Automated Partitioning of Data-Parallel Kernels using Polyhedral Compilation

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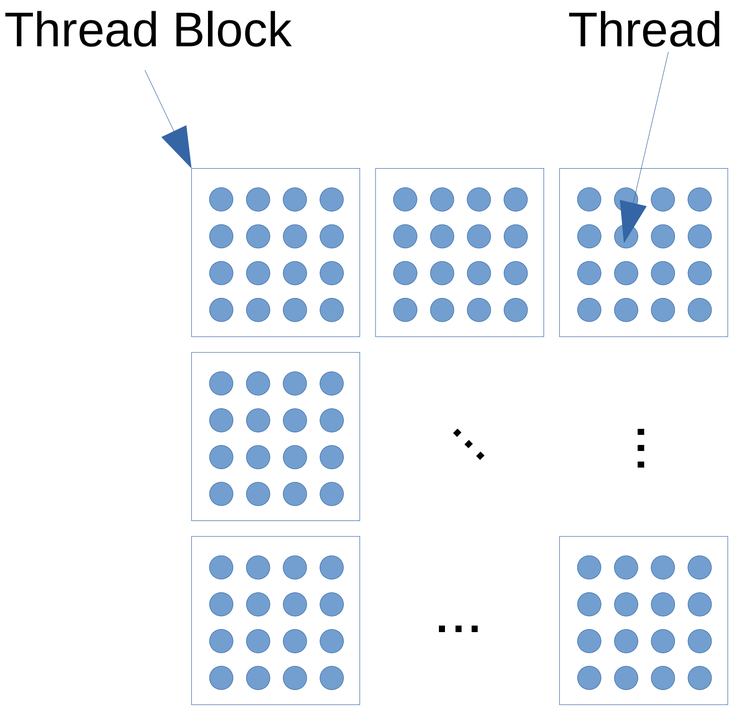
GPGPU Programming

Kernel Code (2D Jacobi)

```
__global__ void stencil(float* A, float* B, int N) {
  idx = threadIdx.x + blockIdx.x * blockDim.x;
  idy = threadIdx.y + blockIdx.y * blockDim.y;
  if (idx >= N || idy >= N) return;
  auto x1 = idx.x-1 >= 0 ? A[idx.y * N + idx.x-1] : 0;
  auto x2 = idx.x+1 < N ? A[idy.y * N + idx.x+1] : 0;
  auto x3 = idx.y-1 >= 0 ? A[(idx.y-1) * N + idx.x] : 0;
  auto x4 = idx.y+1 < N ? A[(idx.y+1)* N + idx.x] : 0;
  auto x5 = A[idx.y* N + idx.x];
  B[idx.y * N + idx.x] = 0.2*(x1+x2+x3+x4+x5);
}
```

Kernel Instance



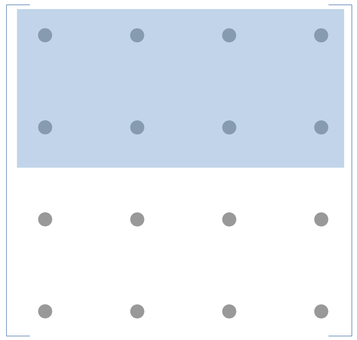
Automated Multi GPU programming?

- Thread Blocks are highly independent
- Kernels often exhibit spatial locality
- Grid provides simple mechanism to manage parallelism

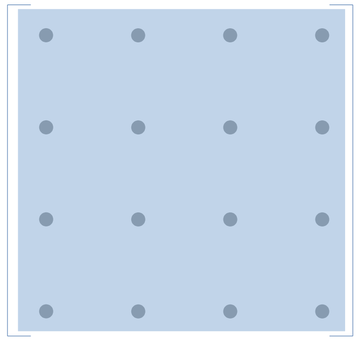
- Why not automate scale out to multiple GPUs?

