A Scalable River Network Simulator for Extreme Scale Computers using the PETSc Library

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Outline

- Introduction
- PETSc/DMNetwork
- Numerical methods
- Test and scaling results
- Future work
Introduction

- Most flow routing models are not suitable river-basin scale and real-time applications
- Muskingum (kinematic) based parallel flow routing model developed
- Does not capture a wave propagation in the upstream direction
  - Backwater effect
  - Overestimate flood peak
  - $S_f \neq S_b$ in case of dam-break

- Scalable River Network Simulator (SRNS) developed to solve SW equations using PETSc/DMNetwork

\[
\frac{1}{g} \frac{\partial u}{\partial t} + \frac{u}{g} \frac{\partial u}{\partial x} + \frac{\partial h}{\partial x} + (S_f - S_b) = 0
\]

Source: NASA (2016)
**PETSc** (Portable Extensible Toolkit for Scientific computation)

- High-performance software for the scalable (parallel) solution of scientific applications
DMNetwork

- It is one of data management packages in PETSc
- Data and topology management for multiphysics PDE-based network problems
  - Circuits, power grid, gas networks, electrical and water distribution
- Design elements
  - Vertex: connection points in topology graph
  - Edge: a connection between vertices
  - Component: physics associated with vertex and edges
Steps for using DMNetwork

1. Set up graph
   - DMNetworkLayoutSetup()

2. Add physics
   - DMNetworkAddComponent()
   - DMNetworkAddNumVariables()

3. Partition
   - DMNetworkDistribute()

4. Solve
   - KSPSetDM/SNESSetDM/TSSetDM()
   - KSPSolve()/SNESolve()/TSSolve()
One-dimensional Free Surface Flow Model

- Flow in a reach simulated

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0
\]

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + \frac{1}{2} gh^2)}{\partial x} = gh(S_b - S_f)
\]

- Flow in a junction

\[
\sum q_i = 0, \forall i
\]

\[
h_i = h_j, \forall i \neq j
\]

- $h$ is water depth
- $u$ is flow velocity
- $z$ bottom elevation
- $S_b$ is bed slope
- $S_f$ is friction term
- $q_i$ is flow rate
Numerical Methods

- Finite volume method used

\[ U_{i}^{n+1} = U_{i}^{n} - \frac{\Delta t}{\Delta x} \left[ F_{i+\frac{1}{2}}^{n} - F_{i-\frac{1}{2}}^{n} \right] \]

\[ U_{i} = [h_{i}, q_{i}], \quad i = 1, \ldots, n_{\text{cells}} \text{ on a reach} \]

- Flux on cell interface is estimated
  - The Godunov method (first order)
  - Second order methods will be implemented
Numerical Methods Cont’d

- Forward Euler used for time stepping

Step 1: Initialization at all grid cells
\[ \frac{dU_i}{dt} = 0 \]

Step 2: Interior reach cells \((i=2 \text{ to } ncell-1)\)
\[ \frac{dU_i}{dt} = -\frac{1}{\Delta x_i} \left[ F_{i+\frac{1}{2}}(t^n) - F_{i-\frac{1}{2}}(t^n) \right] + S_i(t^n) \]

Step 3: Junction cell
\[ \frac{dU_j}{dt} = -\frac{1}{\Delta x} \left[ \sum F_{i+\frac{1}{2}} - \sum F_{i-\frac{1}{2}} \right] \]
Numerical Methods Cont’d

- Post-step processing at $t^{n+1}$

**Step 1: Update ending cell points on a reach**

$h_1 = h_J^{US}$
$q_1 = \frac{q_J^{US}}{n_{out}}$

$h_{n_{cells}} = h_J^{DS}$
$q_{n_{cells}} = \frac{q_J^{US}}{n_{in}}$

$n_{out}$: number of out going reaches at $x_J^{US}$
$n_{in}$: number of incoming reaches at $x_J^{DS}$

**Step 2: Update boundary vertex points**

- Reservoir
- Demand
- Inflow
- Others
Benchmark Test 1: Dam-break Problems (Toro, 2001)

\[ h(x) = \begin{cases} 
    h_L = 1 & 0 < x \leq 10 \\
    h_R = 0.1 & 10 < x \leq 50 
\end{cases} \]

\[ u(x) = \begin{cases} 
    u_L = 2.5 & 0 < x \leq 10 \\
    u_R = 0.0 & 10 < x \leq 50 
\end{cases} \]

Simulated left rarefaction and right shock waves
Benchmark Test 2: Dam-break Problems (Toro, 2001)

\[ h(x) = \begin{cases} 
  h_L = 1 & 0 < x \leq 25 \\
  h_R = 1 & 25 < x \leq 50 
\end{cases} \quad u(x) = \begin{cases} 
  u_L = -5 & 0 < x \leq 25 \\
  u_R = 5 & 25 < x \leq 50 
\end{cases} \]

Simulated left and right rarefactions waves which generate nearly dry bed
Scaling Study

- The Mississippi River simulated for scaling test
- Represents $\frac{1}{8}$ of the total reaches in the conterminous U.S.
- NHDPlus dataset used to setup the river network
- Simulation conducted on Theta at ANL

11.69 petaflops system
4,392 (node) x 64 (cores)
Total cores = 281,088
Scaling Results

SRNS: 28,894,804 unknowns

RAPID (David et al. 2011): Upper Mississippi simulation

![Graph showing time (sec) vs number of cores for SRNS and RAPID](image-url)
Future work

- Implement second order methods to compute flux
- Conduct additional tests to verify the improved implementation
- Simulate the river networks for the conterminous U.S. using subnetwork option provided by DMNetwork
- Couple it with Earth System Models
Thank you!

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