A Stochastic Electricity Market Clearing Formulation with Consistent Pricing Properties*

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We argue that deterministic market clearing formulations introduce strong and arbitrary distortions between day-ahead and expected real-time prices that bias economic incentives and block diversification. We extend and analyze the stochastic clearing formulation proposed by Pritchard et al. (2010) in which the social surplus function induces ℓ_1 penalties between day-ahead and real-time quantities. We prove that the formulation yields price distortions that are bounded by the bid prices, and we show that adding a similar penalty term to transmission flows ensures boundedness throughout the network. We prove that when the price distortions are zero, day-ahead quantities and flows converge to the medians of real-time counterparts. We demonstrate that convergence to expected value quantities can be induced by using a squared ℓ_2 penalty. The undesired effects of price distortions suggest that arguments based on social surplus alone are insufficient to fully appreciate the benefits of stochastic market settlements. We thus propose additional metrics to evaluate these benefits.

Key words: stochastic, electricity, network, market clearing, pricing

1. Introduction

Day-ahead markets enable commitment and pricing of resources to hedge against uncertainty in demand, generation, and network capacities that are observed in real time. The day-ahead market is cleared by independent system operators (ISOs) using deterministic unit commitment (UC) formulations that rely on expected capacity forecasts, while uncertainty is handled by allocating reserves that are used to balance the system if real-time capacities deviate from the forecasts. A large number of deterministic clearing formulations have been proposed in the literature. Representative examples include those of Carrión and Arroyo (2006), Gribik et al. (2011), and Hobs (2001). Pricing issues arising in deterministic clearing formulations have been explored by Wang et al. (2012), Galiana et al. (2003), and O'Neill et al. (2005).

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In addition to guaranteeing reliability and maximizing social surplus, several metrics are monitored by ISOs to ensure that the market operates efficiently. For instance, as is discussed in Ott (2003), the ISO must ensure that market players receive appropriate economic incentives that promote participation. It is also desired that day-ahead and real-time prices are sufficiently close or converge. One of the reasons is that price convergence is an indication that capacity forecasts be effective reflections of real-time capacities. Recent evidence provided by Bowden et al. (2009), Birge et al. (2013), however, has shown that persistent and predictable deviations between day-ahead and real-time prices (premia) exist in certain markets. This can bias the incentives to a subset of players and block the entry of new players and technologies. Consequently, in designing an appropriate pricing scheme, ISOs must achieve fairness. The introduction of purely financial players was intended to eliminate premia, but recent evidence provided by Birge et al. (2013) shows that this has not been fully effective. One hypothesis is that virtual players can exploit predictable price differences in the day-ahead market to create artificial congestion and benefit from financial transmission rights (FTRs).

Prices are also monitored by ISOs to ensure that they do not run into financial deficit (a situation called revenue inadequacy) when balancing payments to suppliers and from consumers. This is discussed in detail in Philpott and Pritchard (2004). In addition, ISOs might need to use uplift payments and adjust prices to protect suppliers from operating at an economic loss. This is necessary to prevent players from leaving the market. As discussed by O'Neill et al. (2005) and Wang et al. (2012), uplift payments can result from using improper characterizations of the system (e.g., nonconvexities) in the clearing model.

Achieving efficient market operations under intermittent renewable generation is a challenge for the ISOs because uncertainty follows complex spatiotemporal patterns not faced before (Constantinescu et al. (2011)). In addition, the power grid is relying more strongly on natural gas and transportation infrastructures, and it is thus necessary to quantify and mitigate uncertainty in more systematic ways (Liu et al. (2009), Zavala (2014)). Doing so will enable the ISOs to efficiently allocate reserves thoughout the network and promote fair and efficient market conditions.

1.1. Previous Work

A wide range of stochastic formulations of day-ahead market clearing and operational UC procedures has been previously proposed. In operational UC models, on/off decisions are made in advance (here-and-now) to ensure that enough running capacity is available at future times to balance the system. The objective of these formulations is to ensure reliability and maximization of social surplus (cost in case of inelastic demands) in intra-day operations. Examples include the works of Takriti et al. (1996), Wang et al. (2008), Constantinescu et al. (2011), Jin et al. (2013), Ruiz et al. (2009), Bouffard et al. (2005). These studies have demonstrated significant improvements in reliability over deterministic formulations. However, these works consistently report marginal improvements in expected social surplus. In addition, these formulations do not explore pricing issues.

Stochastic day-ahead clearing formulations have been proposed by Kaye et al. (1990) and Wong and Fuller (2007). Kaye et al. (1990) analyse day-ahead and real-time markets under uncertainty and argue that day-ahead prices should be set to expected values of the real-time prices. This *price consistency* ensures that the day-ahead market does not bias real-time market incentives in the long run. Such consistency also avoids arbitrage as is argued by Khazaei et al. (2013).Wong and Fuller (2007) propose pricing schemes to achieve appropriate economic incentives (cost recovery) for all suppliers. The pricing schemes, however, rely only partially on dual variables generated by the stochastic clearing model which is adjusted to achieve cost recovery. Consequently, these procedures do not guarantee dual and model consistency.

Morales et al. (2012) propose a stochastic clearing model to price electricity in pools with stochastic producers. Their model co-optimizes energy and reserves and they prove that it leads to revenue adequacy in expectation. In addition, they prove that prices allow for cost recovery in expectation for all players (i.e., no uplifts are needed in expectation) but pricing consistency is not explored.

Pritchard et al. (2010) propose a stochastic formulation that captures day-ahead and real-time bidding of both suppliers and consumers. The formulation maximizes the day-ahead social surplus and the expected value of the real-time corrections by considering the possibility of players bidding in the real-time market. The authors prove that the formulation leads to revenue adequacy in expectation and provide conditions under which adequacy will hold for each scenario. The authors do not explore pricing consistency and economic incentives.

Khazaei et al. (2013) propose a stochastic equilibrium formulation in which players bid parameters of a quadratic supply function to maximize the expected value of their profit function while the ISO uses these parameters to solve the clearing model and generate day-ahead and real-time quantities and prices. It is shown that the equilibrium model generates day-ahead prices that converge to expected value prices and thus achieve consistency. It is also shown that day-ahead quantities converge to expected value quantities and a small case study is presented to demonstrate that the formulation yields higher social surplus and producer profits compared to deterministic clearing. The proposed formulation uses a quadratic supply function and quadratic penalties for deviations between day-ahead and real-time quantities. No network and no capacity constraints are considered.

Morales et al. (2014) propose a bilevel stochastic optimization formulation that uses forecast capacities of stochastic suppliers as degrees of freedom. Using small computational studies, they demonstrate that their framework provides cost recovery for all suppliers and for each scenario. The authors, however, do not discuss the effects of the modified pricing strategy on consumer payments (the demands are treated as inelastic) and do not discuss incentives and fairness issues. In addition, no theoretical guarantees are provided. In particular, it is not guaranteed that a set of day-ahead capacities and prices exist that can achieve cost recovery forboth suppliers and consumers in each scenario. While plausible, we believe that this requires further evidence and theoretical justification.

1.2. Contributions of This Work

In this work, we argue that deterministic formulations generate day-ahead prices that are distorted representations of expected real-time prices. This pricing inconsistency arises because solving a day-head clearing model using summarizing statistics of uncertain capacities (e.g., expected forecasts) does not lead to day-ahead prices that are expected values of the real-time prices. We argue that these price distortions lead to diverse issues such as the need of uplift payments as well as arbitrary and biased incentives that block diversification. We extend and analyze the stochastic clearing formulation of Pritchard et al. (2010) in which linear supply functions for day-ahead and real-time markets are used. The structure of this surplus function has the key property that if real-time bid prices are symmetric relative to the day-ahead prices, an ℓ_1 term arises that penalizes deviations between day-ahead and real-time quantities. We prove that this formulation yields price distortions that are bounded by the smallest real-time bid price and therefore it is very robust. We also prove that when the price distortion is zero, the formulation yields day-ahead quantities and network flows that converge to the median of their real-time counterparts. In addition, we prove that the formulation yields revenue adequacy in expectation and yields zero uplifts in expectation. We provide several case studies to demonstrate the properties of the stochastic formulation. Moreover, we demonstrate that quadratic supply functions induce day-ahead quantities and flows that converge to the means of real-time counterparts.

The paper is structured as follows. In Section 2 we describe the market setting. In Section 3 we present deterministic and stochastic formulations of the day-ahead ISO clearing problem. In Section 4 we present a set of performance metrics to assess the benefits of the stochastic formulation over its deterministic counterpart. In Section 5 we present the pricing properties of the formulation. In Section 6 we present case studies to demonstrate the developments. Pricing properties of quadratic penalty functions are presented in Section 7. Concluding remarks and directions of future work are provided in Section 8.

2. Market Setting

We consider a market setting based on the work of Pritchard et al. (2010) and Ott (2003). A set of suppliers (suppliers) $\mathcal{G}$ and consumers (demands) $\mathcal{D}$ bid into the day-ahead market by providing price bids $\alpha_i^g \geq 0$, $i \in \mathcal{G}$ and $\alpha_j^d \geq 0$, $j \in \mathcal{D}$, respectively. If a given demand is inelastic, we set the bid price to $\alpha_j^d = VOLL$ where $VOLL$ denotes the value of lost load, typically 1,000 \$/MWh. Suppliers and consumers also provide estimates of the available capacities $\bar{g}_i$ and $\bar{d}_j$, respectively. We assume that these capacities satisfy $0 \leq \bar{g}_i \leq Cap_i^g$ and $0 \leq \bar{d}_j \leq Cap_j^d$ where $0 \leq Cap_i^g < +\infty$ is the total installed capacity of the supplier (its maximum possible supply) and $0 \leq Cap_i^d < +\infty$ is the total installed capacity of the consumer (its maximum possible demand). The cleared day-ahead quantities for suppliers and consumers are given by g_i and d_j, respectively. These satisfy $0 \leq g_i \leq \bar{g}_i$ and $0 \leq d_j \leq \bar{d}_j$.

Suppliers and consumers are connected through a network comprising of a set of lines $\mathcal{L}$ and a set of nodes $\mathcal{N}$. For each line $\ell \in \mathcal{L}$ we define its sending node as $snd(\ell) \in \mathcal{N}$ and its receiving node as $rec(\ell) \in \mathcal{N}$. For each node $n \in \mathcal{N}$, we define its set of receiving lines as $\mathcal{L}_n^{rec} \subseteq \mathcal{L}$ and its set of sending lines as $\mathcal{L}_n^{snd} \subseteq \mathcal{L}$. These sets are given by

$$\mathcal{L}_n^{rec} = \{\ell \in \mathcal{L} \,|\, n = rec(\ell)\},\ n \in \mathcal{N} \tag{1a}$$

$$\mathcal{L}_n^{snd} = \{\ell \in \mathcal{L} \,|\, n = snd(\ell)\},\ n \in \mathcal{N}. \tag{1b}$$

Day-ahead capacities $\bar{f}_\ell$ are also typically estimated for the transmission lines. We assume that these satisfy $0 \leq \bar{f}_\ell \leq Cap_\ell^f$. Here, $0 \leq Cap_\ell^f < +\infty$ is the installed capacity of line (its maximum possible value). We define the set of all suppliers connected to node $n \in \mathcal{N}$ as $\mathcal{G}_n \subseteq \mathcal{G}$ and the set of demands connected to node n as $\mathcal{D}_n \subseteq \mathcal{D}$. Subindex $n(i)$ indicates the node at which supplier $i \in \mathcal{G}$ is connected, and $n(j)$ indicates the node at which the demand $j \in \mathcal{D}$ is connected. We use subindex i exclusively for suppliers and subindex j exclusively for consumers.

At the moment the day-ahead market is cleared, the real-time market conditions are uncertain. In particular, we assume that a subset of generation, demand, and transmission line capacities are uncertain. We further assume that discrete distributions comprising a finite number of scenarios Ω and $p(\omega)$ denote the probability of scenario $\omega \in \Omega$. We also require that $\sum_{\omega \in \Omega} p(\omega) = 1$. The expected value of variable $Y(\cdot)$ is given by $\mathbb{E}[Y(\omega)] = \sum_{\omega \in \Omega} p(\omega) Y(\omega)$. If $Y(\omega)$ is scalar-valued, the median is denoted as $\mathbb{M}[Y(\omega)]$ and satisfies the following properties,

$$\mathbb{P}(Y(\omega) \geq \mathbb{M}[Y(\omega)]) = \mathbb{P}(Y(\omega) \leq \mathbb{M}[Y(\omega)]) = \frac{1}{2} \tag{2a}$$

$$\mathbb{M}[Y(\omega)] = \underset{m}{\operatorname{argmin}}\, \mathbb{E}[|Y(\omega) - m|], \tag{2b}$$

where $|\cdot|$ is the absolute value function. The second expression can be used as a definition for a median for the case of a multivariate $Y(\omega)$.

In the real-time market, the suppliers can offer to sell additional generation over the agreed day-ahead quantities at a bid price $\alpha_i^{g,+} \geq 0$. The additional generation is given by $(G_i(\omega) - g_i)_+$ where $G_i(\omega)$ is the cleared quantity in the real-time market and $0 \leq \bar{G}_i(\omega) \leq Cap_i^g$ is the realized capacity under scenario $\omega \in \Omega$. Real-time generation quantities are bounded as $0 \leq G_i(\omega) \leq \bar{G}_i(\omega)$. Here, $(X - x)_+ := \max\{X - x, 0\}$. The suppliers also have the option of buying electricity at an offering price $\alpha_i^{g,-} \geq 0$ to account for any uncovered generation $(G_i(\omega) - g_i)_-$ over the agreed day-ahead quantities. Here, $(X - x)_- = \max\{-(X - x), 0\}$.

