Complementarity-based simulation of Multi-Rigid-Body Dynamics with Contact and Friction

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Nonsmooth multi-rigid-body dynamics

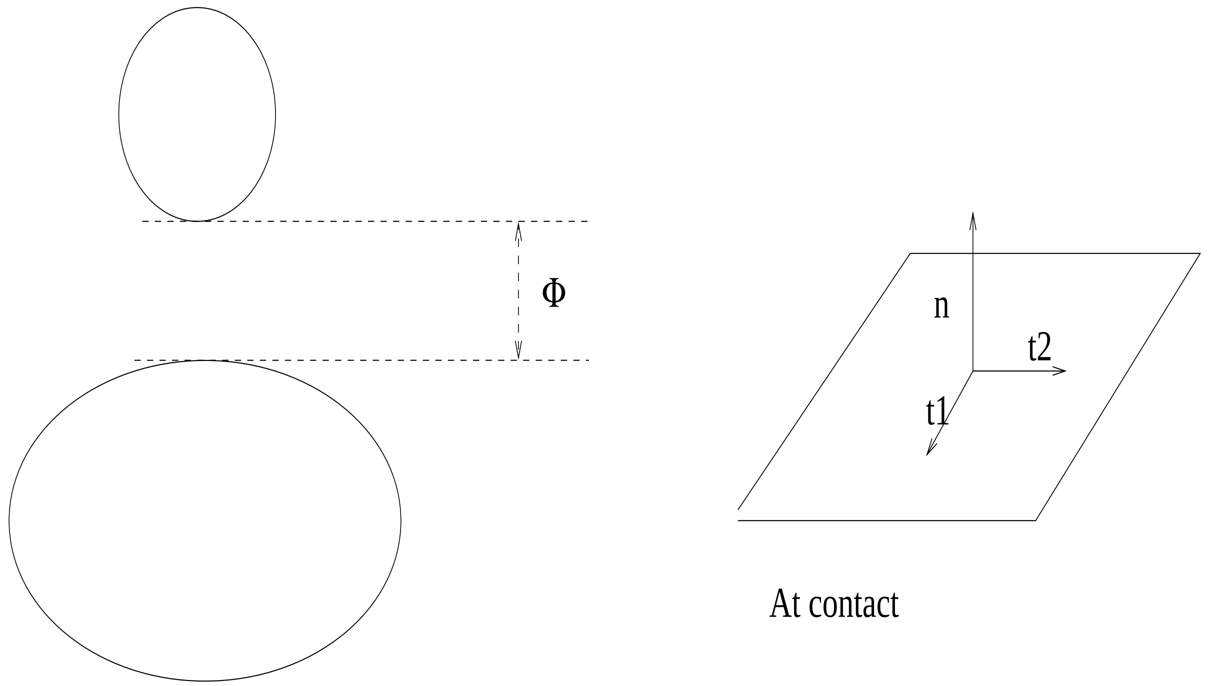
Nonsmooth rigid multibody dynamics (NRMD) methods attempt to predict the position and velocity evolution of a group of rigid particles subject to certain constraints and forces.

- non-interpenetration.
- collision.
- joint constraints
- adhesion
- Dry friction – Coulomb model.
- global forces: electrostatic, gravitational.

■ These we cover in our approach.

Areas that use NRMD

- granular and rock dynamics.
- masonry stability analysis.
- simulation of concrete obstacle response to explosion.
- tumbling mill design (mineral processing industry).
- interactive virtual reality.
- robot simulation and design.



Contact and (Coulomb) Friction Model

- Noninterpenetration Constraints $\Phi(q) \geq 0$ complementary to $c_n \geq 0$ (normal impulse/force). Here q are position parameters.

- Friction force: Here $\widehat{D}(q) = [\widehat{t_1}(q), \widehat{t_2}(q)]$,
$\beta = \text{argmin}_{||\widehat{\beta}|| \leq \mu c_n} v^T \widehat{D}(q) \widehat{\beta}$.

Acceleration Formulation

■ Data, ■ Unknowns.

$$M(q)\frac{d^2q}{dt^2} - \sum_{i=1}^{m} \nu^{(i)} c_\nu^{(i)} - \sum_{j=1}^{p} \left(n^{(j)}(q) c_n^{(j)} + D^{(j)}(q)\beta^{(j)} \right) = k(t, q, \frac{dq}{dt})$$

$$\Theta^{(i)}(q) = 0, \qquad i = 1 \ldots m$$

$$\Phi^{(j)}(q) \geq 0, \quad \textbf{compl. to} \quad c_n^{(j)} \geq 0, \quad j = 1 \ldots p$$

$$\beta^j = \text{argmin}_{\widehat{\beta}(j)} v^T D(q)^{(j)} \widehat{\beta}^{(j)} \quad \textbf{subject to} \quad \left\| \beta^{(j)} \right\| \leq \mu^{(j)} c_n^{(j)}, \; j = 1, \ldots, p$$

Here $\nu^{(\mathbf{i})} = \nabla \mathbf{\Theta}^{(\mathbf{i})}$, $n^{(j)} = \nabla \Phi^{(j)}$.

M:Mass matrix SPD, k:external and inertial force.

It is known that these problems do not have a classical solution even in 2 dimensions, where the discretized cone coincides with the total cone.

Time-stepping scheme

Use Euler method, half-explicit in velocities, linearizing the geometrical constraints. Fundamental variables: velocities and impulses (h $\times$ force).

$$M(v^{l+1} - v^{(l)}) - \sum_{i=1}^{m} \nu^{(i)} c_\nu^{(i)} - \sum_{j \in \mathcal{A}} (n^{(j)} c_n^{(j)} + D^{(j)} \beta^{(j)}) = hk$$

$$\nu^{(i)^T} v^{l+1} = 0, \qquad i = 1..m$$

$$\rho^{(j)} = n^{(j)^T} v^{l+1} \geq 0, \quad \textbf{compl. to} \quad c_n^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\sigma^{(j)} = \lambda^{(j)} e^{(j)} + D^{(j)T} v^{l+1} \geq 0, \quad \textbf{compl. to} \quad \beta^{(j)} \geq 0, \quad j \in \mathcal{A}$$

$$\zeta^{(j)} = \mu^{(j)} c_n^{(j)} - e^{(j)^T} \beta^{(j)} \geq 0, \quad \textbf{compl. to} \quad \lambda^{(j)} \geq 0, \quad j \in \mathcal{A}.$$

