Mihai Anitescu, assigned 03/06/2011, due 03/13/2011 (drop in my mailbox or with Stats Receptionist, mention it is for me).

Stat 310, Part II, Optimization. Homework 4.

Problem 1: (computation;projected gradient for QP)

Implement Gradient Projection Algorithm 16.5 from the textbook (use a version of the truncated conjugated gradient approach that you implemented in Homework 3 to solve subproblem 16.74, instead of checking if the point is inside the trust region, check if it satisfies the bound constraints of 16.74).

Apply it to the following problem:

$$\min_x \frac{1}{2} x^T Q x - x^T f$$
$$l_i \le x_i \le u_i$$

where f=100*ones(n,1), l=0.5*ones(n,1), u=3.5*ones(n,1) and Q is the hessian of the cute problem from previous homework computed at ones(n,1). Start the algorithm from the point 2*ones(n,1). Record the number of matrix-vector multiplications for increasing values of n (start at about 10). For sanity, include a pseudocode of the overall algorithm. **Optional:** Use this algorithm to implement the Augmented Lagrangian Method Described in class for general nonlinear programming.

2. **(Optional)** Consider the constrained optimization problem

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & f(x) \\ s.t. \quad & c_E(x) = 0 \\ & c_I(x) \ge 0 \end{aligned}$$

Assume that the point x^* is the solution of this problem, at which the linear independent constraint qualification (LICQ) holds. Using the necessary optimality conditions for problems with equality constraints applied to the equivalent formulation

$$\begin{aligned} \min_{x\in\mathbb{R}^n, z\in\mathbb{R}^{n_I},} \quad & f(x) \\ s.t. \quad & c_E(x) = 0 \\ & \left[c_I(x)\right]_j - z_j^2 = 0 \quad j = 1,2,\ldots n_I \end{aligned}$$

prove that the first and second-order necessary optimality conditions for problems with inequality constraints assuming that the ones for equality constraints do hold (at x^* (you can assume f,c as regular as needed).

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3. Let $A \in \mathbb{R}^{m \times n}$ have full row rank. Let $Q \in \mathbb{R}^{n \times n}$ be such that $u \neq 0, Au = 0 \Rightarrow u^T Q u > 0$. Prove that
   a. There exists $c > 0$ such that $Q + cA^T A$ is positive definite.
   b. The matrix $\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix}$ is not singular.
   c. Assume that you have an equality-constrained problem for which the LICQ and the second-order sufficient conditions hold. Discuss the implications of a), for the existence of an augmented Lagrangian and of b) for the local well-posedness of the system of nonlinear equations that is formed by the first-order optimality conditions of the problem.