Consumers provide bid prices $\alpha_j^{d,-} \geq 0$ to buy additional demand $(D_j(\omega) - d_j)_+$ in the real-time market, where $D_j(\omega)$ is the cleared quantity and $0 \leq \bar{D}_j(\omega) \leq Cap_j^d$ is the available demand capacity realized under scenario $\omega \in \Omega$. We thus have $0 \leq D_j(\omega) \leq \bar{D}_j(\omega)$. Consumers also have the option of selling the demand decifit $(D_j(\omega) - d_j)_-$ at price $\alpha_j^{d,+} \geq 0$.

The flows cleared in the real-time market are given by $F_\ell(\omega)$ and satisfy $0 \leq F_\ell(\omega) \leq \bar{F}_\ell(\omega)$. Here, $\bar{F}_\ell(\omega)$ is the transmission line capacity realized under scenario $\omega \in \Omega$ and satisfies $0 \leq \bar{F}_\ell(\omega) \leq Cap_\ell^f$. Uncertain line capacities can be used to model $N - x$ contingencies or uncertainties in capacity due to ambient conditions (e.g., ambient temperature affects line capacity).

We also define day-ahead clearing prices (i.e., locational marginal prices) for each node $n \in \mathcal{N}$ as π_n. The real-time prices are defined as $\Pi_n(\omega)$, $\omega \in \Omega$.

3. Clearing Formulations

In this section, we present *energy-only* day-ahead deterministic and stochastic clearing formulations. The term "energy-only" indicates that no unit commitment decisions are made. We consider these simplified formulations in order to focus on important concepts related to pricing and payments to suppliers and consumers. Model extensions are left as a topic of future research.

3.1. Deterministic Formulation

In a deterministic setting, the day-ahead market is cleared by solving the following optimization problem.

$$\min_{d_j, g_i, f_\ell} \quad \sum_{i \in \mathcal{G}} \alpha_i^g g_i - \sum_{j \in \mathcal{D}} \alpha_i^d d_j \tag{3a}$$

$$\text{s.t.} \sum_{\ell \in \mathcal{L}_n^{rec}} f_\ell - \sum_{\ell \in \mathcal{L}_n^{snd}} f_\ell + \sum_{i \in \mathcal{G}_n} g_i - \sum_{i \in \mathcal{D}_n} d_i = 0, \quad (\pi_n) \quad n \in \mathcal{N} \tag{3b}$$

$$-\bar{f}_\ell \leq f_\ell \leq \bar{f}_\ell, \qquad \ell \in \mathcal{L} \tag{3c}$$

$$0 \leq g_i \leq \bar{g}_i, \qquad i \in \mathcal{G} \tag{3d}$$

$$0 \leq d_j \leq \bar{d}_j, \qquad j \in \mathcal{D} \tag{3e}$$

The objective function of this problem is the day-ahead *negative* social surplus. The solution of this problem gives the day-ahead quantities g_i, d_j, flows f_ℓ, and prices π_n. The deterministic formulation assumes a given value for the capacities $\bar{g}_i, \bar{d}_j$, and $\bar{f}_\ell$. Because the conditions of the real-time market are uncertain at the time the day-ahead problem (3) is solved, these capacities are typically assumed to be the most probable ones (e.g., the expected value or *forecast* for supply and demand capacities) or are set based on the current state of the system (e.g., for line capacities). In particular, it is usually assumed that $\bar{g}_i = \mathbb{E}[\bar{G}_i(\omega)]$, $\bar{d}_j = \mathbb{E}[\bar{D}_j(\omega)]$, and $\bar{f}_\ell$ is the most probable state. One can also assume that $\bar{g}_i = Cap_i^g$ and $\bar{d}_j = Cap_j^d$, and $\bar{f}_\ell = Cap_\ell^f$. Such an assumption, however, can yield high economic penalties if the day-ahead dispatched quantities are far from those realized in the real-time market. Similarly, one can also assume conservative values worst-case capacities. This approach, however, can also yield high economic penalties. In this sense, note that day-ahead capacities $\bar{g}_i, \bar{d}_j, \bar{f}_{ell}$ provided by market players can be used as mechanisms to hedge against risk. Doing so, however, gives only limited control because the players need to summarize the entire possible range of real-time capacities in one statistic. In Section 4 we argue that this limitation induces a distortion between day-ahead and real-time prices and biases revenues.

When the capacities become known, the ISO uses fixed day-ahead commited quantities g_i, d_j, f_ℓ, to solve the following real-time clearing problem.

$$\min_{D_j(\cdot),G_i(\cdot),F_\ell(\cdot)} \quad \sum_{i\in\mathcal{G}} \left(\alpha_i^{g,+}(G_i(\omega)-g_i)_+ - \alpha_i^{g,-}(G_i(\omega)-g_i)_-\right) \tag{4a}$$

$$-\sum_{j\in\mathcal{D}} \left(\alpha_j^{d,+}(D_j(\omega)-d_j)_+ - \alpha_j^{d,-}(D_j(\omega)-d_j)_-\right) \tag{4b}$$

$$\text{s.t.} \sum_{\ell\in\mathcal{L}_n^{rec}} F_\ell(\omega) - \sum_{\ell\in\mathcal{L}_n^{snd}} F_\ell(\omega) + \sum_{i\in\mathcal{G}_n} G_i(\omega) - \sum_{j\in\mathcal{D}_n} D_j(\omega) = 0, \quad (\Pi_n(\omega)), \quad n\in\mathcal{N} \tag{4c}$$

$$-\bar{F}_\ell(\omega) \le F_\ell(\omega) \le \bar{F}_\ell(\omega), \qquad \ell\in\mathcal{L} \tag{4d}$$

$$0 \le G_i(\omega) \le \bar{G}_i(\omega), \qquad i\in\mathcal{G} \tag{4e}$$

$$0 \le D_j(\omega) \le \bar{D}_j(\omega), \qquad j\in\mathcal{D} \tag{4f}$$

The objective function of this problem is the real-time negative social surplus. The solution of this problem yields different real-time quantities $G_i(\omega), D_j(\omega)$, flows $F_\ell(\omega)$, and prices $\Pi_n(\omega)$ depending on the scenario $\omega\in\Omega$ realized.

4. ISO Performance Metrics

In this section, we discuss some objectives of the ISOs from a market operations standpoint and use these to motivate a new set of metrics to quantify the benefits of using stochastic formulations over deterministic counterparts. We place special emphasis on the structure of the *social surplus function* and on the issue of *price consistency*. We provide arguments as to why price consistency is a key property in achieving fair incentives. We argue that deterministic formulations do not actually yield price consistency and hence result in a range of undesired effects such as biased payments, revenue inadequacy, and the need for uplifts.

4.1. Social Surplus

Consider the combination of the day-ahead and real-time costs for suppliers and consumers,

$$C_i^g(\omega) = +\alpha_i^g g_i + \alpha_i^{g,+}(G_i(\omega) - g_i)_+ - \alpha_i^{g,-}(G_i(\omega) - g_i)_- \tag{5a}$$

$$C_j^d(\omega) = -\alpha_j^d d_i + \alpha_j^{d,+}(D_j(\omega) - d_j)_- - \alpha_j^{d,-}(D_j(\omega) - d_j)_+. \tag{5b}$$

We analyze the particular case in which the players bid prices satisfy the following *symmetry property*: $\alpha_i^{g,+} - \alpha_i^g = \alpha_i^g - \alpha_i^{g,-} = \Delta\alpha_i^g$ and $\alpha_j^{d,+} - \alpha_j^d = \alpha_j^d - \alpha_j^{d,-} = \Delta\alpha_j^d$. We refer to $\Delta\alpha_i^g$ and $\Delta\alpha_j^d$ as the *incremental bid prices*. To avoid degeneracy, we require that $\Delta\alpha_i^g > 0$ and $\Delta\alpha_j^d > 0$.

Theorem 1. *Assume that the day-ahead and real-time bids satisfy $\alpha_i^{g,+} - \alpha_i^g = \alpha_i^g - \alpha_i^{g,-} = \Delta\alpha_i^g$ and $\alpha_j^{d,+} - \alpha_j^d = \alpha_j^d - \alpha_j^{d,-} = \Delta\alpha_j^d$. The cost functions for suppliers and consumers become*

$$C_i^g(\omega) = +\alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|, \quad i \in \mathcal{G}, \omega \in \Omega \tag{6a}$$

$$C_j^d(\omega) = -\alpha_j^d D_j(\omega) + \Delta\alpha_j^d |D_j(\omega) - d_j|, \quad j \in \mathcal{D}, \omega \in \Omega. \tag{6b}$$

Proof:

$$\begin{aligned}
C_i^g(\omega) &= \alpha_i^g g_i + \alpha_i^{g,+}(G_i(\omega) - g_i)_+ - \alpha_i^{g,-}(G_i(\omega) - g_i)_- \\
&= \alpha_i^g g_i + (\alpha_i^g + \Delta\alpha_i^g)(G_i(\omega) - g_i)_+ - (\alpha_i^g - \Delta\alpha_i^g)(G_i(\omega) - g_i)_- \\
&= \alpha_i^g g_i + \alpha_i^g (G_i(\omega) - g_i)_+ - \alpha_i^g (G_i(\omega) - g_i)_- + \Delta\alpha_i^g (G_i(\omega) - g_i)_+ + \Delta\alpha_i^g (G_i(\omega) - g_i)_- \\
&= \alpha_i^g g_i + \alpha_i^g (G_i(\omega) - g_i) + \Delta\alpha_i^g |G_i(\omega) - g_i)| \\
&= \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i)|.
\end{aligned} \tag{7}$$

The last three equalities follow from the facts that $|X - x| = (X - x)_+ + (X - x)_-$ and $X - x = (X - x)_+ - (X - x)_-$. The same property applies to $C_j^d(\omega)$ (using the appropriate cost terms). □

Definition 1. (**Social Surplus**) We define the *expected negative social surplus* (or *social surplus* for short) as

$$\begin{aligned}
\varphi &:= \mathbb{E}\left[\sum_{i \in \mathcal{G}} C_i^g(\omega) + \sum_{j \in \mathcal{D}} C_j^d(\omega)\right] \\
&= \varphi^g + \varphi^d
\end{aligned} \tag{8}$$

with $C_i^g(\cdot), C_j^d(\cdot)$ defined as in (6) and where φ^g, φ^d are the *expected supply and consumer costs*,

$$\begin{aligned}\varphi^g &:= \mathbb{E}\left[\sum_{i\in\mathcal{G}} C_i^g(\omega)\right] &\text{(9a)}\\ &= \sum_{i\in\mathcal{G}} \left(+\alpha_i^g \mathbb{E}[G_i(\omega)] + \Delta\alpha_i^g \mathbb{E}\left[|G_i(\omega) - g_i|\right]\right)\\ \varphi^d &:= \mathbb{E}\left[\sum_{j\in\mathcal{D}} C_j^d(\omega)\right] &\text{(9b)}\\ &= \sum_{j\in\mathcal{D}} \left(-\alpha_j^d \mathbb{E}\left[D_j(\omega)\right] + \Delta\alpha_j^d \mathbb{E}\left[|D_j(\omega) - d_j|\right]\right).\end{aligned}$$

This particular structure of the expected social suplus function was noticed by Pritchard et al. (2010) and provides interesting insights. From Equation (9), we note that the expected quantities $\mathbb{E}[G_i(\omega)]$, $\mathbb{E}[D_j(\omega)]$ act as forecasts of the day-ahead quantities and are priced by using the day-ahead bids α_i^g, α_d^j (first term). This immediately suggests that it is the *expected cleared quantities* $G_i(\omega), D_j(\omega)$ and *not* the capacities $\bar{g}_i, \bar{d}_j$ that are to be used as forecasts, as is done in the day-ahead formulation (3). The second term penalizes deviations of the real-time quantities from the day-ahead commitments using the incremental bids $\Delta\alpha_i^g$ and $\Delta\alpha_j^d$. The ℓ_1 structure of the second term also suggests that if the expected social surplus function is minimized, day-ahead quantities will tend to converge to the median of the real-time quantities, because of property (2b). A deterministic setting, however, cannot guarantee optimality in this sense because it minimizes the day-ahead and real-time components of the surplus function separately. The expected social surplus for the deterministic formulation is obtained by solving the day-ahead problem (3) followed by the solution of the real-time problem (4) for all scenarios $\omega \in \Omega$. The day-ahead surplus and the expected value of the real-time surplus are then combined to obtain the expected surplus φ^{det}.

A deterministic setting can yield surplus inefficiencies because it cannot properly anticipate the effect of day-ahead decision on real-time market decisions. For instance, certain suppliers can be inflexible in the sense that they cannot modify their day-ahead supply easily in the real-time market (e.g., coal plants). This results in constraints of the form $g_i = G_i(\omega), \omega \in \Omega$ or $d_j = D_j(\omega), \omega \in \Omega$. This inflexibility can trigger inefficiencies because the operator is forced to use expensive units in the real-time market (e.g., combined-cycle) or because load shedding is needed to prevent infeasibilities. Most studies on stochastic market clearing and unit commitment have focused on showing improvements in social surplus over deterministic formulations. Many of those reports, however, report minimal benefits. In Section 6 we demonstrate that even when social surplus differences are negligible, the resulting prices and payments can be drastically different. Consequently, surplus can be a misleading metric. This situation motivates us to consider alternative metrics for monitoring performance.

