding of their functionality, and robustness under experimental data uncertainty and inherent variability.

References

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