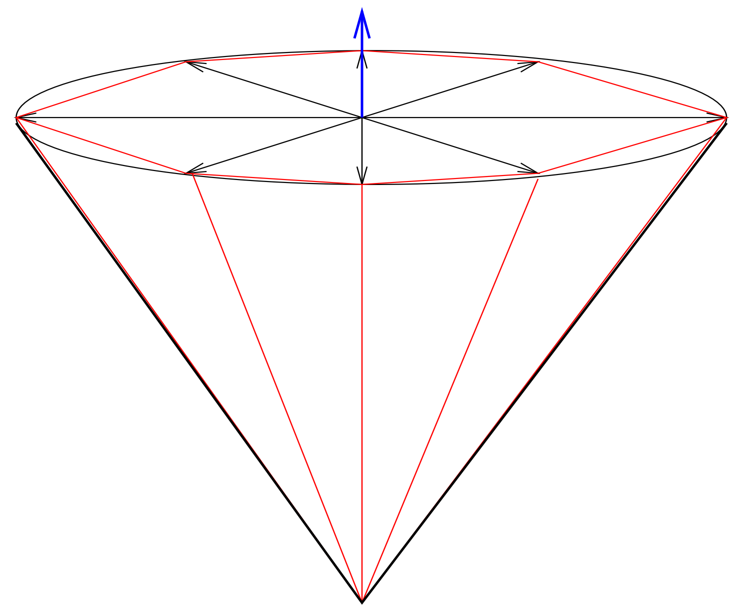
Here $\nu^{(\mathbf{i})} = \nabla \boldsymbol{\Theta}^{(\mathbf{i})}$, $n^{(j)} = \nabla \Phi^{(j)}$. h is the time step. The set $\mathcal{A}$ consists of the active constraints. (Anitescu and Potra,1997) based on (Stewart and Trinkle, 1996),

The time-stepping scheme has a solution although the classical formulation doesn't!

Discretized Friction Model

- d_i(GC) is the column corresponding to $t(\alpha_i)$, $\alpha_i \in [0, \pi]$, $i = 1, 2, \ldots, l$, $D(q) = [d_1, d_2, ...d_l]$.
- To each tangential direction we attach a force $\beta_i \geq 0$, $i = 1, 2, \ldots, l$. We denote by $\beta = (\beta_1, \beta_2, \ldots, \beta_l)$.
- The frictional constraints become

Polygonal cone approximation to the Coulomb cone (3D).



$$\beta = \text{argmin}_{\widehat{\beta} \geq 0} v^T D(q) \widehat{\beta} \quad \text{subject to} \quad \left|\left|\widehat{\beta}\right|\right|_1 \leq \mu c_n.$$

Matrix Form of the Integration Step

$$\begin{bmatrix} M & -\tilde{\nu} & -\tilde{n} & -\tilde{D} & 0 \\ \tilde{\nu}^T & 0 & 0 & 0 & 0 \\ \tilde{n}^T & 0 & 0 & 0 & 0 \\ \tilde{D}^T & 0 & 0 & 0 & \tilde{E} \\ 0 & 0 & \tilde{\mu} & -\tilde{E}^T & 0 \end{bmatrix} \begin{bmatrix} v^{(l+1)} \\ \tilde{c}_\nu \\ \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} -Mv^{(l)} - hk \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix}^T \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} = 0, \qquad \begin{bmatrix} \tilde{c}_n \\ \tilde{\beta} \\ \tilde{\lambda} \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \tilde{\rho} \\ \tilde{\sigma} \\ \tilde{\zeta} \end{bmatrix} \geq 0.$$

Note Replacing 0 by $-\tilde{\mu}$ makes the problem PSD!

How do we expand the reach of the scheme?

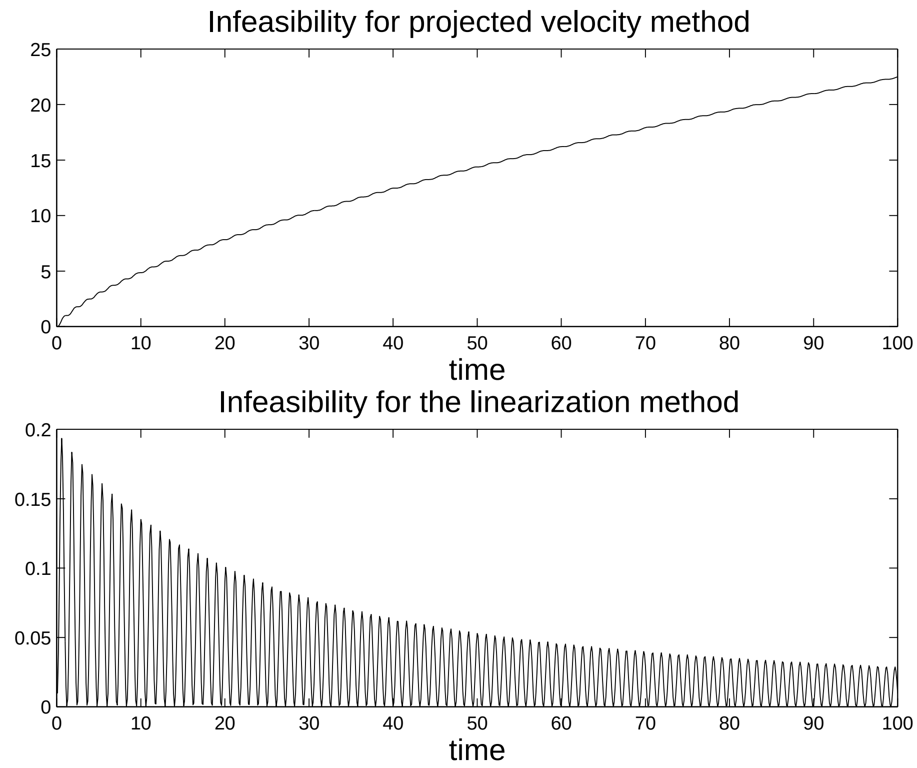
1. Collision detection for integrate-detect-restart approaches are costly. How do we reduce this computational effort?