In the following discussion, we define different metrics based on market behavior in expectation. A practical way of interpreting these *expected metrics* is the following: assume that the market conditions of a given day are repeated over a sequence of days and we collect the results over such period by using each day as an scenario. We then compute a certain metric (like the social welfare) to perform the comparisons between the stochastic and deterministic clearing mechanisms to evaluate performance. In this sense, market behavior in expectation can also interpreted as long run market behavior.

4.2. Pricing Consistency

We seek that the day-ahead prices be consistent representations of the expected real-time prices. In other words, we seek that the *expected price distortions* (also known as *expected price premia*) $\pi_n - \mathbb{E}[\Pi_n(\omega)]$, $n \in \mathcal{N}$ be zero or at least in a bounded neighborhood. This is desired for various reasons that we will explain.

Definition 2. **(Price Distortions)** We define the *expected price distortion* or *expected price premia* as

$$\mathcal{M}_n^{\pi} := \pi_n - \mathbb{E}\left[\Pi_n(\omega)\right], \quad n \in \mathcal{N}. \tag{10}$$

We say that the *price is consistent* at node $n \in \mathcal{N}$ if $\mathcal{M}_n^{\pi} = 0$. In addition, we define the *node average* and *maximum absolute distortions*,

$$\mathcal{M}_{avg}^{\pi} := \frac{1}{|\mathcal{N}|} \sum_{n \in \mathcal{N}} |\mathcal{M}_n^{\pi}| \tag{11a}$$

$$\mathcal{M}_{max}^{\pi} := \max_{n \in \mathcal{N}} |\mathcal{M}_n^{\pi}|. \tag{11b}$$

Pricing consistency is related to the desire that day-ahead and real-time prices converge, as is discused by Ott (2003). Note, however, that it is unrealistic to expect that day-ahead and real-time prices converge in each scenario. This is possible only in the absence of uncertainty (capacity forecasts are perfect such as in the perfect information setting). Any real-time deviation in capacity from a day-ahead forecast will lead to a deviation between day-ahead and real-time prices. It is possible, however, to ensure that *day-ahead and real-time prices converge in expectation.* This situation also implies that any deviation of the real-time price from the day-ahead price is entirely the result of *unpredictable random factors.* This is also equivalent to saying that day-ahead prices converge to the expected value of the real-time prices.

Pricing consistency cannot be guaranteed with deterministic formulations because the day-ahead clearing model forecasts real-time capacities, not real-time quantities. Consequently, players are forced to "summarize" their possible real-time capacities in single statistics $\bar{d}_j, \bar{g}_i, \bar{f}_\ell$. Typically, expected values are used. This summarization, however, is inconsistent because it does not effectively averages out real-time market performance as the structure of the surplus function (9) suggests. In fact, as we show in Section 5, expected values need not be the right statistic to use in the day-ahead market. This is consistent with the observations made by Morales et al. (2014). In addition, we note that certain random variables might be difficult to summarize (e.g., if they follow multimodal and heavy-tailed distributions).

4.3. Suppliers and Consumer Payments

As argued by Kaye et al. (1990), we can also justify the desire of seeking price consistency by analyzing the *payments* to the market players. The payment includes the day-ahead settlement plus the correction payment given at real-time prices, as is the standard practice in market operations. For more details, see Ott (2003) and Pritchard et al. (2010).

DEFINITION 3. **(Payments)** The payments to suppliers and from consumers for scenario $\omega in \Omega$ are defined as follows:

$$\begin{aligned} P_i^g(\omega) &:= g_i\pi_{n(i)} + (G_i(\omega) - g_i)\Pi_{n(i)}(\omega) \\ &= g_i(\pi_{n(i)} - \Pi_{n(i)}(\omega)) + G_i(\omega)\Pi_{n(i)}(\omega), \quad i \in \mathcal{G}, \omega \in \Omega \qquad (12a) \\ P_j^d(\omega) &:= -d_j\pi_{n(i)} - (D_j(\omega) - d_j)\Pi_{n(j)}(\omega) \\ &= d_j(\Pi_{n(i)}(\omega) - \pi_{n(i)}) - D_j(\omega)\Pi_{n(j)}(\omega), \quad j \in \mathcal{D}, \omega \in \Omega. \qquad (12b) \end{aligned}$$

We say that the *expected payments are fair* if they satisfy

$$\mathbb{E}\left[P_i^g(\omega)\right] = +\mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \quad i \in \mathcal{G} \qquad (13a)$$
$$\mathbb{E}\left[P_j^d(\omega)\right] = -\mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}, \qquad (13b)$$

where

$$\mathbb{E}\left[P_i^g(\omega)\right] = +g_i\mathcal{M}^\pi_{n(i)} + \mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \quad i \in \mathcal{G} \qquad (14a)$$
$$\mathbb{E}\left[P_j^d(\omega)\right] = -d_j\mathcal{M}^\pi_{n(j)} - \mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}. \qquad (14b)$$

If the prices are consistent at each node $n \in \mathcal{N}$, the expected payments are fair. The definition of fairness is motivated by the following observations. The price distortion is factored in the expected payments. From (14) we see that price distortions (premia) can bias benefits toward a subset of players. In particular, if the premium at a given node is negative ($\mathcal{M}^\pi_n < 0$), a supplier will not benefit from the day-ahead markey but a consumer will. If $\mathcal{M}^\pi_n > 0$, the oppostive holds true. This situation can prevent consumers from providing price-responsive demands. We can thus conclude that price consistency ensures *payment fairness* with respect to suppliers and consumers.

Price consistency is also desired because, depending on their position toward risk, certain players can benefit more than others from exploiting the premia. If the premia are positive, risk-averse players benefit. If the premia are negative, risk-taking players benefit. Therefore, price consistency also ensures fairness in this sense. In fact, as discussed by Bessembinder and Lemmon (2002), and efficient market setting must ensure that price consistency holds (regardless of the players position toward risk) as the number of players converges to infinity.

Kaye Kaye et al. (1990) argue that setting the day-ahead prices to the expected real-time prices (price consistency) is desirable because it effectively eliminates the day-ahead component of the market. Consequently, the market operates (in expectation) as a pure real-time market. This situation is desirable because it implies that the day-ahead market does not interfere with the incentives provided by real-time markets. This is particularly important for players that benefit from real-time market variability (such as peaking units and price-response demands). This also implies that the ISO does not have any preference to either risk-taking or risk-averse players. We also highight that price consistency does not imply that premia do not exist; they can exist in each scenario but not in expectation.

Deterministic formulations can yield persistent price premia that benefit a subset of players or that can be used for market manipulation. For instance, consider the case in which a wind farm forecast has a very similar mean but very different variance (uncertainty) for several consecutive days. If the expected forecast is used, the day-ahead prices will be consistently the same for all days, thus making them more predictable and biased toward a subset of players. While the use of risk-adaptive reserves can help ameliorate this effect, this approach is not guaranteed to achieve price consistency.

4.4. Uplift Payments

From (14) we see that if the premium at a given node is negative ($\mathcal{M}_n^{\pi} < 0$), negative payments (losses) can be incurred by the suppliers. The reason is that g_i and the term $\mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right]$ are non-negative if the prices are non-negative. Moreover, even if the payments are positive, they might not cover the supplier costs, and the supplier will incur in a loss. This issue is analyzed by Wong and Fuller (2007) and Morales et al. (2012). It is thus desired that suppliers be paid at least as much as what they asked for and it is desired that consumers do not pay more than what they are willing to pay for. This is formally stated in the following definition.

Definition 4. (**Wholeness**) We say that suppliers and consumers are *whole in expectation* if

$$\mathbb{E}\left[P_i^g(\omega)\right] \geq \mathbb{E}\left[C_i^g(\omega)\right], \quad i \in \mathcal{G} \tag{15a}$$

$$-\mathbb{E}\left[P_j^d(\omega)\right] \leq -\mathbb{E}\left[C_j^d(\omega)\right], \quad j \in \mathcal{D}. \tag{15b}$$

If the players are not made whole, they can leave the market this hinders diversification. Uplift payments are routinely used by the ISOs to avoid this situation Galiana et al. (2003), Baldick et al. (2005). Uplift can result from inadequate representations of system behavior such as nonconvexities O'Neill et al. (2005). We will demonstrate that uplifts can also arise from inappropiate statistical representations of real-time market performance as that introduced by deterministic clearing. Consequently, uplift payments are a useful metric to determine the effectiveness of a given clearing formulation.

Definition 5. (**Uplift Payments**) We define the *expected uplift payments* to suppliers and consumers as

$$\mathcal{M}_i^U := -\min\left\{\mathbb{E}[P_i^g(\omega)] - \mathbb{E}[C_i^g(\omega)], 0\right\}, \quad i \in \mathcal{G} \tag{16a}$$

$$\mathcal{M}_j^U := -\min\left\{\mathbb{E}[P_j^d(\omega)] - \mathbb{E}[C_j^d(\omega)], 0\right\}, \quad j \in \mathcal{D}. \tag{16b}$$

We also define the total uplift as $\mathcal{M}^U := \sum_{i\in\mathcal{G}} \mathcal{M}_i^U + \sum_{j\in\mathcal{D}} \mathcal{M}_j^U$.

4.5. Revenue Adequacy

An efficient clearing procedure must ensure that the ISO does not run into financial deficit. In other words, the ISO must have a positive cash flow (payments collected from consumers are greater than the payments given to suppliers). We consider the following expected revenue definition, used by Pritchard et al. (2010), to assess performance with respect to this case.

Definition 6. (**Revenue Adequacy**) The *expected net payment to the ISO* is defined as

$$\begin{aligned}\mathcal{M}^{ISO} &:= \mathbb{E}\left[\sum_{i\in\mathcal{G}} P_i^g(\omega) + \sum_{j\in\mathcal{D}} P_j^d(\omega)\right] \\ &= \sum_{i\in\mathcal{G}} \mathbb{E}[P_i^g(\omega)] + \sum_{j\in\mathcal{D}} \mathbb{E}[P_j^d(\omega)].\end{aligned} \tag{17}$$

We say that the ISO is *revenue adequate in expectation* if $\mathcal{M}^{ISO} \le 0$.

Revenue adequacy guarantees that, in expectation, the ISO will not run into financial deficit. The collected revenue is used to pay for financial transmission rights (FTRs). This topic is analyzed by Philpott and Pritchard (2004). In particular, when a line is congested, a price difference is created between nodes, and this creates a payment to the holder of the FTR. This implies that, the stronger the friction, the more revenue the holder can collect. We define these FTR payments in an analogous manner to the suppliers and consumers payments,

Definition 7. **(FTRs)** The FTR payments for line $\ell \in \mathcal{L}$ and scenario $\omega \in \Omega$ are defined as

$$P_\ell^f(\omega) := f_\ell(\pi_{snd(\ell)} - \pi_{rec(\ell)}) + (F_\ell(\omega) - f_\ell)(\Pi_{snd(\ell)}(\omega) - \Pi_{rec(\ell)}(\omega)). \tag{18}$$

Using this definition we have that

$$\begin{aligned}\mathbb{E}\left[P^f_\ell(\omega)\right] &= f_\ell(\pi_{snd(\ell)} - \pi_{rec(\ell)}) + \mathbb{E}\left[(F_\ell(\omega) - f_\ell)(\Pi_{snd(\ell)}(\omega) - \Pi_{rec(\ell)}(\omega))\right] \\ &= f_\ell\left(\mathcal{M}^\pi_{snd(\ell)} - \mathcal{M}^\pi_{rec(\ell)}\right) + \mathbb{E}\left[F_\ell(\omega)\left(\Pi_{snd(\ell)}(\omega) - \Pi_{rec(\ell)}(\omega)\right)\right]. \end{aligned} \tag{19}$$

This definition is an extension of the deterministic variant presented by Philpott and Pritchard (2004). Note that the forward component of the payment is a function of the price distortion differences $\mathcal{M}^\pi_{snd(\ell)} - \mathcal{M}^\pi_{rec(\ell)}$. We now show the net payment to the ISO can be made equal to the sum of the expected FTR payments over all transmission lines and that the day-ahead component is eliminated if the distortions are zero. To establish this result, consider the following flow balance equations.

$$\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i = 0,\ (\pi_n) \qquad n\in\mathcal{N} \tag{20a}$$

$$\begin{aligned}&\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) \\ &\qquad - \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j) = 0,\ (p(\omega)\Pi_n(\omega)) \qquad \omega\in\Omega, n\in\mathcal{N}\end{aligned} \tag{20b}$$

The first equation is the balance for day-ahead quantities and flows, as in (3b). The second equation corresponds to the real-time market balance, but this is written in terms of differences, as opposed to (4c). This is key to establish the following property.

