Splittable TAO objective for ADMM in PETsc/TAO

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Objectives

Demonstrate Alternating Direction Method of Multipliers' splittable objective implementation in TAO.

- Provide working example of splitable objectives on TAO
- Results comparison between single objective implementation and ADMM implementation
- Future structural splittable objectives in TAO

Introduction

Nowadays, With ever-increasing complexities of data, decentralization of data collection and storage, inevitably brings a need for distributed solution method. Here, we propose that Alternating Direction Method of Multipliers (ADMM), is well suited for distributed, parallel convex solver, which can be easily implemented into existing TAO framework. In essence, ADMM takes the form of decomposition-coordination, of which can be viewed as an attempt to blend the benefits of dual decomposition and augmented Lagrangian methods for constrained optimization [1], Currently TAO supports only one objective. Thus, this project tries to make it more expandable, and provide proof of concept for multiple objectives in TAO, in this example, ADMM.

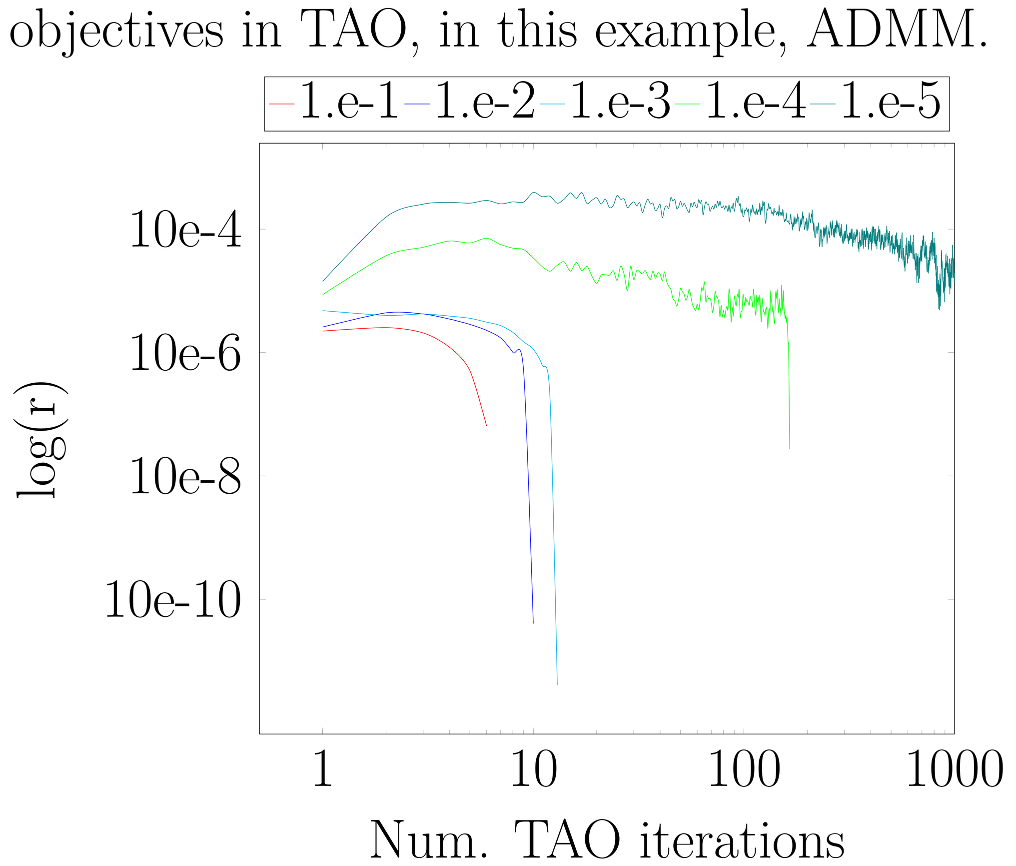

Figure 1:Tomography Example Convergence with Different ε

Algorithms

As mentioned, ADMM is an algorithm that is intended to blend the decomposability of dual ascent with the superior convergence properties of the method of multipliers. The algorithm solves problems in the form [1],

$$\text{minimize } f(x) + g(z)$$
$$\text{subject to } Ax + Bz = c$$

with variables $x \in \mathbf{R}^n$ and $z \in \mathbf{R}^m$, where $A \in \mathbf{R}^{p\times n}$, $B \in \mathbf{R}^{p\times m}$, and $c \in \mathbf{R}^p$. Now, for the method of multipliers, we form the augmented Lagrangian

$$L_p(x, z, y) =$$

$$f(x)+g(z)+y^T(Ax+Bz-c)+(\rho/2)\|Ax+Bz-c\|_2^2$$

with which we can construct the following ADMM iterations

$$x^{k+1} := \text{argmin}_x L_\rho(x, z_k, y_k)$$

$$z^{k+1} := \text{argmin}_z L_\rho(x^{k+1}, z, y_k)$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c)$$