2. Due to linearization, extra effort may be needed to prevent constraint drift. Can it be avoided?

3. The LCP may have a nonconvex solution set (Anitescu and Hart, 2002). Can we construct an efficient convex relaxation?

4. How do we modify the approach to include partially elastic or elastic collisions (e.g. bouncing balls?).

Example of constraint drift



Comparison of constraint error the original method and modified method (to be presented later) for a pendulum example. Ratio of infeasibilities gets to about 10^3!

Linearization method

Consider penalty method (main method currently used in practice).

$$\Phi^{(1)}(q) \geq 0 \ \text{ enforced by penalty force } \ \theta^{(1)}(q) = \gamma^{(1)} \left(\Phi_{-}^{(j)}(q) \right)^2$$

where $\gamma^{(1)}$ is a very large parameter. This coresponds to our main competitor, the penalty method.

Dynamics (for the frictionless case becomes)

$$\begin{aligned} \frac{dq}{dt} &= v. \\ M\frac{dv}{dt} &= k(t,q,v) + \theta^{(1)}(q)\nabla_q \Phi^{(1)}(q). \end{aligned}$$

Apply backward Euler, where $\Phi^{(1)}(q^{(l+1)})$ is replaced by its linearization

$$\Phi^{(1)}(q^{(l+1)}) \approx \Phi^{(1)}(q^{(l)}) + h_l \nabla_q \Phi^{(1)}(q^{(l)})^T v^{(l+1)}$$

Take the limit as time step h_l is fixed and $\gamma^{(1)} \to \infty$ and ....

Linearization method

... we obtain:

$$
\begin{aligned}
q^{(l+1)} &= q^{(l)} + h_l v^{(l+1)}. \\
M\frac{v^{(l+1)} - v^{(l)}}{h_l} &= k(t^{(l)}, q^{(l)}, v^{(l)}) + \sum_{j=1}^{m} c^{(j),(l+1)} \nabla_q \Phi^{(j)}(q^{(l+1)}) \\
0 \le c^{(j),(l+1)} &\perp \Phi^{(j)}(q^{(l)}) + h_l \nabla\Phi(q^{(l)})^T v^{(l+1)} \ge 0
\end{aligned}
$$

- Solution to the "do not backtrack at collision" and constraint stabilization problem: Replace in LCP

$$\nabla\Phi(q^{(l)})^T v^{(l+1)} \ge 0 \Longrightarrow \Phi^{(j)}(q^{(l)}) + \gamma h_l \nabla\Phi(q^{(l)})^T v^{(l+1)} \ge 0.$$

$$\nabla\Theta(q^{(l)})^T v^{(l+1)} = 0 \Longrightarrow \Theta^{(j)}(q^{(l)}) + \gamma h_l \nabla\Theta(q^{(l)})^T v^{(l+1)} = 0.$$

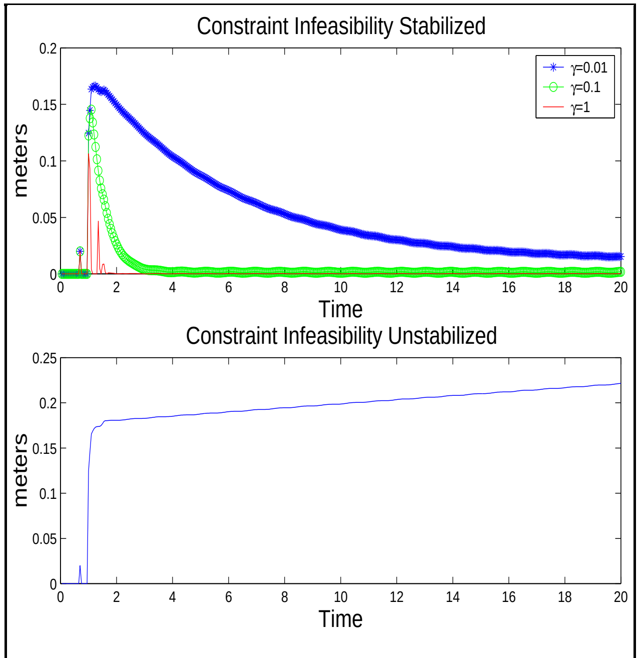
  Here $\gamma \in (0, 1]$. $\gamma = 1$ corresponds to exact linearization.

- This also shows that our methods can be interpreted as linear implicit approach applied to the penalty method.

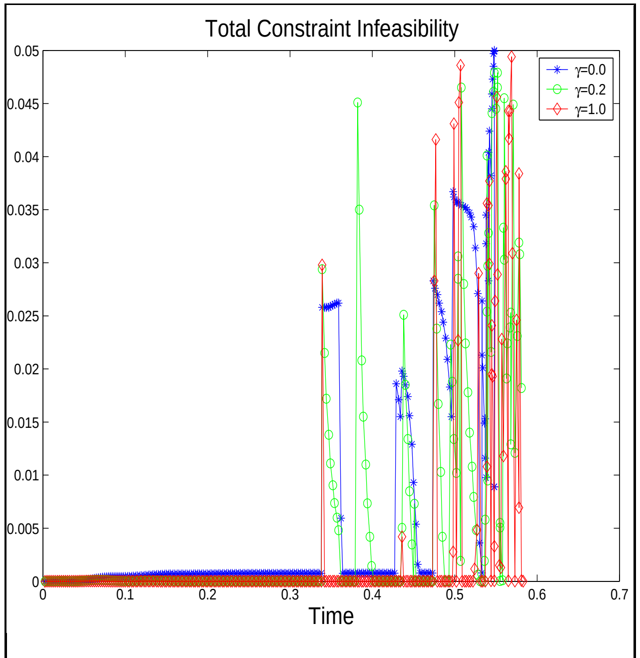
Application: Robotic Grasping Simulator

- GraspIt! Developped by Andrew Miller (Columbia) to accommodate arbitrary hand and robot designs, in a dynamically sound fashion.
- Implements the LCP algorithm with the linearization approach. Uses $\gamma = 0.2$. Timesteps $\approx$ 2 ms (relatively small), because Φ can be computed only for relatively small interpenetration values.
- Has been shown to induce constraint stabilization (prevent constraint drift) (Anitescu, Miller, Hart, 2003).

GraspIt simulation results



Ellipse simulation.



Robot grasp simulation.