THEOREM 2. *Assume the flow balance equations* (20a)-(20b) *are satisfied for a given set of cleared quantities and flows* $g_i, G_i(\cdot), d_j, D_j(\cdot), f_\ell, F_\ell(\cdot)$, *and let* $\mathcal{M}^{ISO}$ *be given by* (17). *Then*

$$\mathcal{M}^{ISO} = -\sum_{\ell\in\mathcal{L}} \mathbb{E}\left[P^f_\ell(\omega)\right]. \tag{21}$$

If, in addition, the price distortion differences satisfy $\left(\mathcal{M}^\pi_{snd(\ell)} - \mathcal{M}^\pi_{rec(\ell)}\right) = 0$ *for all* $\ell\in\mathcal{L}$, *then*

$$\mathcal{M}^{ISO} = -\sum_{\ell\in\mathcal{L}} \mathbb{E}\left[F_\ell(\omega)\left(\Pi_{snd(\ell)}(\omega) - \Pi_{rec(\ell)}(\omega)\right)\right]. \tag{22}$$

Proof: If the flow balances hold, we have

$$\begin{aligned}0 &= \sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{j\in\mathcal{D}_n} d_j\right) \\ &\quad + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) - \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j)\right)\right].\end{aligned} \tag{23}$$

Consequently, for any arbitrary set of prices $\pi_n, \Pi_n(\cdot)$, we have

$$\begin{aligned}
&\sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell \right) + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) \right) \right] \\
&= \sum_{n\in\mathcal{N}} \pi_n \left(\sum_{i\in\mathcal{G}_n} g_i - \sum_{j\in\mathcal{D}_n} d_j \right) + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) - \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j) \right) \right] \\
&= \sum_{i\in\mathcal{G}_n} \pi_{n(i)} g_i + \mathbb{E}\left[\sum_{i\in\mathcal{G}_n} \Pi_{n(i)}(\omega) \left(G_i(\omega) - g_i \right) \right] - \sum_{j\in\mathcal{D}_n} \pi_{n(j)} d_j - \mathbb{E}\left[\sum_{j\in\mathcal{D}_n} \Pi_{n(j)}(\omega) \left(D_j(\omega) - d_j \right) \right] \\
&= \mathcal{M}^{ISO}.
\end{aligned}$$

The last expression is holds from (14) and from the definition (17). We also have that

$$\begin{aligned}
&\sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell \right) + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) \right) \right] \\
&= \sum_{n\in\mathcal{N}} (\pi_n - \mathbb{E}[\Pi_n(\omega)]) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell \right) + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} F_\ell(\omega) - \sum_{\ell\in\mathcal{L}_n^{snd}} F_\ell(\omega) \right) \right].
\end{aligned}$$

We use the following identity:

$$\sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell \right) = \sum_{\ell\in\mathcal{N}} f_\ell (\pi_{rec(n)} - \pi_{snd(n)}). \tag{24}$$

The first result follows by applying this identity to day-ahead and real-time flows and by using the definition of the expected FTR payments and price distortions. The second result follows by setting the price distortions to zero for all $n \in \mathcal{N}$. □

Theorem 2 establishes a connection between the ISO revenue and FTR payments and provides the following insights.

- FTR payments are given by the price differences between nodes times the flows. If the price difference is large, the FTR payment also will be large. Theorem 2 states that if the prices are consistent, we have that the ISO net revenue is the expected value of the FTR payments in the real-time market, which implies that the day-ahead component is eliminated. This property deserves special attention. Recent studies have found persistent positive premia in electricity markets. The ISOs have incorporated purely financial players (virtual players) in an attempt to reduce these price gaps. The evidence provided by Bowden et al. (2009) and Birge et al. (2013), however, demonstrates that this has not been fully effective. A hypothesis of Birge et al. (2013) is that certain financial players manipulate their demand bids in trying to increase their FTR payments by creating artificial congestion. If the day-ahead component of the FTR payments is effectively eliminated, FTR payments become less predictable, and thus the market is more difficult to manipulate. In other words, the market is affected only by unpredictable randomness. Note that it is also possible (as we demonstrate in Section 6) that artificial premia are being introduced into the market because of the inability of deterministic clearing settings to properly average out real-time market conditions.

- If revenue adequacy holds, then $\sum_{\ell\in\mathcal{L}}\mathbb{E}\left[P_\ell^f(\omega)\right]\geq 0$. Consequently, a lack of revenue adequacy will make FTR holders incur a loss.

- Revenue adequacy and fair payments are not mutually exclusive. A given clearing can yield fair payments but not revenue adequacy and viceversa. Consequently, we seek to have the clearing satisfies both revenue adequacy and payment fairness.

- If all prices in the network are equal for each scenario (for any quantities and flows satisfying the network equations), then $\mathcal{M}^{ISO}=0$. The reason is that, if prices are equal, then FTR payments are zero. From Theorem 2 we have that the FTR payments are equal to the net ISO revenue $\mathcal{M}^{ISO}$. This case includes the trivial case in which all lines in the network are uncongested or, equivalently, that the network has a single node.

4.6. Stochastic Formulation

Motivated by the structure of the expected surplus function and of the flow balance equations and the resulting properties, we consider the stochastic market clearing formulation.

$$
\begin{aligned}
\min_{d_j,D_j(\cdot),g_i,G_i(\cdot),f_\ell,F_\ell(\cdot)} \varphi^{sto} := \mathbb{E}&\left[\sum_{i\in\mathcal{G}}\alpha_i^g G_i(\omega)+\Delta\alpha_i^g|G_i(\omega)-g_i|\right]\\
&+\mathbb{E}\left[\sum_{j\in\mathcal{D}}-\alpha_j^d D_j(\omega)+\Delta\alpha_j^d|D_j(\omega)-d_j|\right]\\
&+\mathbb{E}\left[\sum_{\ell\in\mathcal{L}}\Delta\alpha_\ell^f|F_\ell(\omega)-f_\ell|\right] && \text{(25a)}\\
\text{s.t.} \sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i = 0,\ (\pi_n) \qquad n\in\mathcal{N} && \text{(25b)}\\
\sum_{\ell\in\mathcal{L}_n^{rec}}(F_\ell(\omega)-f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}}(F_\ell(\omega)-f_\ell) + \sum_{i\in\mathcal{G}_n}(G_i(\omega)-g_i)&\\
-\sum_{j\in\mathcal{D}_n}(D_j(\omega)-d_j)=0,\ (p(\omega)\Pi_n(\omega)) \qquad \omega\in\Omega, n\in\mathcal{N} && \text{(25c)}\\
-\bar{F}_\ell(\omega)\leq F_\ell(\omega)\leq \bar{F}_\ell(\omega),\ \ \omega\in\Omega,\ell\in\mathcal{L} && \text{(25d)}\\
0\leq G_i(\omega)\leq \bar{G}_i(\omega), \qquad \omega\in\Omega, i\in\mathcal{G} && \text{(25e)}\\
0\leq D_i(\omega)\leq \bar{D}_j(\omega), \qquad \omega\in\Omega, j\in\mathcal{D} && \text{(25f)}
\end{aligned}
$$

Note that the objective function can be written as

$$\varphi^{sto} := \varphi + \sum_{\ell \in \mathcal{L}} \Delta\alpha_\ell^f \mathbb{E}\left[|F_\ell(\omega) - f_\ell|\right], \tag{26}$$

where φ is the negative surplus function defined in (8) and $\Delta\alpha_\ell^f > 0$ are penalty parameters.

The stochastic setting provides a natural mechanism to *anticipate* the effects of day-ahead decisions on real-time market corrections, and it minimizes the expected social surplus directly, which includes contributions from the day-ahead and real-time markets. This optimality property gives rise to several important pricing and payment properties, as we will see in the following section.

The above formulation is partially based on the one proposed by Pritchard et al. (2010). The common features are the following.

- The real-time prices (duals of the network balance (25c)) have been weighted by their corresponding probabilities. This feature will enable us to construct the Lagrange function of the problem in terms of expectations.
- The network balance in the real-time market is written in terms of the residual quantities $(G_i(\omega) - g_i)$, $(D_j(\omega) - d_j)$, and flows $(F_\ell(\omega) - f_\ell)$. This feature will be key in obtaining consistent prices and it emphasizes the fact that the real-time market is a market of corrections. As shown in Theorem 2, this structure also guarantees consistency between the ISO net revenue and the FTR payments.

The differences between the proposed formulation and the one presented by Pritchard et al. (2010) are the following.

- The formulation does not impose bounds on the day-ahead quantities and flows. In Section 5 we will prove that the penalization terms render bounds for the day-ahead quantities and flows redundant. In Section 6 we will demonstrate that considering bounds on day-ahead variables together with penalization terms can lead to price inconsistency.
- The objective function penalizes deviations between day-ahead and real-time flows in addition to supply and demand quantities. In Section 5 we will show that this is key to ensure consistent pricing throughout the network.
- We allow for randomness in the transmission line capacities. In Section 5 we will see that doing so has no effect on the underlying properties of the model.

We refer to the solution of the stochastic formulation (25) as the *here-and-now* solution to reflect the fact that a single implementable decision must be made now in anticipation of the uncertain future and that day-ahead quantities and flows are scenario-independent. We also consider the (ideal, non-implementable) *wait-and-see* (WS) solution. For details, refer to Birge and Louveaux (1997). In the WS setting, we assume that the capacities for each scenario are actually known at the moment of decision. In other words, we assume availability of perfect information. In order to obtain the WS solution, the clearing problem (25) is solved by allowing first-stage decisions g_i, d_j, f_ℓ to be scenario-dependent. It is not difficult to prove that in this case, each scenario generates day-ahead prices and quantities that are equal to real-time counterparts because no corrections are necessary. We denote the expected social surplus obtained under perfect information as φ_{WS}^{sto}.

The penalty structure of the social surplus function opens the possibility of considering different formulations. For instance, quadratic penalties have been proposed by Pritchard et al. (2010) and Khazaei et al. (2013), that result from using a quadratic supply and demand costs function. Comparisons of the proposed framework with quadratic penalty formulations are presented in Section 7.

To motivate the following discussions, we use a stochastic formulation with no network constraints:

$$\min_{d_j, g_i, G_i(\cdot), D(\cdot)} \ \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] + \mathbb{E}\left[\sum_{j\in\mathcal{D}} -\alpha_j^d D_j(\omega) + \alpha_j^d |D_j(\omega) - d_j|\right] \tag{27a}$$

$$\text{s.t.} \sum_{i\in\mathcal{G}} g_i = \sum_{j\in\mathcal{D}} d_j \quad (\pi) \tag{27b}$$

$$\sum_{i\in\mathcal{G}} (G_i(\omega) - g_i) = \sum_{j\in\mathcal{D}} (D_j(\omega) - d_j) \quad \omega\in\Omega \quad (p(\omega)\Pi(\omega)) \tag{27c}$$

$$0 \le G_i(\omega) \le \bar{G}_i(\omega), \quad i\in\mathcal{G}, \omega\in\Omega \tag{27d}$$

$$0 \le D_j(\omega) \le \bar{D}(\omega), \quad j\in\mathcal{D}, \omega\in\Omega. \tag{27e}$$

This formulation assumes infinite transmission capacity. In this case, the entire network collapses into a single node; consequently, a single day-ahead price π and real-time price $\Pi(\omega)$ are used.

5. Properties of Stochastic Clearing

In this section, we prove that the stochastic formulations yield bounded price distortions and that these distortions can be made arbitrarily small, leading to payment fairness. In addition, we prove that day-ahead quantities are bounded by real-time quantities and that they converge to the medians of the real-time quantities when the distortions are zero. Further, we prove that the formulation yields revenue adequacy and zero uplifts in expectation.

5.1. No Network Constraints

To simplify the presentation, we begin with the single-node formulation and then generalize the results to the case of network constraints. We recall that the partial Lagrange function of the single node problem (27) is given by

$$\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi, \Pi(\cdot)) =$$

$$\begin{aligned}&\mathbb{E}\left[\sum_{i\in\mathcal{G}}\alpha_i^g G_i(\omega)+\Delta\alpha_i^g|G_i(\omega)-g_i|\right]-\mathbb{E}\left[\sum_{j\in\mathcal{D}}\alpha_j^d D_j(\omega)-\alpha_j^d|D_j(\omega)-d_j|\right]\\&-\pi\left(\sum_{i\in\mathcal{G}}g_i-\sum_{j\in\mathcal{D}}d_j\right)-\mathbb{E}\left[\Pi(\omega)\left(\sum_{i\in\mathcal{G}}(G_i(\omega)-g_i)-\sum_{j\in\mathcal{D}}(D_j(\omega)-d_j)\right)\right].\end{aligned}\tag{28}$$

Note that the contribution of the balance constraints can be written in expected value form if we weight the Lagrange multipliers of the balance equations (prices) by the probabilities $p(\omega)$.

