Adjoint Sensitivity Analysis for Wind Power Generation

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***Abstract*—In this note we present an approach to estimate the adjoint sensitivity of wind power generation using numerical weather prediction (NWP) models. This approach augments the planning and operations of wind farms by improving the accuracy of wind forecasts obtained by either physics-based NWP or statistical data-based models. The proposed analysis can be used to determine the simulation domain size and resolution, and suitable observation placement. We illustrate the method of determining sensitivities of wind speed at wind-farm locations with respect to current ambient conditions in a northern Texas region using real data and atmospheric conditions.**

***Index Terms*—wind power, sensitivity analysis, weather forecasting**

I. Introduction

Accurate wind speed forecasts are essential in estimating the amount of wind-power produced [1], [2]. The wind forecast can be obtained by using time-series prediction (TSP) methods that employ historical data (e.g., AR, ANN), by using physics-based numerical weather prediction (NWP) models, or a combination of both. In [2] we showed that large adoption levels require accurate predictions for cost-efficient operations. Arguably, optimal grid integration of wind farms is essential in the robust management of this energy sector.

In this study we present an adjoint sensitivity analysis (ASA) method that augments the wind prediction using either statistical or physics-based methods. ASA is a method used to determine the sensitivity of a model state or parameter (e.g., future wind speed) with respect to input states (e.g., present ambient conditions). In the context of wind power generation, we show how ASA can be used to determine the simulation domain size and resolution, the quantities that should be modeled more accurately, and suitable locations for sensor placement when using NWP simulations. Furthermore, this analysis can reveal meaningful observations to be used in the AR-type/ANN models. In this study we give a brief mathematical presentation of ASA and illustrate its potential in a northern Texas region using real data and atmospheric conditions.

II. Adjoint Sensitivity Analysis

Consider a numerical model $\mathcal{M}$ that evolves an initial state x_{t_0} to a given final time t_N (e.g., 24 hours ahead):

$$x^k = \mathcal{M}\left(t_{k-1}, x^{k-1}, p\right)\,,\; x^0 = x_i(t_0,p)\,,\; k \in \{1\dots N\}\,, \quad (1)$$

where p are model parameters. For instance, $\mathcal{M}$ may represent the discretization operator of a partial differential equation. Sensitivity analysis reveals how a model solution is affected by small perturbations in the model variables and parameters [3]. We write the sensitivity of the solution x with respect to parameter p_i as $S_i(t) = \frac{\partial x(t)}{\partial p_i}$ or scaled to be unitless, $S_i(t) = \frac{\partial x(t)}{\partial p_i}\frac{p_i}{x(t)}$ [4]. Just as the model state x^{t_0} is evolved through $\mathcal{M}$, the sensitivity S_i is evolved by the gradient (also known as tangent linear) model

$$S_i^k = \frac{\partial \mathcal{M}}{\partial x}(x^{t_{k-1}}, p)S^{k-1} + \frac{\partial \mathcal{M}}{\partial p_i}(x^{t_{k-1}}, p)\,,\; S_i^0 = \frac{\partial x^{t_0}}{\partial p_i}\,,$$

where $t_k \in [t_0, t_N]$. We are interested in the effect that the initial condition at location i, $p_i \equiv x_i^{t_0} := x_i(t_0)$, has at some targeted locations in the final system state, x^{t_N}. Therefore, the sensitivity takes the form

$$S_i = \frac{\partial x^{t_N}}{\partial x_i^{t_0}}\frac{x_i^{t_0}}{x^{t_N}}\,. \quad (2)$$

and its evolution is described by

$$S_i^k = \frac{\partial \mathcal{M}}{\partial x}(x^{t_{k-1}})S^{k-1}\,,\; S_i^0 = \frac{\partial x^{t_0}}{\partial x_i^{t_0}}\,.$$

This is useful if one is interested in what effect a small perturbation at a single source location would have on the future states, x^{t_k}. Alternatively, one could consider the inverse or *adjoint process* [5] of observing some target state in the state space at future times and inferring what states in the initial conditions have a strong influence on that target state [4]. We aim to find the regions in the initial state to which target points at later times are most sensitive. Therefore, the sensitivities are computed in terms of a cost function, that is, a function of the state at the final time,

$$\Psi(x^{t_N}(x^{t_0})) \in \mathbb{R}\,, \quad \frac{\partial \Psi}{\partial x^{t_0}} = \left[\frac{\partial \Psi}{\partial x_1^{t_0}} \;\cdots\; \frac{\partial \Psi}{\partial x_M^{t_0}}\right]^T \in \mathbb{R}^M,$$

where M is the dimension of the initial state vector. By using the chain rule, one obtains

$$\frac{\partial \Psi(x^{t_N})}{\partial x_i^{t_0}} = \frac{\partial \Psi(x^{t_N})}{\partial x^{t_N}}\frac{\partial x^{t_N}}{\partial x^{t_0}} = \frac{\partial \Psi(x^{t_N})}{\partial x^{t_N}}S_i^{t_N}\,. \quad (3)$$