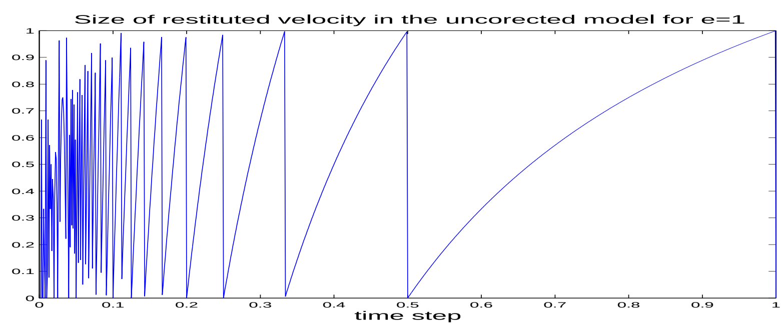
- Note that the smaller the γ, the slower the constraint stabilization.
- On the other hand, the smaller the γ, the smaller the effect of the infeasibility on the energy. However, this does not seem to affect the computations in a major way in the cases we have described.

Treating Partially Elastic Collisions

- A portion of the collision impulse (Poisson law) or normal velocity (Newton law) is restituted to the system.
- Newton's law is more efficient computationally because it can enforced in one LCP (if collision has occured in the previous step):

$$\frac{\Phi(q^{(l)})}{h} + \nabla\Phi^{(j)^T}(q^{(l)})v^{(l+1)} + e_N\nabla\Phi^{(j)^T}(q^{(l-1)})v^{(l)} \geq 0.$$

- However, odd results appear if we apply it in connection with the linearization method. Example: one particle colliding with a wall.



Treating Partially Elastic Collisions

- A simple fix, which works in most cases, (though not provable) is to lag the normal velocity by 1.

- For the LCP with linearization, the change becomes

$$\frac{Phi(q^{(l)}}{h}\nabla\Phi^{(j)^T}(q^{(l)})v^{(l+1)} + e_N\nabla\Phi^{(j)^T}(q^{(l-2)})v^{(l-1)} \geq 0.$$

- For one isolated collision it can be shown that this rule captures the exact behavior in the limit as the time step goes to 0. But no theory exists in general for this case.

A convex relaxation approach.

The original scheme may have a nonconvex solution set no matter how small the friction coefficient. A convex relaxation scheme:

$$M(v^{l+1} - v^{(l)}) - \sum_{i=1}^{m} \nu^{(i)} c_\nu^{(i)} - \sum_{j \in \mathcal{A}} \left(n^{(j)} c_n^{(j)} + D^{(j)} \beta^{(j)}\right) = hk$$

$$\nu^{(i)^T} v^{l+1} + \frac{\Theta^{(i)}(q^{(l)})}{h_l} = 0, \qquad i = 1, \ldots, m$$

$$n^{(j)^T} v^{l+1} + \frac{\Phi^{(j)}(q^{(l)})}{h_l} - \mu^{(\mathbf{j})} \lambda^{(\mathbf{j})} \geq 0, \quad \text{compl. to} \quad c_n^{(j)} \geq 0,\ j \in \mathcal{A}$$

$$\lambda^{(j)} e^{(j)} + D^{(j)T} v^{l+1} \geq 0, \quad \text{compl. to} \quad \beta^{(j)} \geq 0,\ j \in \mathcal{A}$$

$$\mu^{(j)} c_n^{(j)} - e^{(j)^T} \beta^{(j)} \geq 0, \quad \text{compl. to} \quad \lambda^{(j)} \geq 0,\ j \in \mathcal{A}.$$

Recall, $\nu^{(\mathbf{i})} = \nabla \boldsymbol{\Theta}^{(\mathbf{i})}$, $n^{(j)} = \nabla \Phi^{(j)}$, $q^{(l+1)} = q^{(l)} + hv^{(l+1)}$.

■ Data, ■ Unknowns, ■ Convex relaxation term.

Convex relaxation results

$$\min_{v} \quad \frac{1}{2}v^T M^{(l)} v + \widehat{k}^{(l)^T} v$$

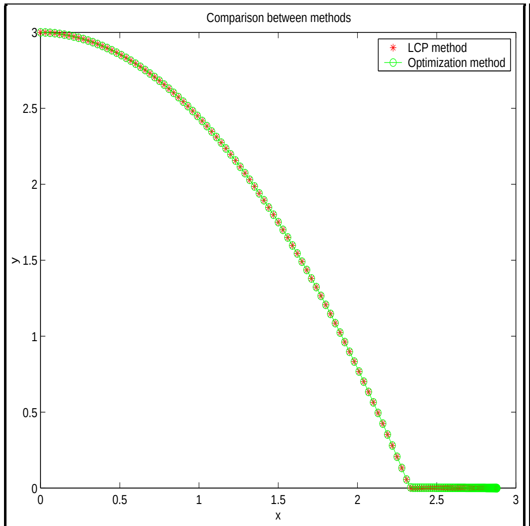
$$\text{subject to} \quad n^{(j)^T} v + \mu^{(j)} d_i^{(j)^T} v \geq -\Gamma^{(j)} - \frac{\mathbf{\Phi^{(j)}(q^{(l)})}}{\mathbf{h_l}}$$

$$i = 1, 2, \ldots, m_C^{(j)}, \quad j \in \mathcal{A}$$

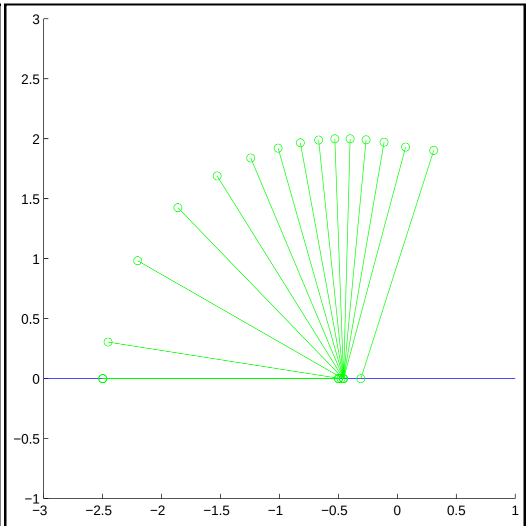
$$\nu^{(i)^T} v = -\frac{\mathbf{\Theta^{(i)}(q^{(l)})}}{\mathbf{h_l}}, \quad i = 1, 2, \ldots, m.$$

■ Data, ■ Unknowns.

