Solving the Load Flow and Helmholtz Equations using PETSc

Domenico Lahaye and helping friends

DIAM - TU Delft

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Helping Friends

- Reijer Idema (graduated November 2012)
- Abdul Sheikh Hannan (graduated November 2013)
- Romain Thomas (PhD started October 2012)
- Martijn de Jong (PhD started October 2012)
- Prof. Kees Vuik
- ...

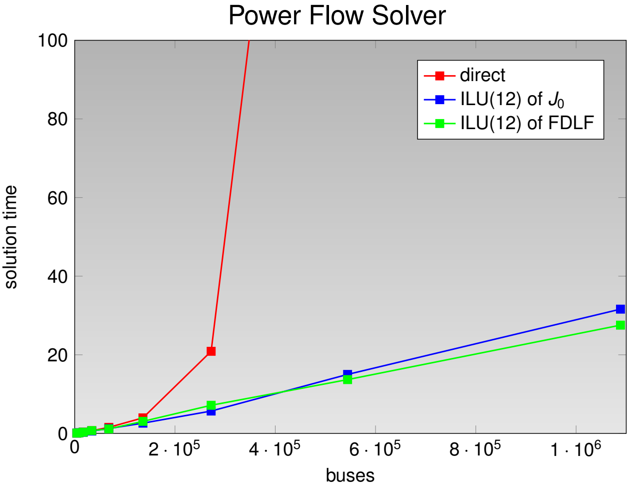
Outline

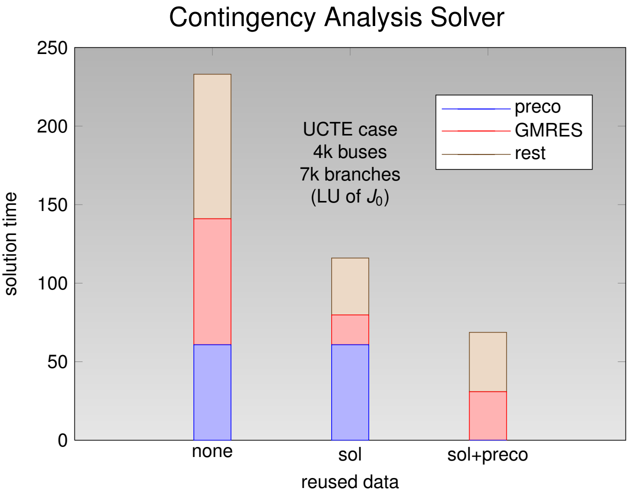
Future Power Systems = Power Webs

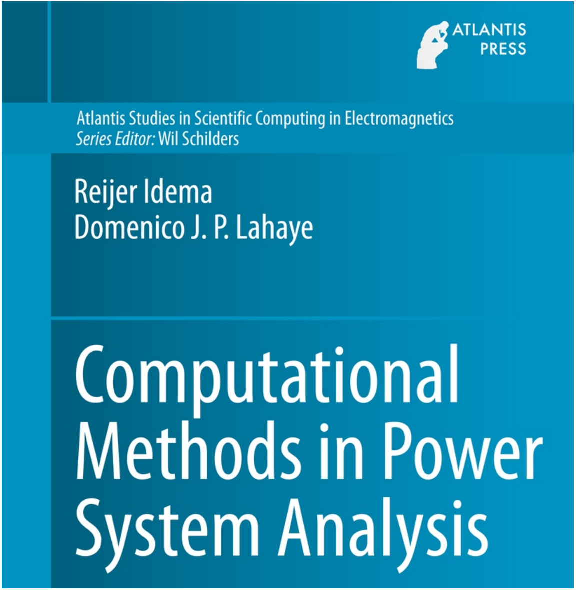
- large: European study (UCTE) model with 10,000+ nodes
- bi-directional: market mechanisms
- uncertainty: time-variable production by renewables
- operated closer to limits: electrical vehicle charging