Following [4], one can extend (3) for all time indices

$$\frac{\partial \Psi(x^{t_N})}{\partial x_i^{t_0}} = \frac{\partial \Psi(x^{t_N})}{\partial x^{t_N}}\frac{\partial x^{t_N}}{\partial x^{t_{F-1}}}\cdots\frac{\partial x^{t_1}}{\partial x^{t_0}}\frac{\partial x^{t_0}}{\partial x_i^{t_0}}\,,$$

$$\frac{\partial x^{t_k}}{\partial x^{t_{k-1}}} = \frac{\partial \mathcal{M}}{\partial x}(x^{t_{k-1}})\,,\; \frac{\partial x^{t_0}}{\partial x_i^{t_0}} = \delta_i x^{t_0}\,;\; i \in \{1\dots M\}\,.$$

Alternatively, by transposing, the adjoint process evolves the sensitivity in reverse order:

$$\left(\frac{\partial \Psi(x^{t_N})}{\partial x^{t_0}}\right)^T = \left(\frac{\partial x^{t_1}}{\partial x^{t_0}}\right)^T\cdots\left(\frac{\partial x^{t_N}}{\partial x^{t_{F-1}}}\right)^T\left(\frac{\partial \Psi(x^{t_F})}{\partial x^{t_F}}\right)^T\,.$$

If the following equations are satisfied [4]:

$$\lambda^{t_{k-1}} = \left(\frac{\partial x^{t_k}}{\partial x^{t_{k-1}}}\right)^T\lambda^{t_k} = \left(\frac{\partial \mathcal{M}}{\partial x}(x^{t_{k-1}})\right)^T\lambda^{t_k} = \mathbf{M}^T{}_{k-1}\lambda^{t_k}\,,$$

$$\lambda^{t_N} = \left(\frac{\partial \Psi(x^{t_N})}{\partial x^{t_N}}\right)^T, \quad k \in \{1\dots N\}\,,$$

it can be shown that the adjoint variables or influence functions λ^{t_k} [5] represent the gradients of the cost function with respect to perturbations in the state at earlier times $\lambda^{t_k} = \left(\frac{\partial \Psi(x^{t_N})}{\partial x^{t_k}}\right)^T = \nabla_{x^{t_k}}\Psi(x^{t_N})$. Note that we evolve the adjoint variable λ^{t_k} backwards in time, starting at the final time and taking steps with the *adjoint model* $\mathbf{M}^T = \left(\frac{\partial \mathcal{M}}{\partial x}\right)^T$ back to the initial time. As we did in Equation

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(2), we can also consider the scaled adjoint sensitivity $\widehat{\lambda}$, which can be physically interpreted as the percentage change in the cost function when the variable $x_i^{t_k}$ is changed:

$$\widehat{\lambda}_i^{t_k} = \frac{\partial \Psi(x^{t_N})}{\partial x_i^{t_k}} \frac{x_i^{t_k}}{\Psi(x^{t_N})} \,. \tag{4}$$

Large sensitivity values indicate areas of influence, that is, locations where errors or perturbations in the current state will produce significant changes in the target sites and time as described through the cost function.

III. Results and Discussion

We illustrate the ASA method on a real test case and employ the Weather Research and Forecasting (WRF) model [6], a state-of-the-art NWP system that will take the place of $\mathcal{M}$ (see [2] for implementation details). A simplified WRF model has been run through a source-to-source program called Transformation of Algorithm in FORTRAN (TAF) to automatically produce both gradients ($\mathbf{M}$) and adjoints of the gradients ($\mathbf{M}^T$).

We estimate the sensitivity of the wind speed $\Psi = \sqrt{U^2+V^2}$, where U and V are the W-E and S-N wind components, respectively. The initial adjoint values at the final time are therefore given by the gradient of the scaled cost function

$$\widehat{\lambda}^{t_N} = \frac{\partial \Psi}{\partial \bullet}\frac{\bullet}{\Psi} = \frac{(\bullet)^2}{U^2+V^2} \,, \quad \bullet = \{U, V\} \,. \tag{5}$$

The scaled version is useful because it allows us to compare sensitivities in model states with different units of measure, for instance, wind speed and temperature.