GPU Data Distribution: Matrix Multiplication

GPU 1



x



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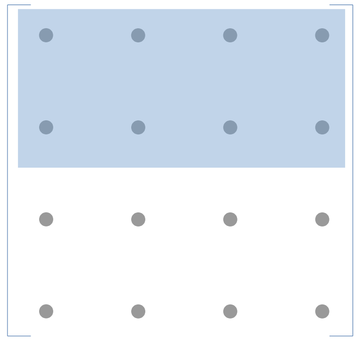
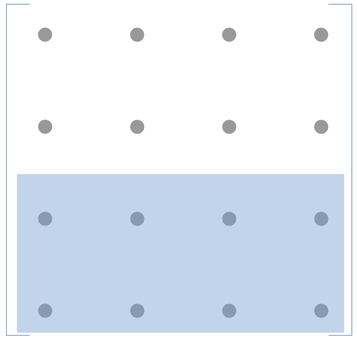


GPU 2



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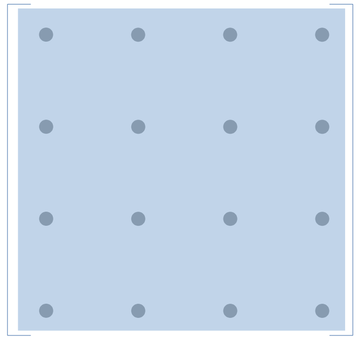

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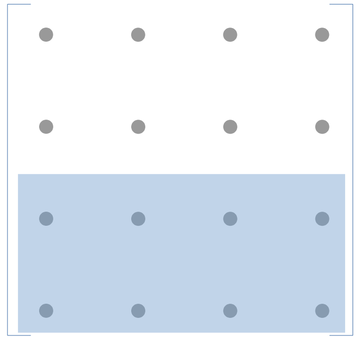


Polyhedral Analysis

- Mathematical model for loop optimizations
- Data dependencies are represented as sets and maps of n-dimensional integer points
- Sets described via linear constraints, maps as affine functions
- Sets and maps can be combined (union, difference, ...) and queried (min, max, ...)

```
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < i; ++j) {
        S(i, j, i + j);
    }
}
```

```
D(S) := [n] -> { [i, j] : 0 <= i < n & 0 <= j < i }
   S :=       { [i, j] -> [i, j, i+j] }
```

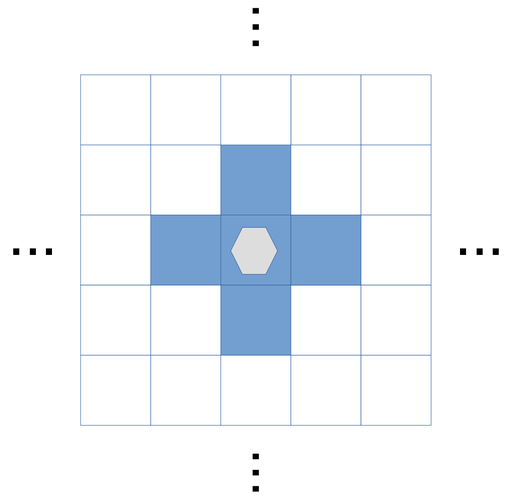
Polyhedral Analysis: 2D Jacobi

```
__global__ void stencil(float* A, float* B, int N) {
  idx = threadIdx.x + blockIdx.x * blockDim.x;
  idy = threadIdx.y + blockIdx.y * blockDim.y;
  if (idx >= N || idy >= N) return;
  auto x1 = idx.x-1 >= 0 ? A[idx.y * N + idx.x-1] : 0;
  auto x2 = idx.x+1 < N ? A[idx.y * N + idx.x+1] : 0;
  auto x3 = idx.y-1 >= 0 ? A[(idx.y-1) * N + idx.x] : 0;
  auto x4 = idx.y+1 < N ? A[(idx.y+1)* N + idx.x] : 0;
  auto x5 = A[idx.y* N + idx.x];
  B[idx.y * N + idx.x] = 0.2*(x1+x2+x3+x4+x5);
}
```

Extracted Information:

- Access patterns of a single thread
- Data type of array arguments
- Array dimensionality

Polyhedral Analysis: 2D Jacobi



```
Read(A) := [N] -> { [x, y] -> [x-1, y] : x-1 >= 0;
                    [x, y] -> [x+1, y] : x+1 < N;
                    [x, y] -> [x, y-1] : y-1 >= 0;
                    [x, y] -> [x, y+1] : y+1 < N;
                    [x, y] -> [x, y]; }
Dim(A) := [N] -> { N, inf }
ElType(A) := float

Write(B) := { [x, y] -> [x, y]; }
Dim(B) := [N] -> { N, inf }
ElType(B) := float
```

We can predict the memory accesses of all threads of a kernel!