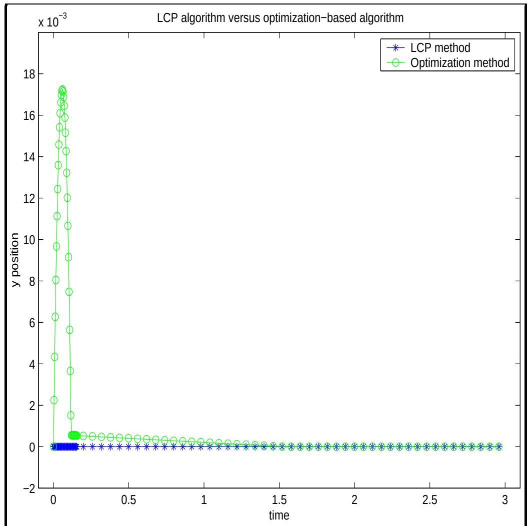
- The velocity solution of the convex relaxation LCP is a solution of this QP for $\Gamma^{(j)} = 0$, $j \in \mathcal{A}$.
- For sufficiently small friction, a fixed point iteration that chooses $\Gamma^{(j)}$ as a function of the preceding velocity will converge to the velocity solution of the original (possibly nonconvex), scheme.
- The distance between solutions of relaxed and unrelaxed scheme does not exceed $\mathbf{C_{geometry}} \max_{j \in \mathcal{A}} \left\{ \mu^{(j)} \lambda^{(j)} \right\}$.



Dropped particle



Painleve example



Sliding particle

$h_k = \frac{0.1}{2^k}, \mu = 0.3$

k	$h_k \quad \left\|y_{QP} - y_{LCP}\right\|_2$
0	5.6314784e-002
1	1.7416198e-002
2	6.7389905e-003
3	2.1011170e-003
4	7.6112319e-004
5	2.6647317e-004
6	9.2498029e-005
7	3.2649217e-005

$h_k = \frac{0.1}{2^k}, \mu = 0.75$

k	$h_k \quad \left\|y_{QP} - y_{LCP}\right\|_2$
0	1.5736018e+000
1	7.2176724e-001
2	1.4580267e-001
3	9.2969637e-002
4	5.5543025e-003
5	4.3982975e-003
6	3.7537593e-003
7	3.7007014e-004

No convergence, but small absolute error.

Time-stepping results

Under some standard assumptions, including pointed friction cone, and $e = 0$, for any fixed time interval T, there exists an $H > 0$, and a $V > 0$ and $C > 0$ such that if $h_l < H, \forall l$, for either the original or the relaxed scheme we have that

1. $v^{(l)} \leq V, \forall l.$

2. The infeasibility, defined as

$$I(q) = \max_{1 \leq i \leq m,\ 1 \leq j \leq n_{total}} \left\{ 0, \left| \Theta^{(i)}(q) \right|, \Phi^{(j)}(q) \right\}$$

satisfies $I(q^{(l+1)}) \leq C \left|\left| h_l v^{(l+1)} \right|\right|^2, \forall l.$

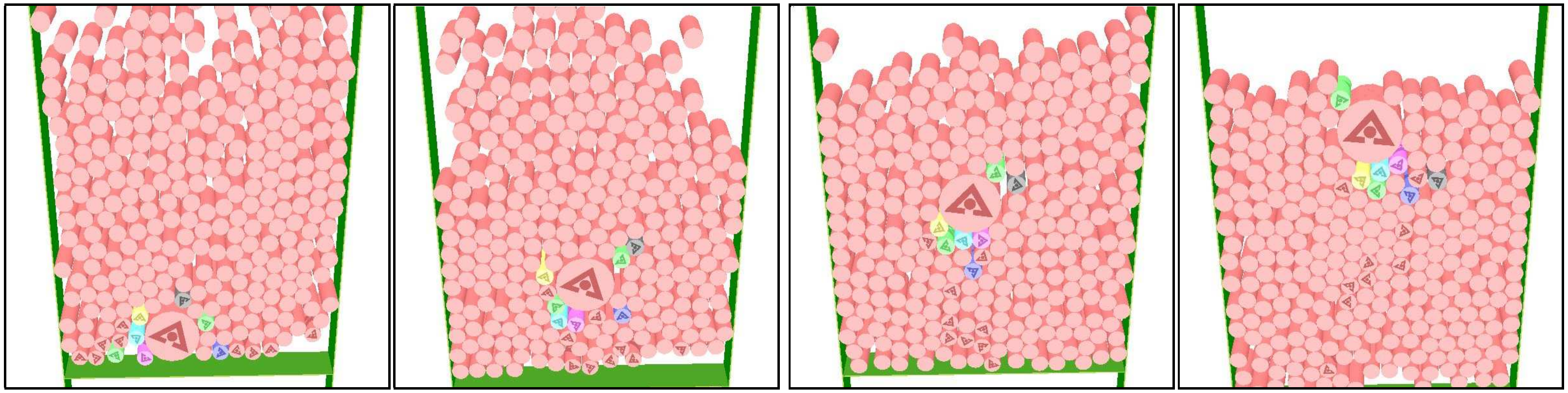
a) The last conclusion demonstrates constraint stabilization.

b) $H = \infty$ if geometrical data has uniformly bounded second derivatives.

