Stat 310, Part II, Optimization. Homework 3.

Problem 1: (computation; trust region Steihaug CG)

Implement the trust-region CG-Steihaug Algorithm 7.2 from the textbook (in conjunction with the trust-region management algorithm 4.1). Choose *Bk* to be the exact Hessian. Use the same ε_k rule from Algorithm 7.1 Apply the code to solve Fenton's function starting at both [3 2] and [3 4]. Report the total number of matrix-vector multiplications and the total number of function evaluations.

Apply the algorithm to the "cute" function from the previous homework. Experiment with increasing values of the size of the problem (start with 10). Choose the stopping tolerance on the size of the gradient to be relatively large (say 1e-3 to 1e-4, but experiment with it as well).

Problem 2: (theory, optimality conditions). Problem 12.19 from the textbook.

Consider the problem:

$$\min_{x\in\mathbb{R}^2} f(x) = -2x_1 + x_2 \text{ subject to } \begin{cases} (1-x_1)^3 - x_2 \geq 0 \\ x_2 + 0.25x_1^2 - 1 \geq 0 \end{cases}$$

The optimal solution is $x^* = (0,1)^T$ at which both constraints are active.

1. Do the LICQ conditions hold at this point?
2. Are the KKT conditions satisfied?
3. Write down the sets $\mathcal{F}(x^*)$ and $\mathcal{C}(x^*,\lambda^*)$
4. Are the second-order necessary conditions satisfied? Are the second-order sufficient conditions satisfied ?