Polyhedral Code Generation: 2D Jacobi

Goal: for a group of threads, list all array elements they read or write

1) Intersect (limit) domain of memory access map to threads satisfying:

```
[xmin, xmax, ymin, ymax] -> { [x, y] : xmin <= x < xmax and ymin <= y < ymax; }
```

The image of the map now contains all memory accesses

2) Generate efficient code that lists these elements

Polyhedral Code Generation: 2D Jacobi

For 2D Jacobi, we get a 2-level result

1) Loop that lists all rows

```
void rows(int xmin, int xmax, int ymin, int ymax, int N) {
  if (xmin >= 1) __row(xmin - 1);
  for (int c0 = xmin; c0 < xmax; c0 += 1) __row(c0);
  if (N >= xmax + 1) __row(xmax);
}
```

2) First and last element in each row

```
int first(int row, int xmin, int xmax, int ymin, int ymax) {
  return row >= xmin && xmax >= row + 2 ? ymin - 1 : xmin == row + 1 ? ymin : row - xmax + ymin
}

int last(int row, int xmin, int xmax, int ymin, int ymax) {
  return row + 1 >= xmax ? -row + xmax + ymax - 1 : xmin == row + 1 ? ymax - 1 : ymax
}
```

* Polyhedral libraries usually implement AST generation

Host Code Modifications

A custom tool (using a regex matcher) transforms code like this

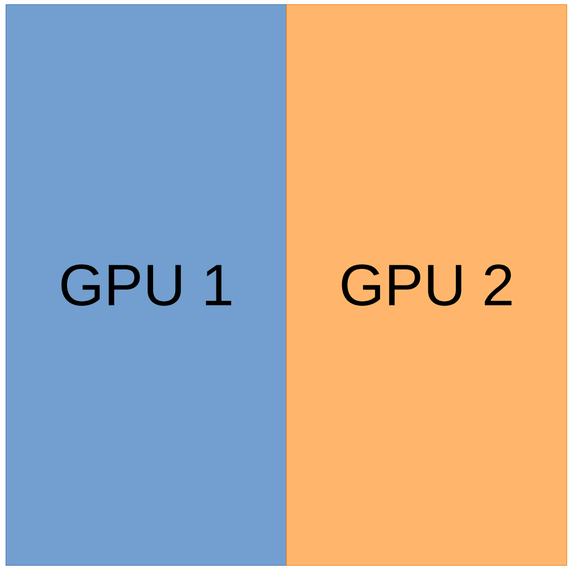
```
int main() {
  ...
  kernel<<<threadGrid, threadBlocks>>>(arr_in, arr_out, N);
  ...
}
```

Into code like this

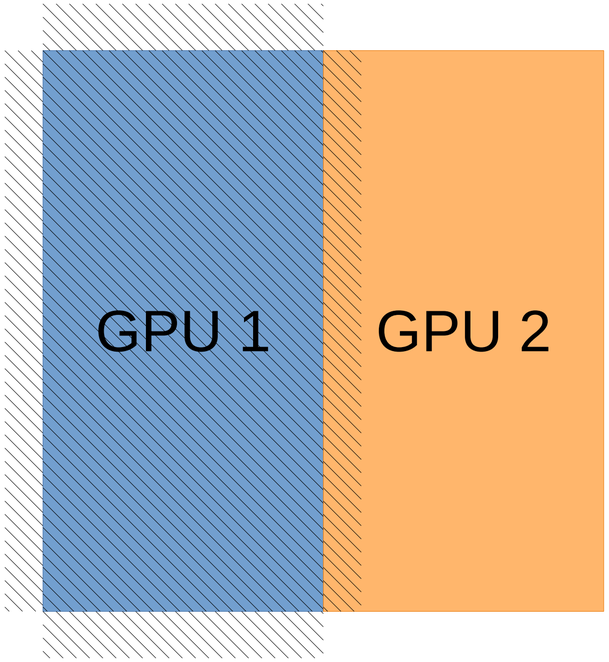
```
int main() {
  ...
  for (int 0; i < numGPUs(); ++i) {
    auto newGrid = calcGrid(threadGrid);
    redistribute_data(arr_in, newGrid);
    kernel<<<newGrid, threadBlocks>>>(arr_in, arr_out, N);
    update_distribution(arr_out, newGrid);
  }
  ...
}
```