Granular matter

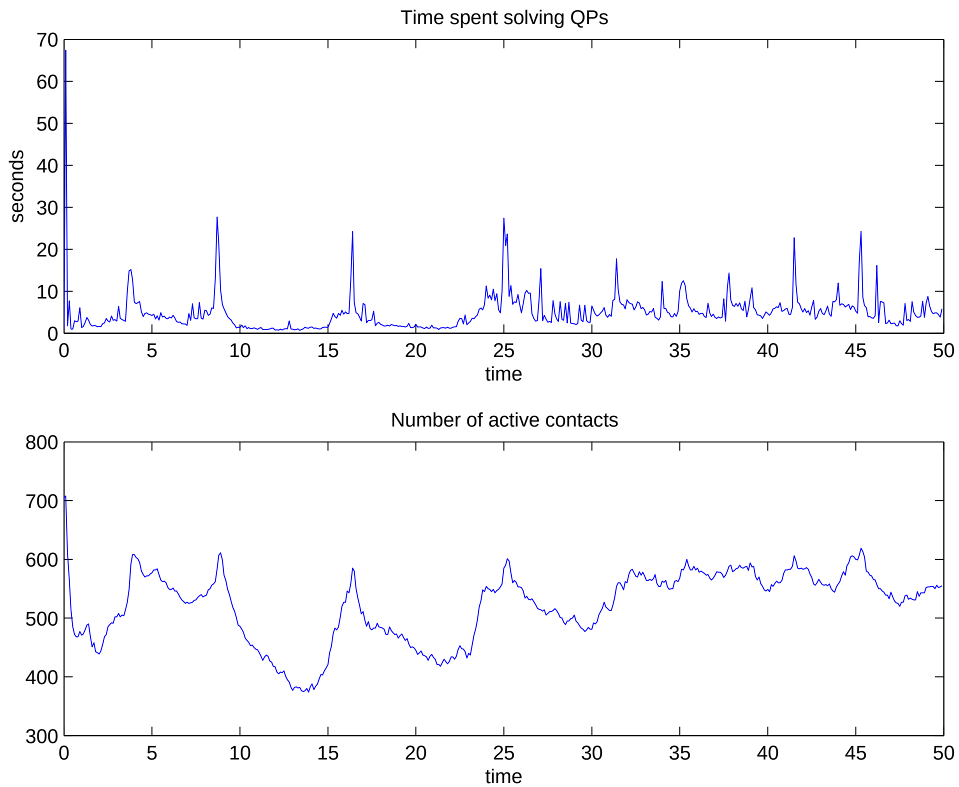
- Sand, Powders, Rocks, Pills are examples of granular matter.
- The range of phenomena exhibited by granular matter is tremendous. Size-based segregation, jamming in grain hoppers, but also flow-like behavior.
- There is still no accepted continuum model of granular matter.
- Direct simulation methods (discrete element method) are still the most general analysis tool, but they are also computationally costly.
- The favored approach: the penalty method which works with time-steps of microseconds for moderate size configurations.

Brazil nut effect simulation



- Time step of 100ms, for 50s. 270 bodies.
- Convex Relaxation Method. One QP/step. No collision backtrack.
- Friction is 0.5, restitution coefficient is 0.5.
- Large ball emerges after about 40 shakes. Results in the same order of magnitude as MD simulations (but with 4 orders of magnitude larger time step).

Brazil nut effect simulations performance



Conclusions and future work

- We define a method that achieves constraint stabilization while solving only one linear complementarity problem per step.
- Our method does not need to stop and detect collisions explicitly.
- The method has been extended to a parametric version that is used in a robotic grasp simulator (Miller and Christiansen, 2002) and (Anitescu, Miller and Hart, 2003). A version of it is used by a commercial simulator.
- Future work: Nonsmooth particles and stabilization proof for nonzero coefficient of restitution. Fast QP Solver. Convergence for relaxation scheme as $h \rightarrow 0$?

Assumption: pointed friction cone assumption

Friction cone: set of all possible constraint reaction impulses:

$$FC(q) = \left\{ t = \tilde{\nu} c_\nu + \tilde{n} c_n + \tilde{D}\tilde{\beta} \,\middle|\, c_n \geq 0,\ \tilde{\beta} \geq 0, \right.$$
$$\left. \left\| \beta^{(j)} \right\|_1 \leq \mu^{(j)} c_n^{(j)},\ \forall j \in \mathcal{A} \right\}$$

- **Definition** (Pang and Stewart 1999) $FC(q)$ is pointed if

$$0 = \tilde{\nu} c_\nu + \tilde{n} c_n + \tilde{D}\tilde{\beta} \in FC(q),\ c_n \geq 0,\ \tilde{\beta} \geq 0 \Rightarrow \left(c_\nu, c_n, \tilde{\beta} \right) = 0$$

- This assumption is essential in ensuring convergence to a differential inclusions as $h_l \rightarrow 0$ (Stewart 2000) and solvability of time-stepping scheme under more general conditions (Pang and Stewart 1999).
- This assumption implies that the configuration is disassemblable at q (Anitescu, Cremer and Potra 1995).

Main Result: assumptions and setup

- Definition: The active set $\mathcal{A} = \{j | 1 \leq j \leq n, \quad \Phi^{(j)}(q^{(l)}) \leq \hat{\epsilon}\}$.
- The friction cone is uniformly pointed for all configurations.
- The geometrical data of the problem, are twice continously differentiable in a neighborhood of the feasible set F.

- The external force increases at most linearly in position and velocity.
- The ratio between succesive time steps is uniformly lower bounded $\frac{h_l}{h_{l-1}} \geq \frac{1}{c_h}$
- The mass matrix is constant (Newton-Euler body coordinates).
- There is no initial infeasibility and the coefficient of restitution is 0.

Main result, continued

Then for any fixed time interval T, there exists an $H > 0$, and a $V > 0$ and $C > 0$ such that if $h_l < H$, $\forall l$, we have that

1. $v^{(l)} \leq V$, $\forall l$.

2. If $j \notin \mathcal{A}$ then $\Phi^{(j)}(q^{(l+1)}) \geq 0$.

3. The infeasibility, defined as

$$I(q) = \max_{1 \leq i \leq m,\, 1 \leq j \leq n_{total}} \left\{ 0, \left| \Theta^{(i)}(q) \right|, \Phi^{(j)}(q) \right\}$$

satisfies $I(q^{(l+1)}) \leq C \left|\left| h_l v^{(l+1)} \right|\right|^2$, $\forall l$.

a) The last conclusion demonstrates constraint stabilization.

b) $H = \infty$ if geometrical data has uniformly bounded second derivatives.