Further, ADMM can be writtein in a different form, by including the linear and quadratic terms in the augmented Lagrangian and scaling the dual variable, where $u = (1/\rho)y$ is the scaled dual variable, and defining residual $r = Ax + Bz - c$, we get

$$x^{k+1} := \text{argmin}_x \left(f(x) + (\rho/2)\|Ax + Bz^k - c + u^k\|_2^2\right)$$

$$z^{k+1} := \text{argmin}_x \left(g(x) + (\rho/2)\|Ax_{k+1} + Bz - c + u^k\|_2^2\right)$$

$$u^{k+1} := u^k + Ax^{k+1} + Bz^{k+1} - c$$

In practice, for PETsc/TAO, the typical form becomes,

$$\min_x f(x) + \rho\|x - z + u\|_2^2$$

$$\min_x R(z) + \rho\|x - z + u\|_2^2$$

$$u^{k+1} := u^k + Ax^{k+1} + Bz^{k+1} - c$$

Convergence

- Residual convergence. $r^k \to 0$ as $k \to \infty$
- Objective convergence. $f(x^k) + g(z^k) \to p^*$ as $k \to \infty$
- Dual variable convergence. $y^k \to y^*$ as $k \to \infty$, where y^* is a dual optimal point.

In practice, for PETSc/TAO, the break conditions are

$$\|x - z\| < \sqrt{n} * \text{ABSTOL}$$

$$\|\rho(z_{k+1} - z_k)\| < \|\rho * u\| * \text{RELTOL}$$

Results

For comparision, tomography recovery was conducted with and without using ADMM.