To illustrate the results that can be obtained from such an analysis, we employ WRF with real data and perform simulations in June ’06. In Fig. 1 we show the wind-speed sensitivity with respect to the wind speed in a northern Texas region (with a rich density of wind farms) 6 and 12 hours before the target time. The larger the value, the more sensitive is the final-time target solution to the current state. In other words, the sensitivity at the final time (5) is propagated backwards 6 and 12 hours and gives a measure of influence of the initial condition on the final target state 6 and 12 hours ahead. The direction of the wind sensitivity is illustrated in Fig. 2, in which different sources are shown to affect the target area on different days. This analysis points to the dynamic size of the domain necessary for such a simulation to efficiently achieve accurate forecasts. In Fig. 3a we show the sensitivity with respect to temperature field that indicates the temperature-wind relationship. In Fig. 3b we show averaged most sensitive locations over a month that point to the best locations for wind-speed sensors to improve wind-speed predictions 12 hours ahead.

The high sensitivity regions illustrated in this study indicate areas with high impact on the future wind speed conditions. It is therefore important that these regions be resolved accurately by the NWP models and observed by TSP methods.

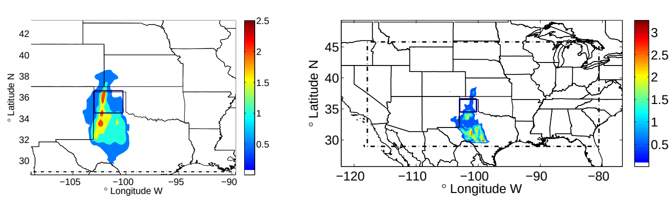
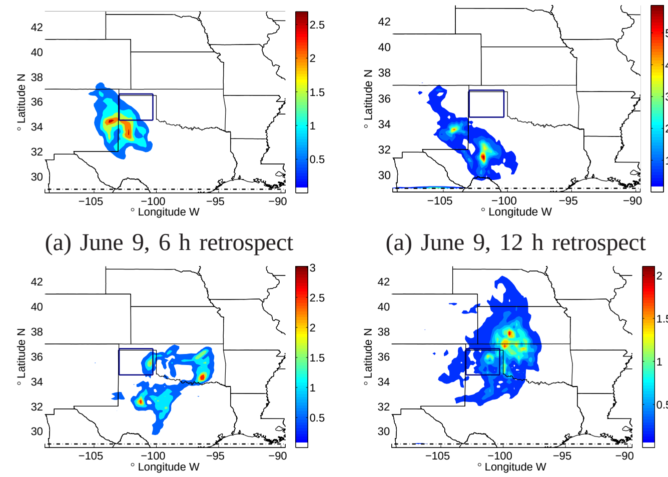
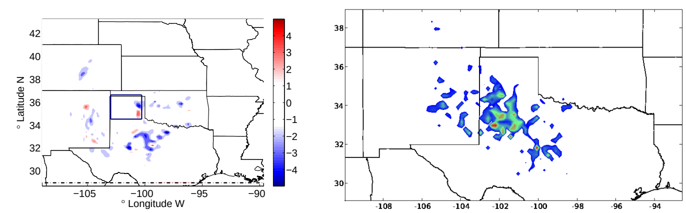


Fig. 1. Wind-speed sensitivity with respect to the wind speed in a region in northern Texas (solid line caret) with 6 hours (zoomed-in, left) and 12 hours (right) before the final time accumulated on all height levels.



(a) June 9, 6 h retrospect (a) June 9, 12 h retrospect (a) June 12, 12 h retrospect (a) June 23, 12 h retrospect

Fig. 2. Wind-speed sensitivity with respect to the wind speed 6 and 12 hours before the final time on June 9, 12, and 23, 2006.



a) Temperature sensitivity b) Average sensitivity

Fig. 3. Wind-speed sensitivity with respect to the temperature 12 hours before the final time (a) and average of the most sensitive locations of the wind field 12 hours before the final time for June 2006 (b).

We have presented an adjoint sensitivity analysis that can be used to indicate regions in the physical and state space that should be better resolved or observed to improve forecast activities for robust planning and operation of wind farms. Specific information that can be extracted from such an analysis includes resolution, domain size, and modeled states for NWP approaches. Furthermore, a good indication of the best sensor location and type can be obtained. If the forecast is done through TSP, one can obtain sensor types and locations that are likely to improve the outcome of AR-type and ANN prediction techniques.

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