Load Flow Equations

- x: voltage amplitude and phase at each node in network
- $F(x) = 0$: non-linear system matching generation with demand
- $J_i x_i = F_i$: linear system at i-th Newton iteration
- in the past solved by direct methods
- currently solved approximately by ILU/GMRES
- implemented in PETSc


Power Flow Solver
solution time
buses
0
20
40
60
80
100
0
$2 \cdot 10^5$
$4 \cdot 10^5$
$6 \cdot 10^5$
$8 \cdot 10^5$
$1 \cdot 10^6$
direct
ILU(12) of J_0
ILU(12) of FDLF


Contingency Analysis Solver
250
200
150
100
50
0
solution time
UCTE case
4k buses
7k branches
(LU of J_0)
preco
GMRES
rest
none
sol
sol+preco
reused data


ATLANTIS
PRESS
Atlantis Studies in Scientific Computing in Electromagnetics
Series Editor: Wil Schilders
Reijer Idema
Domenico J. P. Lahaye
Computational
Methods in Power
System Analysis

Outline

Helmholtz Equation

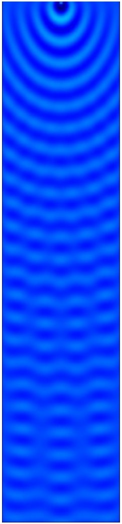
$$-\Delta \mathbf{u}(x,y) - k^2 \, \mathbf{u}(x,y) = \mathbf{g}(x,y) \; \text{ on } \Omega$$

Dirichlet and/or Sommerfeld on $\partial\Omega$

finite differences or elements

$A\,u = f$ sparse complex symmetric

all standard solvers fail



Complex Shifted Laplace Preconditioner

preconditioning by damping

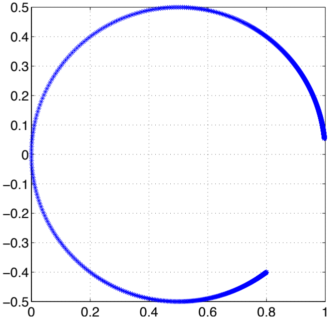
$M : -\Delta \mathbf{u} - (1 + \beta_2 \, i) k^2 \, \mathbf{u}$

M-solve using multigrid

$M^{-1} A$ favorable spectrum

standard in many applications

Erlangga e.a. 2006



Complex Shifted Laplace Preconditioner

Number of outer Krylov iterations

	Wavenumber					
Grid	$k = 10$	$k = 20$	$k = 30$	$k = 40$	$k = 50$	$k = 100$
$n = 32$	**10**	17	28	44	70	13
$n = 64$	10	**17**	28	36	45	173
$n = 96$	10	17	**27**	35	43	36
$n = 128$	10	17	27	**35**	43	36
$n = 160$	10	17	27	35	**43**	25
$n = 320$	10	17	27	35	42	**80**

Complex Shifted Laplace Preconditioner

Good News

- SLP preconditioner renders spectrum favorable to Krylov

However ...

- eigenvalues rush to zero as k increases
- outer Krylov convergence limited by near-null space

Can deflation improve?

Deflation using Multigrid Vectors

Deflation perspective

- replace preconditioned system $M^{-1} A = M^{-1} b$
- by deflated preconditioned system $P^T M^{-1} A = P^T M^{-1} b$
- deflation vectors Z and Galerkin coarse grid matrix $E = Z^T A Z$
- deflation operator $P = I - A Q$ where $Q = Z E^{-1} Z^T$
- P: projection (later modified to shift to 1)
- Z: columns of the coarse to fine grid interpolation