THEOREM 3. *Consider the single-node stochastic clearing problem* (27)*, and assume that the incremental bid costs satisfy* $\Delta\alpha_j^d>0$*,* $j\in\mathcal{D}$ *and* $\Delta\alpha_i^g>0$*,* $i\in\mathcal{G}$*. The price distortion* $\mathcal{M}^\pi=\pi-\mathbb{E}[\Pi(\omega)]$ *is bounded as*

$$|\mathcal{M}^\pi|\leq\Delta\alpha,\tag{29}$$

where

$$\Delta\alpha=\min\left\{\min_{i\in\mathcal{G}}\Delta\alpha_i^g,\min_{j\in\mathcal{D}}\Delta\alpha_j^d\right\}.\tag{30}$$

Proof: The stationarity conditions of the partial Lagrange function with respect to the day-ahead quantities d_j,g_i are given by

$$\partial_{d_j}\mathcal{L}=0=\Delta\alpha_j^d\mathbb{P}(d_j\geq D_j(\omega))-\Delta\alpha_j^d\mathbb{P}(d_j\leq D_j(\omega))+\pi-\mathbb{E}\left[\Pi(\omega)\right]\quad j\in\mathcal{D}\tag{31a}$$

$$\partial_{g_i}\mathcal{L}=0=\Delta\alpha_i^g\mathbb{P}(g_i\geq G_i(\omega))-\Delta\alpha_i^g\mathbb{P}(g_i\leq G_i(\omega))-\pi+\mathbb{E}\left[\Pi(\omega)\right]\quad i\in\mathcal{G},\tag{31b}$$

where $\mathbb{P}(A)$ denotes the probability of event A. To obtain these relationships, we used the property

$$\partial_x|X-x|=\begin{cases}+1\text{ if }X<x\\-1\text{ if }X>x.\end{cases}\tag{32}$$

From this we have that

$$\begin{aligned}\partial_x\mathbb{E}[|X(\omega)-x|]&=\mathbb{E}\left[\mathbf{1}_{X(\omega)<x}-\mathbf{1}_{X(\omega)>x}\right]\\&=\mathbb{E}\left[\mathbf{1}_{X(\omega)<x}+\mathbf{1}_{X(\omega)=x}-\mathbf{1}_{X(\omega)=x}-\mathbf{1}_{X(\omega)>x}\right]\\&=\mathbb{P}(X(\omega)\leq x)-\mathbb{P}(X(\omega)\geq x).\end{aligned}\tag{33}$$

Here, $\mathbf{1}_A$ is the indicator function of event A. Rearranging expression (31a), we obtain

$$\begin{aligned}0&=\Delta\alpha_j^d\mathbb{P}(d_j\geq D_j(\omega))-\Delta\alpha_j^d\mathbb{P}(d_j\leq D_j(\omega))+\pi-\mathbb{E}\left[\Pi(\omega)\right]\\&=\Delta\alpha_j^d\mathbb{P}(d_j\geq D_j(\omega))-\Delta\alpha_j^d(1-\mathbb{P}(d_j\geq D_j(\omega))+\pi-\mathbb{E}\left[\Pi(\omega)\right]\\&=2\Delta\alpha_j^d\mathbb{P}(d_j\geq D_j(\omega))-\Delta\alpha_j^d+\pi-\mathbb{E}\left[\Pi(\omega)\right].\end{aligned}\tag{34}$$

Similarly, (31b) becomes

$$0 = 2\Delta\alpha_i^g \mathbb{P}(g_i \geq G_i(\omega)) - \Delta\alpha_i^g - \pi + \mathbb{E}\left[\Pi(\omega)\right]. \tag{35}$$

If we rearrange expressions (34)-(35), we obtain

$$\mathbb{P}(d_j \geq D_j(\omega)) = \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d} \tag{36a}$$

$$\mathbb{P}(g_i \geq G_i(\omega)) = \frac{\Delta\alpha_i^g + \pi - \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_i^g}. \tag{36b}$$

Because $\sum_{\omega\in\Omega} p(\omega) = 1$, we know that $0 \leq \mathbb{P}(d_j \geq D_j(\omega)) \leq 1$ and $0 \leq \mathbb{P}(g_i \geq G_i(\omega)) \leq 1$. We thus have,

$$0 \leq \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d} \leq 1 \tag{37a}$$

$$0 \leq \frac{\Delta\alpha_i^g + \pi - \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_i^g} \leq 1. \tag{37b}$$

These constraints have a feasible solution if $\Delta\alpha_j^d, \Delta\alpha_i^g > 0$. The above relationships are equivalent to,

$$-\Delta\alpha_j^d \leq \pi - \mathbb{E}\left[\Pi(\omega)\right] \leq \Delta\alpha_j^d \tag{38a}$$

$$-\Delta\alpha_i^g \leq \pi - \mathbb{E}\left[\Pi(\omega)\right] \leq \Delta\alpha_i^g. \tag{38b}$$

or, equivalently,

$$|\pi - \mathbb{E}\left[\Pi(\omega)\right]| \leq \Delta\alpha_j^d, \qquad j \in \mathcal{D} \tag{39a}$$

$$|\pi - \mathbb{E}\left[\Pi(\omega)\right]| \leq \Delta\alpha_i^g, \qquad i \in \mathcal{G}. \tag{39b}$$

The proof is complete. □

The price distortion is bounded by the smallest of all $\Delta\alpha_i^g$ and $\Delta\alpha_j^d$, which we denote as $\Delta\alpha$. This implies that if we let $\Delta\alpha$ (i.e., at least one incremental bid) be sufficiently small, then *we can make the price distortion $\mathcal{M}^\pi$ arbitrarily small.* This is a highly desirable feature because it makes the price distortion robust to market manipulation. Note also that the bound is independent of the cleared quantities, which also reflects the robust behavior induced by the ℓ_1 penalties. Boundedness of the price distortion also eliminates the day-ahead component of the suppliers and consumer payments and thus achieves payment fairness. We highlight that the condition $\Delta\alpha_j^d, \Delta\alpha_i^g > 0$ assumed that Theorem 3 is also required for nondegeneracy of the solution.

We now prove that the day-ahead quantities d_j, g_i obtained from the stochastic clearing model are implicitly bounded by the minimum and maximum real-time quantities. Consequently, no day-ahead bounds are needed.

THEOREM 4. *Consider the single-node stochastic clearing problem* (27)*, and let the assumptions of Theorem 3 hold. The day-ahead quantities are bounded by the real-time quantities as*

$$\min_{\omega\in\Omega} D_j(\omega) \leq d_j \leq \max_{\omega\in\Omega} D_j(\omega),\; j\in\mathcal{D} \tag{40a}$$

$$\min_{\omega\in\Omega} G_i(\omega) \leq g_i \leq \max_{\omega\in\Omega} G_i(\omega),\; i\in\mathcal{G}. \tag{40b}$$

Proof: From (38), consider the case in which the price distortion hits the lower bound for a given demand, $\pi - \mathbb{E}[\Pi(\omega)] = -\Delta\alpha_j^d$. From (36a) we have

$$0 = 2\Delta\alpha_j^d \mathbb{P}(d_j \geq D_j(\omega)) - 2\Delta\alpha_j^d, \tag{41}$$

and $\mathbb{P}(d_j \geq D_j(\omega)) = 1$. This implies that $d_j \geq D_j(\omega)$, $\forall\omega\in\Omega$ and $d_j \geq \min_{\omega\in\Omega} D_j(\omega) \geq 0$. If $\pi - \mathbb{E}[\Pi(\omega)] = -\Delta\alpha_j^d$, we have $\mathbb{P}(d_j \leq D_j(\omega)) = 1$. This implies that $d_j \leq D_j(\omega)$, $\forall\omega\in\Omega$ and $d_j \leq \max_{\omega\in\Omega} D_j(\omega)$. We thus conclude that d_j is bounded from below by $\min_{\omega\in\Omega} D_j(\omega)$ and from above by $\max_{\omega\in\Omega} D_j(\omega)$. The same procedure can be followed to prove that g_i is bounded from below by $\min_{\omega\in\Omega} G_i(\omega)$ and from above by $\max_{\omega\in\Omega} G_i(\omega)$. $\square$

The implicit bound on the day-ahead quantities d_j, g_i is a key property of the stochastic model proposed because it implies that we do not have to choose day-ahead capacities $\bar{g}_i, \bar{d}_j$ (e.g., summarization statistics). These are automatically set by the model through the scenario information. This is important because, as we have mentioned, obtaining proper summarizing statistics for complex probability distributions might not be trivial. This also prevents the possibility of players manipulating their day-ahead capacities.

We now prove that *if the price distortion is zero, the day-ahead quantities converge to the median of the real-time quantities.*

THEOREM 5. *Consider the stochastic clearing problem* (27) *and let the assumptions of Theorem 3 hold. If the price distortion is zero at the solution, then*

$$d_j = \mathbb{M}[D_j(\omega)],\; j\in\mathcal{D} \tag{42a}$$

$$g_i = \mathbb{M}[G_i(\omega)],\; i\in\mathcal{G}. \tag{42b}$$

Proof: From (36a)-(36b) we have that if $\pi = \mathbb{E}[\Pi(\omega)] = 0$, then $\mathbb{P}(d_j \geq D_j(\omega)) = \frac{1}{2}$, which implies $d_j = \mathbb{M}[D_j(\omega)]$, $j\in\mathcal{D}$. We also have that $\mathbb{P}(g_i \geq G_i(\omega)) = \frac{1}{2}$, which implies $g_i = \mathbb{M}[G_i(\omega)]$, $i\in\mathcal{G}$. $\square$

This result implies that the day-ahead quantities d_j, g_i can obe guaranteed to converge to the expected values of the real-time quantities $\mathbb{E}[D_j(\omega)], \mathbb{E}[G_i(\omega)]$ only for the case in which the probability distributions are symmetric. The reason is that, in such a case, the mean and the median coincide. In other words, *the expected value is not necessarily the right statistic to be used for the* *capacities in the day-ahead market.* In fact, for our model, it is also not necessarily the median (although from the model properties we see that such behavior can be expected if the price distortion is zero). We emphasize that convergence to median quantities is the result of the ℓ_1 penalty structure of the surplus function. In Section 7 we show that a quadratic penalty yields day-ahead quantities converging to the expected value of the real-time quantities.

We now prove that the stochastic formulation yields zero uplifts in expectation. Revenue adequacy is not considered because this is a single-node problem. We use the strategy followed by Morales et al. (2012). For this discussion, we denote a minimizer of the partial Lagrange function (subject to the constraints (27d) and (27e)) as $d_j^*, D_j(\cdot)^*, g_i^*, G_i(\cdot)^*, \pi^*, \Pi^*(\cdot)$. Because the problem is convex, we know that the optimal prices $\pi^*, \Pi^*(\cdot)$ satisfy

$$(d_j^*, D_j(\cdot)^*, g_i^*, G_i^*(\cdot)) = \underset{d_j, D_j(\cdot), g_i, G_i(\cdot)}{\operatorname{argmin}} \quad \mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) \quad \text{s.t.} \quad (27\text{d}) - (27\text{e}). \tag{43}$$

Moreover, at fixed $\pi^*, \Pi^*(\cdot)$, the partial Lagrange function can be separated as

$$\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) = \sum_{i\in\mathcal{G}} \mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) + \sum_{j\in\mathcal{D}} \mathcal{L}_j^d(d_j, D_j(\cdot), \pi^*, \Pi^*(\cdot)), \tag{44}$$

where

$$\mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) := \mathbb{E}[C_i^g(\omega)] - \mathbb{E}[P_i^g(\omega)],\ i \in \mathcal{G} \tag{45a}$$

$$\mathcal{L}_j^d(d_j, D_j(\cdot), \pi^*, \Pi^*(\cdot)) := \mathbb{E}[C_j^d(\omega)] - \mathbb{E}[P_j^d(\omega)],\ j \in \mathcal{D}. \tag{45b}$$

Consequently, one can minimize the partial Lagrange function by minimizing (45) independently.

Theorem 6. *Consider the single-node clearing problem* (27)*, and let the assumptions of Theorem 3 hold. Any minimizer* $d_j^*, D_j(\cdot)^*, g_i^*, G_i(\cdot)^*, \pi^*, \Pi^*(\cdot)$ *of* (27) *yields zero uplift payments in expectation:*

$$\mathcal{M}_i^U = 0, \quad i \in \mathcal{G} \tag{46a}$$

$$\mathcal{M}_j^U = 0, \quad j \in \mathcal{D}. \tag{46b}$$

Proof: It suffices to show that $\mathbb{E}[C_i^g(\omega)] \geq \mathbb{E}[P_i^g(\omega)]$ for all $i \in \mathcal{G}$ and $\mathbb{E}[P_j^d(\omega)] \leq \mathbb{E}[C_j^d(\omega)]$ for all $j \in \mathcal{D}$. For fixed $\pi^*, \Pi^*(\omega)$, the candidate solution $d_j = 0, D_j(\cdot) = 0, g_i = 0, G_i(\cdot) = 0$ is feasible for (43) with values $\mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot) = 0,\ i \in \mathcal{G}$ and $\mathcal{L}^d(d_j, D_j(\cdot), \pi^*, \Pi^*(\cdot) = 0,\ j \in \mathcal{D}$. Because the candidate is suboptimal we have $\mathcal{L}_i^g(g_i^*, G_i^*(\cdot), \pi^*, \Pi^*(\cdot)) \leq \mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) = 0$ and $\mathcal{L}_j^d(d_j^*, D_j^*(\cdot), \pi^*, \Pi^*(\cdot)) \leq 0$. Equation (45) and the definition of $\mathcal{M}_i^U$ and $\mathcal{M}_j^U$ in (16) give the result. $\square$

5.2. Network Constraints

Having established some insights into the properties of the stochastic model, we now turn our attention to the full stochastic problem with network constraints (25) and generalize our results.