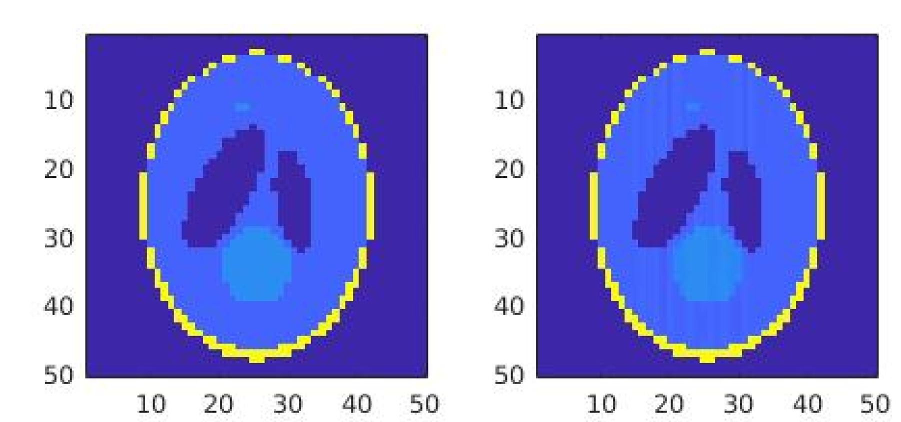

Figure 2:Ground Truth and L1 Dictionary BRGN recovery

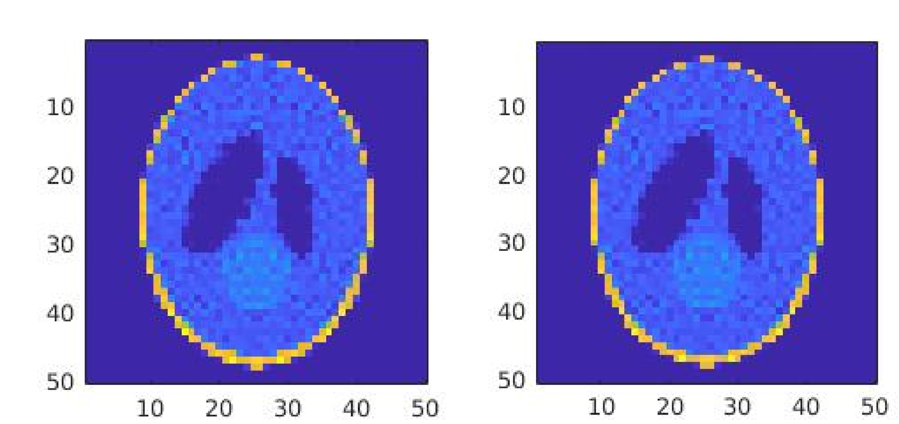

Figure 3:L2 Proximal Norm and ADMM Recovery

Results

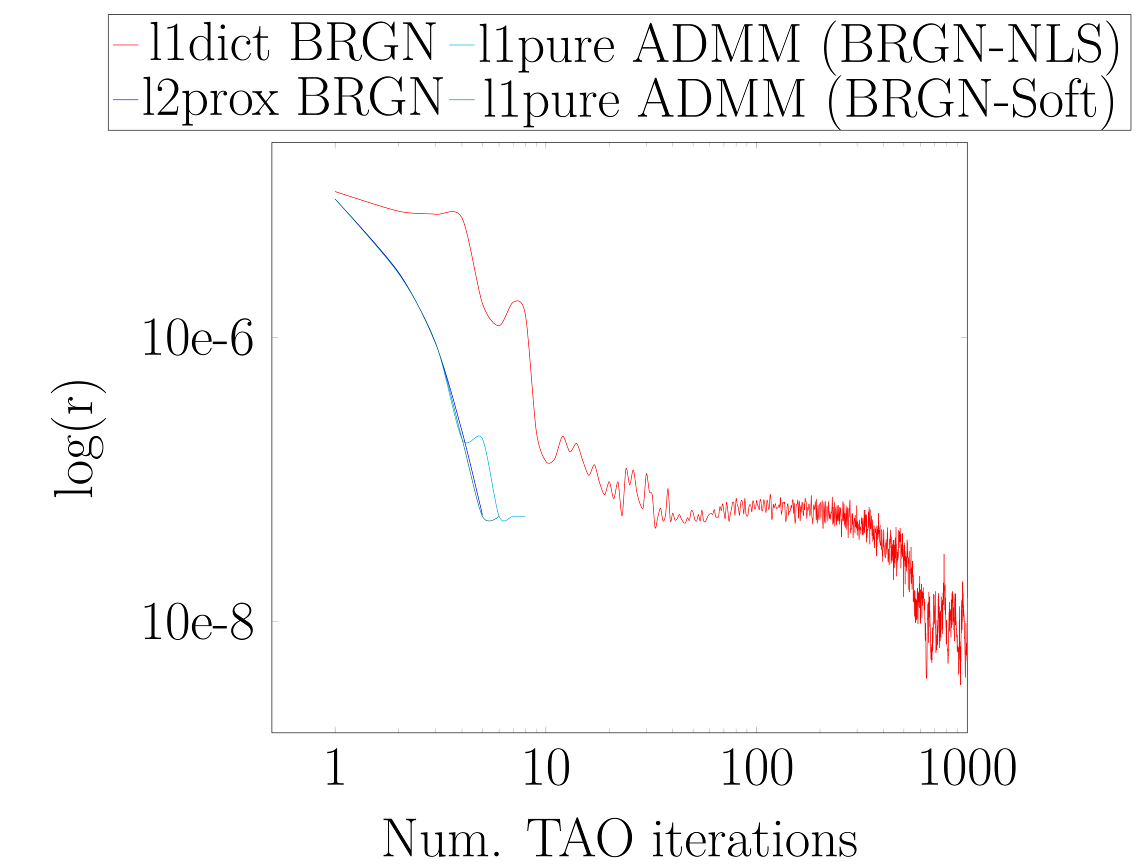

Figure 4:Tomography Conv. with diff. Methods at ε 1e-8

Conclusion

Splittable objective, namely ADMM, was implemented in PETsc/TAO, and its outcomes were analyzed, and compared with existing software artifact. For the tomography example, clearly L1 dictionary was the most efficient regularizer. Yet, here, ADMM proved to be equal or better efficiency than existing L2 proxiaml regularizer, and showed that indeed separable objective works with existing TAO framework. For future work, integration of ADMM feature with possibility of warm start optimization feature could be considered. As the current PETsc/TAO development team's native implementation for splittable objective is under way, future work could include incorporting splittable objectives and native ADMM support for TAO - one of which may include PETSCFN feature-interface.

References

[1] Stephen Boyd, et al. *Distributed optimization and statistical learning via the alternating direction method of multipliers.* Foundations and Trends in Machine Learning, Berkeley, California, 2010