  good approx to near-null space for $k\,h$ fixed

Deflation using Multigrid Vectors

Multigrid perspective

- replace smoother $I - M^{-1} A$

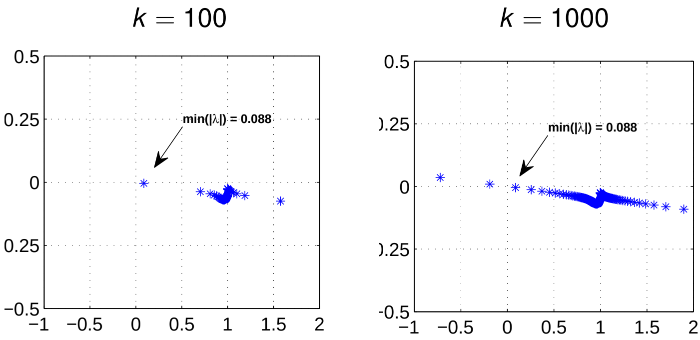
  (M complex shifted-Laplacian, M^{-1} as before)

- by smoother + coarse grid solve $(I - Q A)\ (I - M^{-1} A)$

  ($Q = Z E^{-1} Z^T$ coarse grid solve, E^{-1} new element)

- Fourier two-grid analysis for
  - 1D problem with Dirichlet bc
  - uniform coarsening
  - E and M inverted exactly

Spectrum Deflated Preconditioned Operator



tighter clusters at low frequency

spread due to E^{-1} at high frequency

Avoiding Eigenvalue Spread at High Wavenumber

- deflate friend of A instead of A (unsuccessful)
- choose other deflation vectors (under investigation)

Deflation using Multigrid Vectors

Multilevel Extension

- composite two-level preconditioner $P^T M^{-1} A = P^T M^{-1} b$
- deflation operator $P = I - A Q$ where $Q = Z E^{-1} Z^T$
- coarse grid Helmholtz operator $E = Z^T A Z$
- apply idea recursively to apply P
- multilevel Krylov method (Erlangga-Nabben 2009)

Convergence Outer Krylov Acceleration

Number of outer Krylov iterations with/without deflation

Grid	$k = 10$	$k = 20$	$k = 30$	$k = 40$	$k = 50$	$k = 100$
$n = 32$	**5/10**	8/17	14/28	26/44	42/70	13/14
$n = 64$	4/10	**6/17**	8/28	12/36	18/45	173/163
$n = 96$	3/10	5/17	**7/27**	9/35	12/43	36/97
$n = 128$	3/10	4/17	6/27	**7/35**	9/43	36/85
$n = 160$	3/10	4/17	5/27	6/35	**8/43**	25/82
$n = 320$	3/10	4/17	4/27	5/35	5/42	**10/80**

Less iterations and therefore speedup

(Abdul Sheikh Hannan, D.L. and Kees Vuik, NLA, 2013).

Numerical Results

3D problem with wedge-like contrast in wavenumber
using 20 grid points per wavelength

Wave number k	Solve Time		Iterations	
	PREC	DEF+PREC	PREC	DEF+PREC
5	0.09	0.24	9	11
10	1.07	1.94	15	12
20	16.70	18.89	32	16
30	73.82	78.04	43	21
40	1304.2	214.7	331	24
60	xx	989.5	xx	34

speedup in CPU of by a factor 6

(Abdul Sheikh Hannan, D.L. and Kees Vuik, submitted to JCP).

Numerical Results

2D Marmousi Problem
using 20 grid points per wavelength

Frequency f	Solve Time		Iterations	
	PREC	DEF+PREC	PREC	DEF+PREC
1	1.23	5.08	13	7
10	40.01	21.83	106	8
20	280.08	131.30	177	12
40	20232.6	3997.7	340	21

speedup in CPU of by a factor 5

Conclusions

Newton-Krylov Load Flow Solver

- scalable much faster for large problems
- flexible reuse of data in contingency analysis

Deflated Shifted-Laplacian Helmholtz Solver

- less iterations than shifted-Laplacian
- faster than shifted-Laplacian solver for sufficiently large problems

Conclusions

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Happy 20th Birthday PETSc!