It is well known that stochastic formulations yield a better expected social surplus. This follows from the well-known inequality (see Birge and Louveaux (1997)):

$$\varphi_{WS}^{sto} \leq \varphi^{sto} \leq \varphi^{det}. \tag{47}$$

This follows from the fact that the stochastic formulation will lead to a lower recourse cost (real-time penalty costs) than will the deterministic solution because the deterministic day-ahead problem does not anticipate recourse actions. The wait-and-see setting can perfectly anticipate real-time market conditions and therefore its real-time penalties are zero. This makes it the optimal, but nonimplementable, policy.

We now establish boundedness of the price distortions throughout the network. To establish our result, we need the following definitions. We define the subset $\bar{\mathcal{N}} \subseteq \mathcal{N}$ containing all nodes at which at least one supplier or consumer is connected. We recall that the partial Lagrange function problem (25) is given by

$$\begin{aligned}
&\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot), \pi_n, \Pi_n(\cdot)) = \\
&\quad \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] - \mathbb{E}\left[\sum_{j\in\mathcal{D}} \alpha_j^d D_j(\omega) - \alpha_j^d |D_j(\omega) - d_j|\right] + \sum_{\ell\in\mathcal{L}} \Delta\alpha_\ell^f \mathbb{E}\left[|F_\ell(\omega) - f_\ell|\right] \\
&- \sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i\right) \\
&- \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) - \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j)\right)\right].
\end{aligned} \tag{48}$$

THEOREM 7. *Consider the stochastic clearing model* (25) *and assume that the incremental bid costs satisfy* $\Delta\alpha_j^d > 0$, $j \in \mathcal{D}$, $\Delta\alpha_i^g > 0$, $i \in \mathcal{G}$, *and* $\Delta\alpha_\ell^f > 0$, $\ell \in \mathcal{L}$. *The price distortions* $\mathcal{M}_n^\pi$, $n \in \mathcal{N}$ *are bounded as*

$$|\mathcal{M}_n^\pi| \leq \Delta\alpha_n,\; n \in \bar{\mathcal{N}}, \tag{49a}$$

$$|\mathcal{M}_{snd(\ell)}^\pi - \mathcal{M}_{rec(\ell)}^\pi| \leq \Delta\alpha_\ell^f,\; \ell \in \mathcal{L}, \tag{49b}$$

where

$$\Delta\alpha_n = \min\left\{\min_{i\in\mathcal{G}_n} \Delta\alpha_i^g, \min_{j\in\mathcal{D}_n} \Delta\alpha_j^d\right\},\; n \in \bar{\mathcal{N}}. \tag{50}$$

Proof: The stationarity conditions of the partial Lagrange function with respect to the day-ahead quantities g_i, d_j are given by

$$\partial_{d_j}\mathcal{L} = 0 = \Delta\alpha_j^d \mathbb{P}(d_j \geq D_j(\omega)) - \Delta\alpha_j^d \mathbb{P}(d_j \leq D_j(\omega)) + \pi_{n(j)} - \mathbb{E}\left[\Pi_{n(j)}(\omega)\right] \quad j \in \mathcal{D} \tag{51a}$$

$$\partial_{g_i}\mathcal{L} = 0 = \Delta\alpha_i^g \mathbb{P}(g_i \geq G_i(\omega)) - \Delta\alpha_i^g \mathbb{P}(g_i \leq G_i(\omega)) - \pi_{n(i)} + \mathbb{E}\left[\Pi_{n(i)}(\omega)\right] \quad i \in \mathcal{G}, \tag{51b}$$

where we recall that $n(i)$ is the node at which supplier i is connected and $n(j)$ is the node at which demand j is connected. Following the same bounding procedure used in the proof of Theorem 3, we obtain

$$-\Delta\alpha_j^g \leq \mathcal{M}_{n(j)}^{\pi} \leq \Delta\alpha_j^d, \; j \in \mathcal{D} \tag{52a}$$

$$-\Delta\alpha_i^g \leq \mathcal{M}_{n(i)}^{\pi} \leq \Delta\alpha_i^g, \; i \in \mathcal{G}. \tag{52b}$$

Because $\Delta\alpha_n$ is the smallest incremental bid price at node $n \in \bar{\mathcal{N}}$, we obtain the bound

$$-\Delta\alpha_n \leq \mathcal{M}_n^{\pi} \leq \Delta\alpha_n, \; n \in \bar{\mathcal{N}}. \tag{53}$$

We now establish the second result. From the stationarity conditions with respect to the day-ahead flows we obtain

$$0 = \Delta\alpha_\ell^f \mathbb{P}(f_\ell \geq F_\ell(\omega)) - \Delta\alpha_\ell^f \mathbb{P}(f_\ell \leq F_\ell(\omega)) + \pi_{snd(\ell)} - \mathbb{E}\left[\Pi_{snd(\ell)}\right] - \pi_{rec(\ell)} + \mathbb{E}\left[\Pi_{rec(\ell)}\right], \; \ell \in \mathcal{L}. \tag{54}$$

Because $0 \leq \mathbb{P}(f_\ell \geq F_\ell(\omega)) \leq 1$ and $\mathbb{P}(f_\ell \geq F_\ell(\omega)) = 1 - \mathbb{P}(f_\ell \leq F_\ell(\omega))$, we have

$$\mathbb{P}(f_\ell \geq F_\ell(\omega)) = \frac{\Delta\alpha_\ell^f + \mathcal{M}_{snd(\ell)}^{\pi} - \mathcal{M}_{rec(\ell)}^{\pi}}{2\Delta\alpha_\ell^f} \tag{55}$$

and

$$-\Delta\alpha_\ell^f \leq \mathcal{M}_{snd(\ell)}^{\pi} - \mathcal{M}_{rec(\ell)}^{\pi} \leq \Delta\alpha_\ell^f, \quad \ell \in \mathcal{L}. \tag{56}$$

The proof is complete. □

If $\Delta\alpha_\ell^f$ are made arbitrarily small, then the difference between the distortions at adjacent nodes can be made arbitrarily small. From Theorem 2 we have that this is enough to guarantee that the day-ahead component of the FTR payments is eliminated. The boundedness of the difference between distortions also implies that if the price distortion on one of the extremes of the line is bounded, then the price distortion at the other extreme will be bounded. Thus, the price distortions remain bounded throughout the network by the incremental bid costs at the nodes $n \in \bar{\mathcal{N}}$, which guarantees payment fairness thoughout the network. To illustrate this, consider the case in which the distortion at the sending node of a given line is bounded as $-\Delta\alpha_{snd(\ell)} \leq \mathcal{M}^{\pi}_{snd(\ell)} \leq \Delta\alpha_{snd(\ell)}$ and that the distortion difference is bounded as $-\Delta\alpha^f_\ell \leq \mathcal{M}^{\pi}_{snd(\ell)} - \mathcal{M}^{\pi}_{rec(\ell)} \leq \Delta\alpha^f_\ell$. We thus have

$$-\Delta\alpha^f_\ell - \Delta\alpha_{snd(\ell)} \leq \mathcal{M}^{\pi}_{rec(\ell)} \leq \Delta\alpha^f_\ell + \Delta\alpha_{snd(\ell)}. \tag{57}$$

If $\Delta\alpha^f_\ell$ is sufficiently small then the distortion at the receiving and sending nodes have the same bounds; and if the distortion $\Delta\alpha_{snd(\ell)}$ is made sufficiently small, then both distortions can be made arbitrarily small. We now state two results that are natural extensions of Theorems 5 and 4.

THEOREM 8. *Consider the stochastic clearing model* (25)*, and let the assumptions of Theorem 7 hold. The day-ahead quantities and flows are bounded by the real-time quantities and flows as*

$$\min_{\omega\in\Omega} D_j(\omega) \leq d_j \leq \max_{\omega\in\Omega} D_j(\omega),\ j\in\mathcal{D} \tag{58a}$$

$$\min_{\omega\in\Omega} G_i(\omega) \leq g_i \leq \max_{\omega\in\Omega} G_i(\omega),\ i\in\mathcal{G} \tag{58b}$$

$$\min_{\omega\in\Omega} F_\ell(\omega) \leq f_\ell \leq \max_{\omega\in\Omega} F_\ell(\omega),\ \ell\in\mathcal{L}. \tag{58c}$$

Proof: For the suppliers and demands, we can use the same procedure used in the proof of Theorem 4. The bounds on the day-ahead flows follow the same argument as well. From (54) we have the implicit bound $0 \leq \mathbb{P}(f_\ell \geq F_\ell(\omega)) \leq 1$, because $\sum_{\omega\in\Omega} p(\omega) = 1$. If $\mathbb{P}(f_\ell \geq F_\ell(\omega)) = 1$ then we have $f_\ell \geq \min_{\omega\in\Omega} F_\ell(\omega)$. If $\mathbb{P}(f_\ell \leq F_\ell(\omega)) = 1$, then $f_\ell \leq \max_{\omega\in\Omega} F_\ell(\omega)$. $\square$

THEOREM 9. *Consider the stochastic clearing problem* (25)*, and let the assumptions of Theorem 7 hold. If the price distortions $\mathcal{M}^{\pi}_n$, $n\in\mathcal{N}$ are zero at the solution, then*

$$d_j = \mathbb{M}[D_j(\omega)],\ j\in\mathcal{D} \tag{59a}$$

$$g_i = \mathbb{M}[G_i(\omega)],\ i\in\mathcal{G} \tag{59b}$$

$$f_\ell = \mathbb{M}[F_\ell(\omega)],\ \ell\in\mathcal{L}. \tag{59c}$$

Proof: From (51) we have that if $\mathcal{M}^{\pi}_n$, $n\in\bar{\mathcal{N}}$, then $\mathbb{P}(d_j \geq D_j(\omega)) = \frac{1}{2}$, which implies $d_j = \mathbb{M}[D_j(\omega)]$, $j\in\mathcal{D}$. We also have that $\mathbb{P}(g_i \geq G_i(\omega)) = \frac{1}{2}$, which implies $g_i = \mathbb{M}[G_i(\omega)]$, $i\in\mathcal{G}$. From (55) we see that this implies that $\mathcal{M}^{\pi}_{snd(\ell)}, \mathcal{M}^{\pi}_{rec(\ell)} = 0$ and $\mathbb{P}(f_\ell \geq F_\ell(\omega)) = \frac{1}{2}$. $\square$

We treat the penalty terms purely as a means to constrain the day-ahead flows and induce the desired pricing properties. Our results indicate that this can be done with no harm by allowing $\Delta\alpha^f_\ell$ to be sufficiently small. Moreover, making these arbitrarily small gurantees that the expected social surplus of the stochastic problem (26) satisfies $\varphi^{sto} \approx \varphi$. The alternative, is to simply impose day-ahead bounds on the form $-\bar{f}_\ell \leq f_\ell \leq \bar{f}_\ell$ and to eliminate the ℓ_1 penalty terms on the flows. In this case, however, we cannot guarantee that the difference between price distortions at adjacent nodes is bounded, as we illustrate in the next section. In addition, similar to the case of day-ahead quantities, imposing day-ahead bounds on flows would require us to choose a proper statistic for $\bar{f}_\ell$, which might not be trivial to do.

We now prove revenue adequacy and zero uplift payments in expectation for the network-constrained formulation. We denote a minimizer of the partial Lagrange function (48) (subject to the constraints (25d) and (25f)) as $d_j^*, D_j^*(\cdot), g_i^*, G_i^*(\cdot), f_\ell^*, F_\ell^*(\cdot), \pi_n^*, \Pi_n^*(\cdot)$. Because the problem is convex, we know that the prices $\pi_n^*, \Pi_n^*(\cdot)$ satisfy

$$
\begin{aligned}
(d_j^*, D_j(\cdot)^*, g_i^*, G_i^*(\cdot), f_\ell^*, F_\ell^*(\cdot)) = \underset{d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot)}{\operatorname{argmin}} \quad & \mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot), \pi^*, \Pi^*(\cdot)) \\
\text{s.t.} \quad & (25\text{d}) - (25\text{f}).
\end{aligned}
\tag{60}
$$

Moreover, at $\pi_n^*, \Pi_n^*(\cdot)$, the partial Lagrange function can be separated as

$$
\begin{aligned}
&\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot), \pi_n^*, \Pi_n^*(\cdot)) = \\
&\sum_{i\in\mathcal{G}} \mathcal{L}_i^g(g_i, G_i(\cdot), \pi_n^*, \Pi_n^*(\cdot)) + \sum_{j\in\mathcal{D}} \mathcal{L}_j^d(d_j, D_j(\cdot), \pi_n^*, \Pi_n^*(\cdot)) - \mathcal{L}^f(f_\ell, F_\ell(\cdot), \pi^*, \Pi^*(\cdot)).
\end{aligned}
\tag{61}
$$

where the first two terms are defined in (45) and

$$
\begin{aligned}
&\mathcal{L}^f(f_\ell, F_\ell(\cdot), \pi_n^*, \Pi_n^*(\cdot)) = \\
&\sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell \right) + \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) \right) \right]
\end{aligned}
\tag{62}
$$

Consequently, one can minimize the partial Lagrange function by maximizing (62) and minimizing (45) independently. From Theorem 2 note that $\mathcal{L}^f(f_\ell, F_\ell(\cdot), \pi^*, \Pi^*(\cdot)) = \mathcal{M}^{ISO}$.