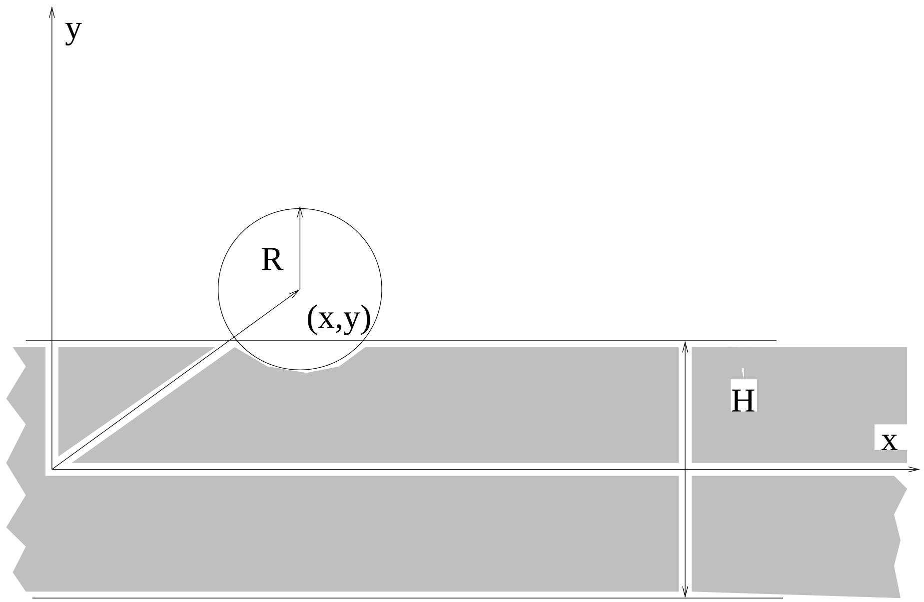
Main technical tool

$$\begin{array}{rrcl} \min_{v} & \frac{1}{2}v^T M^{(l)} v + \widehat{k}^{(l)^T} v & & \\ \text{subject to} & n^{(j)^T} v + \mu^{(j)} d_i^{(j)^T} v & \geq & -\Gamma^{(j)} - \frac{\mathbf{\Phi^{(j)}(q^{(l)})}}{\mathbf{h_l}} \\ & & & i = 1, 2, \ldots, m_C^{(j)}, \quad j \in \mathcal{A} \\ & \nu^{(i)^T} v & = & -\frac{\mathbf{\Theta^{(i)}(q^{(l)})}}{\mathbf{h_l}}, \quad i = 1, 2, \ldots, m. \end{array}$$

- The solution of the LCP for both the original and convex relaxation scheme is a solution of this QP for some $\Gamma^{(j)} \geq 0$, $j \in \mathcal{A}$ and $\widehat{k}^{(l)}$.
- The QP satisfies MFCQ if and only if the friction cone is pointed.
- There exists a $c > 0$, such that, for all l,

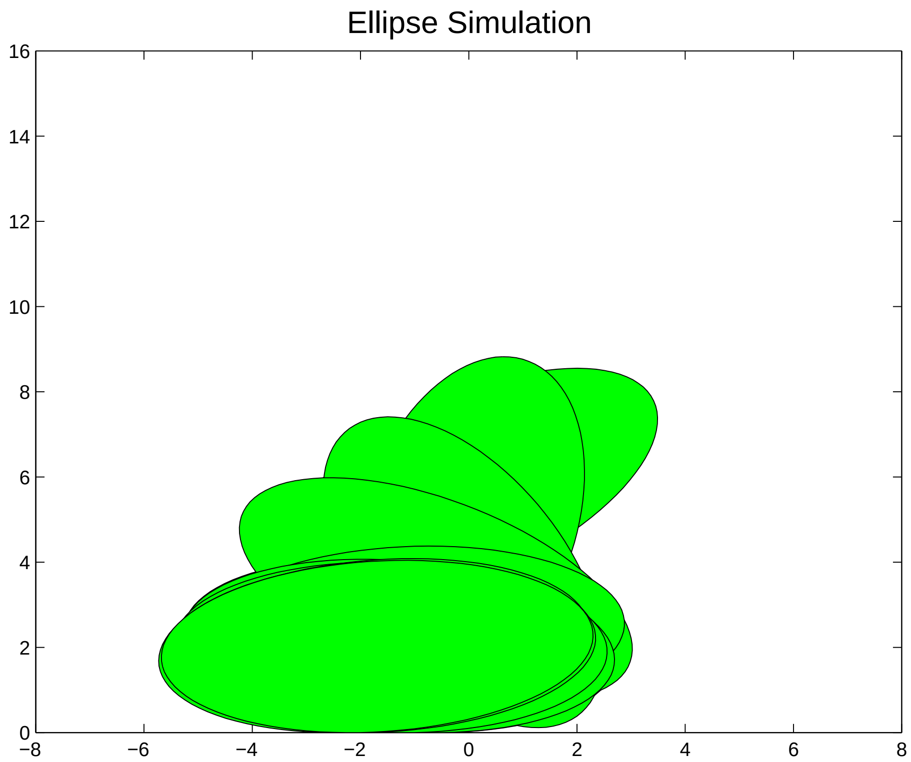
$$\begin{array}{rcl} v^{(l+1)^T} M^{(l)} v^{(l+1)} & \leq & v^{(l)} M^{(l)} v^{(l)} + h_l^2 k^{(l)} M^{(l)^{-1}} k^{(l)} \\ & + & 2h_l v^{(l)^T} k^{(l)} + c \frac{I(q^{(l)})^2}{h_{l-1}^2} \\ I(q^{(l+1)}) & \leq & c h_l^2 v_{l+1}^2, \text{ if } [q^{(l)}, q^{(l+1)}] \in \mathcal{N}(F) \end{array}$$