Data redistribution

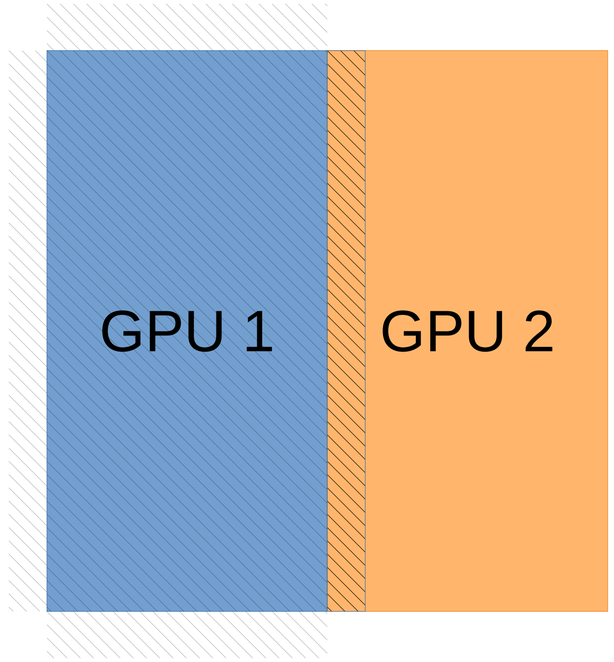
Output Matrix computed in last iteration (input in current iteration):



Overlay read pattern of input matrix (for GPU 1):



Data copied from GPU2 to GPU 1:

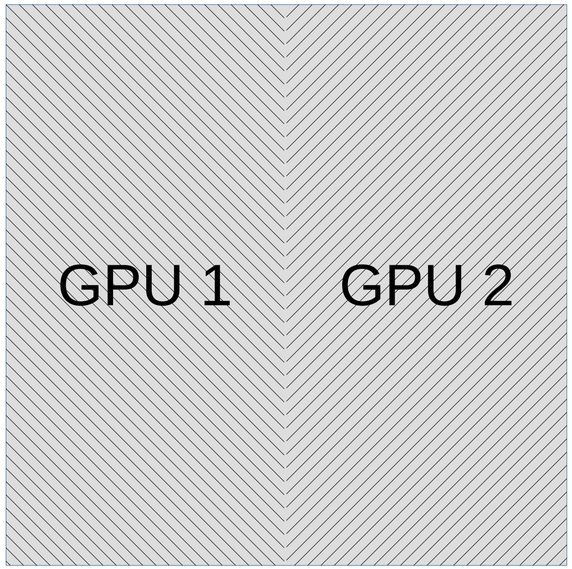


Required information for this is contained in generated code

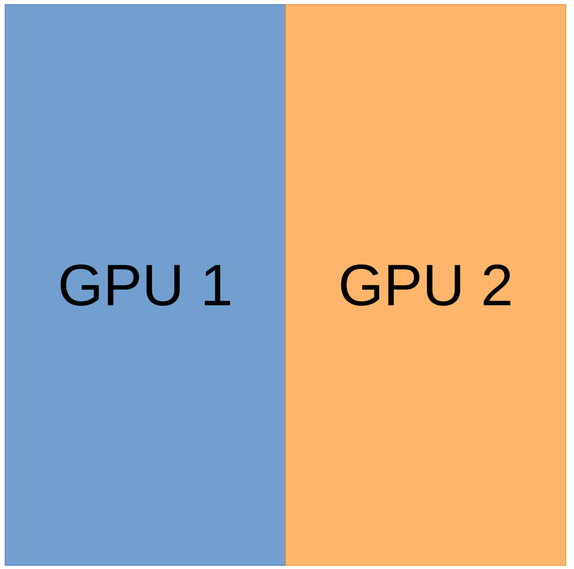
Updating data distribution

Output Matrix data distribution before first iteration:

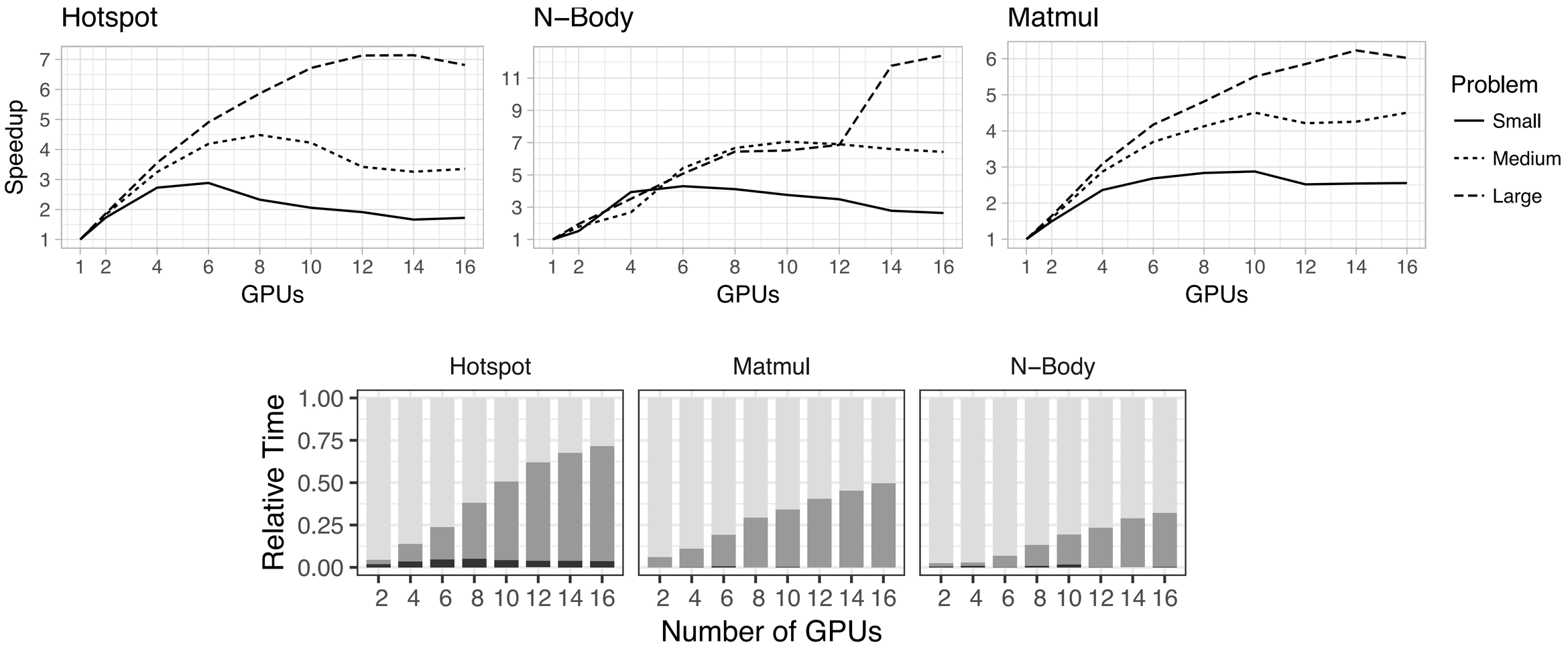
Overlay write patterns of output matrix for GPU 1 and GPU 2:



Output matrix data distribution after first iteration:



Benchmarking Results



Insights

- When polyhedral compilation works, it works well
  - Powerful and accurate model of the application
  - Limitations for irregular kernels

- Scaling out to multiple GPUs is performant for simple kernels
  - GPU applications often already optimized for locality

- Real life GPU kernels are complex
  - Robust implementation is a serious engineering effort