THEOREM 10. *Consider the stochastic clearing problem* (25)*, and let the assumptions of Theorem 7 hold. Any minimizer* $d_j^*, D_j(\cdot)^*, g_i^*, G_i(\cdot)^*, f_\ell^*, F_\ell^*, \pi_n^*, \Pi_n^*(\cdot)$ *of* (25) *yields zero uplift payments for all players and revenue adequacy in expectation:*

$$\mathcal{M}^{ISO} \leq 0 \tag{63a}$$

$$\mathcal{M}_i^U = 0, \quad i \in \mathcal{G}. \tag{63b}$$

Proof: At fixed $\pi_n^*, \Pi_n^*(\cdot)$ we note that $f_\ell = 0, F_\ell(\cdot) = 0$ is a feasible candidate solution for the maximization of $\mathcal{L}^f(f_\ell, F_\ell(\cdot), \pi_n^*, \Pi_n^*(\cdot))$ and that, at this suboptimal point, this term is zero. We thus have $\mathcal{L}^f(f_\ell^*, F_\ell^*(\cdot), \pi_n^*, \Pi_n^*(\cdot)) \leq 0$, which, from Theorem 2, implies $\mathcal{M}^{ISO} \leq 0$. □

We highlight that the introduction of the penalty term for flows does not affect revenue adequacy and cost recovery because the partial Lagrange function remains separable for fixed prices.

6. Computational Studies

In this section, we illustrate the different properties of the stochastic model proposed. We also demonstrate that the stochastic model outperforms the deterministic one in all the metrics proposed. We also highlight that arguments based on social surplus alone can be misleading in assessing the benefits of stochastic formulations.

6.1. System I

We first consider system I sketched in Figure 1. The system has two deterministic suppliers on nodes 1 and 3 and a stochastic supplier on node 2. The stochastic supplier has three possible capacity scenarios $G_2(\omega) = \{25, 50, 75\}$ MWh of equal probabilities $p(\omega) = \{1/3, 1/3, 1/3\}$. For deterministic clearing, the day-ahead capacity limit $\bar{g}$ for the wind supplier will be set to 50 MWh, the expected value forecast. The demand in node 2 is deterministic and inelastic at a level of 100 MWh. We use $\alpha^d = VOLL =$1000\$/MWh as the bid price and an incremental bid price of $\Delta\alpha^d = 0.001$. The bid prices α_i^g for the suppliers are {10,0.1,20} \$/MWh, and the incremental bid prices $\Delta\alpha_i^g$ are {1.0,0.01,2.0} \$/MWh. The transmission capacities of lines $1 \to 2$ and $3 \to 2$ are deterministic and set to $\bar{F}_{1\to 2} = \{25, 25, 25\}$ MWh and $\bar{F}_{2\to 3} = \{50, 50, 50\}$, respectively. The line capacities have been designed such that the system becomes stressed in the scenario in which the stochastic supplier delivers only 25 MWh. In this scenario, both transmission lines become congested, and real-time prices will reach high values.

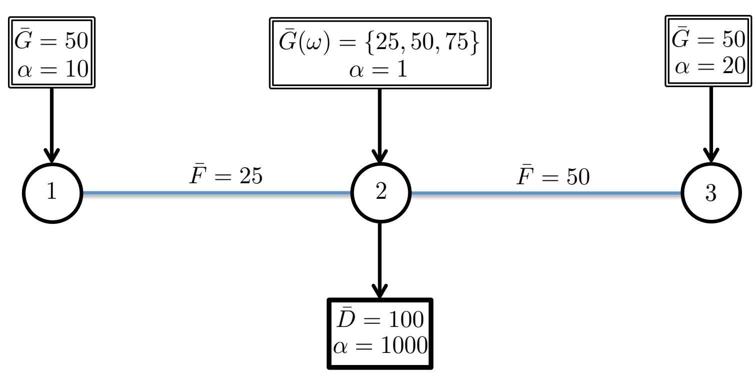

Figure 1 Scheme of System I.

We compare the performance of the deterministic, stochastic here-and-now, and the stochastic wait-and-see (WS) settings. The results are presented in Table 2. We compare the expected surplus for the suppliers φ^g as well as prices and quantities. Because the demand is inelastic, the surplus for the consumers φ^{load} is a constant. Consequently, we show only φ^g. For the deterministic setting, the expected supply surplus φ^g is \$783, and the day-ahead prices π_n are {10,20,20} \$/MWh. The price difference between the first two nodes results from the binding day-ahead flow for line $1 \to 2$ line at 25 MWh. In the real-time market, the prices for each scenario $\Pi_n(\omega)$ are {10,803,412}, {14,21,21}, and {10,16,16} \$/MWh with expected value $\mathbb{E}[\Pi_n(\omega)] =${10,280,150}. There is a strong distortion in the prices, indicated by the metrics $\mathcal{M}^{\pi}_{avg} = 130$ and $\mathcal{M}^{\pi}_{max} = 260$. The deterministic supplier biases the solution toward the expected capacity of the wind supplier of 50 MWh and is overly optimistic about conditions in the real-time market.

We now analyze the clearing of the stochastic formulation. The day-ahead prices are {10,276,148} and the real-time prices are {10,790,406}, {10,20,20}, {10,18,18} with expected value {10,276,148}. The price distortion metrics $\mathcal{M}^{\pi}_{avg}$, $\mathcal{M}^{\pi}_{max}$ are both zero. We note that the expected surplus as well as the day-ahead and real-time quantities for the stochastic and deterministic formulations are the same. The reason is that the *the deterministic and stochastic formulations have the same primal solution.* This situation might lead the practitioner to believe that no benefits are obtained from the stochastic formulation. The prices obtained, however, are completely different. Hence once can see that arguments based on social surplus can be misleading.

The different prices obtained with both formulations lead to *drastically different payment distributions* among the market participants. As seen in Table 2, for the deterministic setting the suppliers obtain expected payments $\mathbb{E}[P_i^g(\omega)]$ of {250,-5553,3799} \$. The wind supplier receives negative payments, and requires an uplift to enable cost recovery. In this case, the expected cost $\mathbb{E}[C_i^g(\omega)]$ for the wind supplier is \$5 and thus requires an expected uplift $\mathcal{M}_i^U$ of \$5,548. For the stochastic formulation, the expected payments are \${250,7316,6886}. The wind supplier has positive payments and no uplift is required. This situation illustrates that the stochastic setting diversifies resources more efficiently.

From Table 1 we note that the stochastic WS solution is consistent in that it leads to no corrections of quantities in the real-time market and it yields the same day-ahead and real-time prices. Thus, *we can guarantee convergence of day-ahead and real-time prices for each scenario only in the presence of perfect information.* From Table 2 we can see that even if the social surplus is better for the WS solution compared with the here-and-now solution, the expected payments are higher only for the stochastic supplier. This result illustrates that we cannot, in general, expect higher payments for all participants if we have perfect information. It is important, however, to monitor the payment distribution for the case of perfect information in order to validate the performance of the stochastic model.

We also compare the ISO revenue for the different formulations. Note that all formulations are revenue adequate in expectation. The stochastic here-and-now solution reaches similar revenues as the wait-and-see counterpart. Both formulations collect an order of magnitude more revenue than does the deterministic counterpart.

Table 1 System I. Comparison of quantities, prices, and social surplus.

	g_i	π_n	$G_i(\omega)$	$\Pi_n(\omega)$	$\mathbb{E}[\Pi_n(\omega)]$	φ^g
Deterministic	{25,50,25}	{10,20,20}	{25,25,50} {25,50,25} {25,75,0}	{15,802,412} {15,22,22} {15,18,18}	{15,280,150}	835
Stochastic	{25,50,25}	{10,276,148}	{25,25,50} {25,50,25} {25,75,0}	{10,790,406} {10,20,20} {10,18,18}	{10,276,148}	835
Stochastic-WS	{25,25,50} {25,50,25} {25,75,0}	{10,821,420} {10,20,20} {10,20,20}	{25,25,50} {25,50,25} {25,75,0}	{10,803,412} {10,20,20} {10,20,20}	{10,281,150}	800

Table 2 System I. Comparison of suppliers and ISO revenues.

	$\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	$\mathcal{M}^{ISO}$
Deterministic	{250,**-5553**,3799}	{250,5,533}	-3504
Stochastic	{250,7316,6886}	{250,5,533}	-12955
Stochastic-WS	{250,7555,7055}	{250,5,500}	-13360

6.1.1. Bounds on Day-ahead Quantities We now demonstrate that adding bounds on the day-ahead flows, as opposed to adding ℓ_1 penalty terms, can affect the pricing properties of the stochastic model. The price distortions $\pi - \mathbb{E}[\Pi(\omega)]$ obtained using day-ahead flow bounds are $\{-1.5, 0, 0\}$ while those obtained with the penalty term using a penalty of $\Delta\alpha^f = 0.001$ are $\{0.001, 0.001, 0.001\}$. The penalty term achieves the desired pricing property. The day-ahead flows obtained with the ℓ_1 penalty formulation are $\{25, 25\}$; these are the medians of the real-time flows which are $\{25, 50\}$ for scenario 1, $\{25, 25\}$ for scenario 2, and $\{25, 0\}$ for scenario 3. This also implies that the day-ahead flows are bounded and therefore day-ahead bounds are redundant.

6.1.2. Effect of Incremental Bid Prices We now illustrate the effect that the incremental bid prices have on the price distortion. Consider the case in which the demand in the central node is also stochastic and with scenarios $D(\omega) = \{100, 50, 25\}$. The stochastic supplier scenarios are again $G(\omega) = \{25, 50, 75\}$. We set the incremental bid price for the stochastic supplier $\Delta\alpha_2^g$ to 1.0 and vary the demand incremental bid price $\Delta\alpha^d$ on the range $\{1.0, 0.001\}$. In Table 3, we present the price distortions as a function of $\Delta\alpha^d$. The distortion remains bounded by the incremental bid and can be made arbitrarily small as we decrease the incremental bid. The result is consistent with the properties established.

6.2. System II

We now consider the more complex system presented in Figure 2. This is an adapted version of the system presented in Pritchard et al. (2010). The system has two stochastic suppliers in nodes 2 and 4 and three deterministic suppliers in nodes 1, 3, and 5. The stochastic suppliers can have 5

Table 3 System I. Effect of incremental bid $\Delta\alpha^d$ on maximum price distortion.

$\Delta\alpha^d$	$\mathcal{M}^{\pi}_{\infty}$
1.0	0.43
0.1	0.058
0.01	0.006
0.001	0.0006

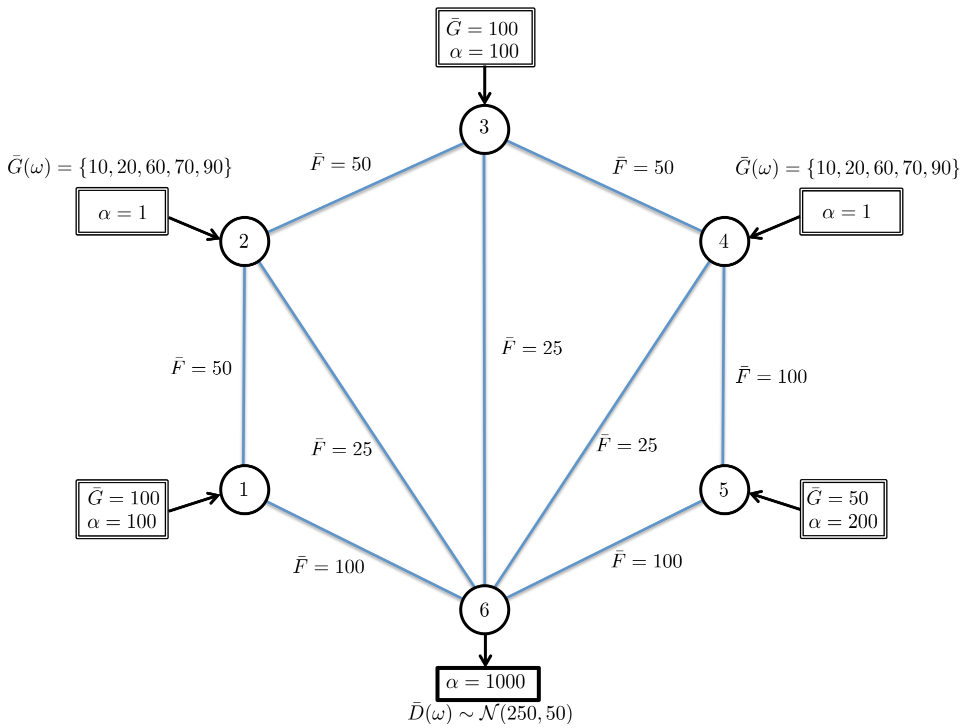

Figure 2 Scheme of System II.

possible capacities $\{10, 20, 60, 70, 90\}$ MWh. The total number of scenarios is 25. These are obtained by permuting all possible capacities and we assigned equal probabilities. The demand in node 6 is stochastic as well (distributed normally) and it is treated as inelastic. We set $\alpha^d = VOLL$ and $\Delta\alpha^d = 0.001$. We also set a penalty for the flows of $\Delta\alpha^f_\ell = 0.001$.

6.2.1. Price Distortion and Uplift Payments The results are presented in Tables 4 and 5. We first note that the price distortion for the deterministic setting is large, reaching values as large as -273 \$/MWh. Also note that the distortion (premia) is small and positive in nodes 1, 2, 6 and large and negative in nodes 3, 4, and 5. This is inefficient because it biases incentives towards a subset of players. In addition, the day-ahead prices are all equal to 100 \$/MWh because clearing the day-ahead market using expected values for the demand and for the stochastic suppliers yields a solution with no transmission congestion. The system is overly optimistic about performance in the real-time market where multiple scenarios exhibit transmission congestion, but the deterministic setting cannot foresee this. This situation is reflected in the ISO revenue collected, which is two orders of magnitude lower for the deterministic formulation compared with the stochastic formulation.

The stochastic formulation has almost the same expected social surplus as the deterministic formulation, but the price distortion is eliminated. Again, expected social surplus is a misleading metric. The largest price distortion is equal to the demand incremental bid price of 0.001, as expected. In Table 5 we see that payments for both formulations are similar except for the 4th supplier, which is a stochastic supplier. This supplier receives a negative payment and requires uplift under deterministic clearing. The uplift is eliminated by using the stochastic formulation. The expected payments and expected ISO revenue collected with the stochastic here-and-now solution are close to those of the perfect information solution.

Table 4 System II. Comparison of day-ahead prices and surplus with deterministic and stochastic formulations.

	φ	$\mathcal{M}_n^{\pi}$	π_n
Deterministic	-217529	{9,9,9,-205,-208,-273}	{100,100,100,100,100,100}
Stochastic	-217628	{0.001,0.001,0.001,0,0,0}	{96,96,96,307,310,374}
Stochastic-WS	-218266	-	-

Table 5 System II. Comparison of suppliers and ISO revenues with deterministic and stochastic formulations.

	$\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	$\mathcal{M}^{ISO}$
Deterministic	{7149,4974,5736,**-2083**,11001}	{6863,52,5491,52,3201}	-1937
Stochastic	{7116,4876,5139,8318,11001}	{7116,52,5139,52,3200}	-60750
Stochastic-WS	{6794,4760,5120,8362,10910}	{6794,49,5120,49,2910}	-60300

6.2.2. Median vs. Mean In Table 6 we compare the day-ahead quantities g_i with the medians $\mathbb{M}[G_i(\omega)]$ and the means $\mathbb{E}[G_i(\omega)]$ of the real-time quantities. As can be seen, convergence is achieved for all suppliers.

Table 6 System II. Day-ahead, median of real-time quantities, and mean of real-time quantities for suppliers.

	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5
g_i	68	60	42	60	0
$\mathbb{M}[G_i(\omega)]$	68	60	42	60	0
$\mathbb{E}[G_i(\omega)]$	70	50	49	49	15

6.2.3. Reliability Constraints We now consider the case in which there are random line failures. We consider 25 scenarios and we assume that each one of the lines $1 \to 6$, $2 \to 6$, $3 \to 6$, $4 \to 6$, and $5 \to 6$ fails in at least five scenarios. All scenarios have equal probability. The results are presented in Table 7.

The deterministic setting becomes revenue inadequate in this case, whereas the stochastic setting is revenue adequate and achieves an expected ISO revenue that is close to that of the perfect information setting. An average price distortion of 150 $/MWh and a maximum distortion of 658 $/MWh were obtained for the deterministic setting, indicating a pronounced effect of line failures on prices. The stochastic formulation eliminates the distortion and the need for uplift payments. Note also that the fourth supplier (stochastic) again faces a negative revenue under deterministic clearing and an uplift payment is needed. This again illustrates that deterministic clearing can affect resource diversification because it consistently biases the payments towards a subset of players.

Table 7 System II. Comparison of suppliers and ISO revenues with deterministic and stochastic formulations.

	$\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	$\mathcal{M}^{ISO}$
Deterministic	{5803,4723,5100,**-919**,9627}	{5231,50,3876,47,3387}	**40570**
Stochastic	{5107,3955,3683,7371,9623}	{5107,51,3683,46,3383}	-118103
Stochastic-WS	{4951,3888,3422,7170,9479}	{4951,47,3422,45,3079}	-118283

6.3. IEEE-118 System

We now demonstrate the properties of the stochastic setting in a more complex network. The IEEE-118 system comprises 118 nodes, 186 lines, 91 demand nodes, and 54 suppliers. We have 3 stochastic suppliers (4, 27, and 52), and we assign an installed capacity of 300 MWh to all of them. This represents 5% of the total generation capacity. We use 10 randomly generated scenarios for the stochastic suppliers assuming that they are independent, and we allow for fluctuations as large as 50% of the installed capacity. The demands are assumed to be deterministic and inelastic, and we set $\Delta\alpha_j^d = 0.001$. We set a penalty penalty parameter for the flows of $\Delta\alpha_\ell^f = 0.001$.

The results are presented in Table 8. The deterministic setting again requires uplift payments. Interestingly, the uplift payments are needed only for the stochastic suppliers (4 and 52). This again illustrates that deterministic clearing can block resource diversification. The price distortions for the deterministic setting can be as large as 280 $/MWh. The stochastic formulation eliminates the uplift payments and eliminates the price distortion. The maximum distortion is $\mathcal{M}_\infty^\pi = 0.0017$. Note that this is smaller than 0.002, the sum of $\Delta\alpha^f$ and $\Delta\alpha^d$.

Table 8 IEEE-118 System. Comparison of suppliers and ISO revenues with deterministic and stochastic formulations.

	$\mathcal{M}^U$	φ^g	$\mathcal{M}^\pi_{avg}$	$\mathcal{M}^\pi_{max}$	$\mathcal{M}^{ISO}$
Deterministic	-13,797	344,142	16	280	-833,656
Stochastic	0	343,959	0.0005	0.0017	-818,250
Stochastic-WS	0	343,578	0	0	-818,583

If we relax $\Delta\alpha_j^d$ to a value of 10, the maximum distortion is $\mathcal{M}^\pi_{max} = 0.17$. If we also relax $\Delta\alpha_i^g$ (multiplying by a factor of 10), the maximum distortion becomes $\mathcal{M}^\pi_{max} = 1.4$. This illustrates that the pricing property remains fairly robust to variations of the incremental bid prices. This is remarkable feature of the model (given the complexity of the network) and a direct result of the ℓ_1 penalties. If we set day-ahead bounds on flows instead of ℓ_1 penalties, the maximum distortion obtained is $\mathcal{M}^\pi_{max} = 3.2$.

The difference in social surplus between deterministic and stochastic formulations is marginal (less than 1%). Also note that the penalty parameters for the flows can be set to arbitrarily small values because they have no economic interpretation. Consequently, they do not affect the social surplus significantly. The stochastic here-and-now solution results in collected ISO revenues that are more consistent with the wait-and-see clearing.

7. Quadratic Penalty Formulation

In this section, we provide some insights into the effect of adding a quadratic penalty into the objective function. We demonstrate that doing so results in day-ahead prices that converge to the expected value of the real-time prices. In addition, the day-ahead quantities converge to the expected values of the real-time quantities.

7.1. Properties

Consider the following simplified model with no network constraints.

$$\min_{d_j, D_j(\cdot), G_i(\cdot),} \; \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \frac{1}{2}\Delta\alpha_i^g (G_i(\omega) - g_i)^2\right] + \mathbb{E}\left[\sum_{j\in\mathcal{D}} -\alpha_j^d D_j(\omega) + \frac{1}{2}\Delta\alpha_j^d (D_j(\omega) - d_j)^2\right] \tag{64a}$$

$$\text{s.t.} \sum_{i\in\mathcal{G}} g_i = \sum_{j\in\mathcal{D}} d_j \quad (\pi) \tag{64b}$$

$$\sum_{i\in\mathcal{G}} G_i(\omega) - g_i = \sum_{j\in\mathcal{D}} D_j(\omega) - d_j, \quad \omega\in\Omega \quad (p(\omega)\Pi(\omega)) \tag{64c}$$

$$0 \le G_i(\omega) \le \bar{G}_i(\omega), \quad i\in\mathcal{G}, \quad \omega\in\Omega \tag{64d}$$

$$0 \le D_j(\omega) \le \bar{D}_j(\omega), \quad j\in\mathcal{D}, \quad \omega\in\Omega. \tag{64e}$$

The optimality conditions with respect to the day-ahead quantities are given by,

$$0 = \Delta\alpha_j^d \mathbb{E}[D_j(\omega) - d_j] + \pi - \mathbb{E}[\Pi(\omega)], \quad j \in \mathcal{D} \tag{65a}$$

$$0 = \Delta\alpha_i^g \mathbb{E}[G_i(\omega) - g_i] - \pi + \mathbb{E}[\Pi(\omega)], \quad i \in \mathcal{G} \tag{65b}$$

At a solution we know that $\sum_{i\in\mathcal{G}} G_i(\omega) - g_i = \sum_{j\in\mathcal{D}} D_j(\omega) - d_j$ holds for all $\omega \in \Omega$. Consequently, $\mathbb{E}\left[\sum_{i\in\mathcal{G}} G_i(\omega) - g_i\right] = \mathbb{E}\left[\sum_{j\in\mathcal{D}} D_j(\omega) - d_j\right]$ must also hold. This condition is equivalent to

$$\sum_{i\in\mathcal{G}} \mathbb{E}\left[G_i(\omega) - g_i\right] = \sum_{j\in\mathcal{D}} \mathbb{E}\left[D_j(\omega) - d_j\right]. \tag{66}$$

Substituting the optimality conditions in this expression, we obtain

$$(\pi - \mathbb{E}[\Pi(\omega)]) \sum_{i\in\mathcal{G}} \frac{1}{\Delta\alpha_i^g} = -(\pi - \mathbb{E}[\Pi(\omega)]) \sum_{j\in\mathcal{D}} \frac{1}{\Delta\alpha_j^d}. \tag{67}$$

If $\Delta\alpha_i^g, \Delta\alpha_j^d > 0$, this relationship can hold only if $\pi = \mathbb{E}[\Pi(\omega)]$. From the optimality conditions we have that this implies that $\mathbb{E}[G_i(\omega) - g_i] = 0, \mathbb{E}[D_j(\omega) - d_j] = 0$ or $g_i = \mathbb{E}[G_i(\omega)]$ and $d_j = \mathbb{E}[D_j(\omega)]$. The day-ahead quantities converge to the mean. This also implies that no bounds are needed for g_i and d_j. Generalizing these pricing and bounding properties to a more general network model is challenging because the probabilistic bounds induced by the ℓ_1 penalties do not hold any more. We explore the behavior of the quadratic penalty model empirically in a more general network setting.

7.2. Case Studies

We first use system II sketched in Figure 2 to perform the tests. For the quadratic penalty formulation we use the squared roots of the incremental costs used for the ℓ_1 formulations: $\sqrt{\Delta\alpha_i^g}, \sqrt{\Delta\alpha_j^d}$. We impose bounds only on the day-ahead flows.

The resulting day-ahead generation quantities g_i are $\{66, 48, 58, 45, 16\}$ and the expected real-time quantities $\mathbb{E}[G_i(\omega)]$ are $\{6, 48, 58, 44, 15\}$. The day-ahead prices π_n are $\{100, 100, 100, 281, 282, 350\}$, and the expected the real-time prices $\mathbb{E}[\Pi_n(\omega)]$ are $\{100, 100, 100, 282, 283, 351\}$. The day-ahead f_ℓ and expected real-time flows $\mathbb{E}[F_\ell(\omega)]$ are $\{-27, 1, 20, 40, 20, 69, 16, 85, -93\}$ and $\{-25, 1, 21, 39, 21, 66, 16, 81, -92\}$, respectively. While there are slight differences between the flows, quantities, and prices, we can see that the model gives the desired pricing behavior.

The price distortions at all nodes for the IEEE-118 system can also be eliminated by using the quadratic penalty formulation. In addition, day-ahead and expected real-time quantities converge. The price distortions remain zero even if the incremental bid prices are increased by a factor of 10 and decreased by a factor of 0.1. This indicates that the formulation is robust. We have observed that uplifts are needed in some instances but these remain fairly small. We have also tested the quadratic penalty formulation using penalties on the day-ahead flows instead of bound constraints and we have found similar performance.

These results give some indication that one can derive stochastic clearing formulations that can achieve the desired pricing properties for different cost functions.

8. Conclusions and Future Work

We have demonstrated that deterministic market clearing formulations introduce strong and arbitrary distortions between day-ahead and expected real-time prices that bias incentives and block diversification. We present a stochastic formulation capable of eliminating these issues. The formulation is based on a social surplus function that accounts for expected costs and penalizes deviations between day-ahead and real-time quantities using ℓ_1 penalties. We show that the formulation yields day-ahead prices that are close to expected real-time prices. In addition, we show that day-ahead quantities and flows tend to converge to the median of real-time counterparts. We have also demonstrated that a quadratic penalty approach can induce convergence to expected value quantities, flows, and prices.

Future work requires extending the model in multiple directions. First, it is necessary to capture the progressive resolution of uncertainty by using multi-stage models and to incorporate ramping constraints and unit commitment decisions. Second, it is necessary to construct formulations that design day-ahead decisions that approach ideal wait-and-see behavior. Morales et al. (2014) demonstrate that this might be possible to do by using bi-level formulations, but a more detailed analysis is needed. Third, the proposed stochastic model is computationally more challenging than existing models available in the literature because it incorporates the detailed network in the first-stage. This leads to problems that much larger first-stage dimensions which are difficult to decompose and parallelize. Consequently, scalable strategies are needed.

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