Example of non differentiability of the signed distance



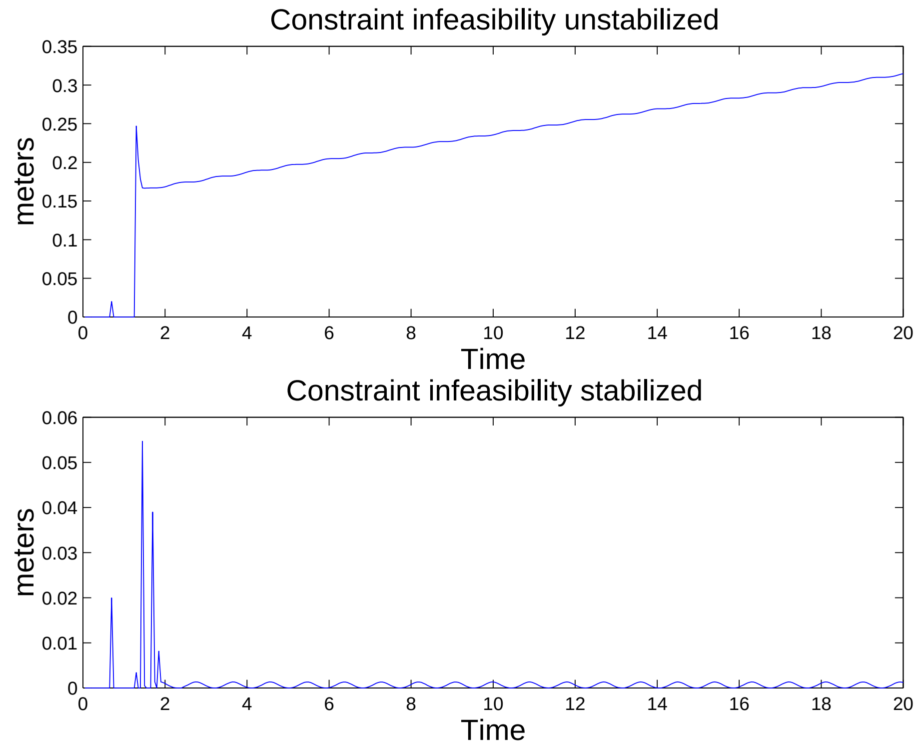
- Signed distance: $d_{12}(q) = |y| - R - \frac{H}{2}$ is not differentiable everywhere!.
- It is, however, differentiable over the set $d_{12}(q) \geq -\epsilon$ for any $\epsilon < R + \frac{H}{2}$. . We are OK if the infeasibility is not too large.

Elliptic body simulation



We present ten frames of the simulation of an elliptic body that is dropped on the table. There is an initial angular velocity of 3, the body has axes 4 and 8 and is dropped from a height of 8.

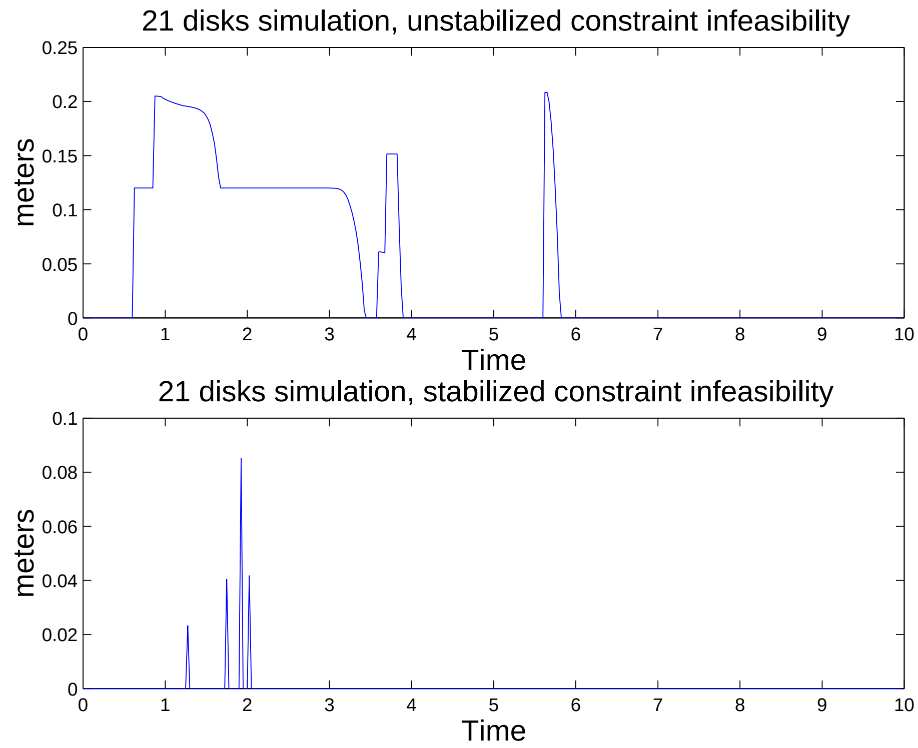
Infeasibility behavior unstabilized versus stabilized metod



We see that drift becomes catastrophic for the unstabilized method, whereas remains in a narrow range for the stabilized method.

Constraint stabilization is accomplished!

Infeasibility comparison for 21 body simulation



21 disks of radius 3 on a plank starting from the cannoball arrangement at rest. Stabilized method still has a much lower infeasibility. The time-step is 50ms, and the LCP is solved in at most 30 ms at every step.

Stabilized time-stepping scheme

$$M(v^{l+1} - v^{(l)}) - \sum_{i=1}^{m} \nu^{(i)} c_\nu^{(i)} - \sum_{j \in \mathcal{A}} \left(n^{(j)} c_n^{(j)} + D^{(j)} \beta^{(j)}\right) = hk$$

$$\nu^{(i)^T} v^{l+1} + \frac{\boldsymbol{\Theta}^{(\mathbf{i})}(\mathbf{q}^{(\mathbf{l})})}{\mathbf{h_l}} = 0, \qquad i = 1, \ldots, m$$

$$n^{(j)^T} v^{l+1} + \frac{\boldsymbol{\Phi}^{(\mathbf{j})}(\mathbf{q}^{(\mathbf{l})})}{\mathbf{h_l}} - \left(\mu^{(\mathbf{j})} \lambda^{(\mathbf{j})}\right) \geq 0, \quad \textbf{compl. to} \quad c_n^{(j)} \geq 0,\ j \in \mathcal{A}$$

$$\lambda^{(j)} e^{(j)} + D^{(j)T} v^{l+1} \geq 0, \quad \textbf{compl. to} \quad \beta^{(j)} \geq 0,\ j \in \mathcal{A}$$

$$\mu^{(j)} c_n^{(j)} - e^{(j)^T} \beta^{(j)} \geq 0, \quad \textbf{compl. to} \quad \lambda^{(j)} \geq 0,\ j \in \mathcal{A}.$$

Recall, $\nu^{(\mathbf{i})} = \nabla \boldsymbol{\Theta}^{(\mathbf{i})}$, $n^{(j)} = \nabla \Phi^{(j)}$, $q^{(l+1)} = q^{(l)} + hv^{(l+1)}$.

The term $\left(-\mu^{(\mathbf{j})} \lambda^{(\mathbf{j})}\right)$ coresponds to the convex relaxation algorithm proposed by (Anitescu and Hart, 2002). Stabilization works w or w/o it.

The need for constraint stabilization

The positions are updated by $q^{(l+1)} = q^{(l)} + h_l v^{(l+1)}$.

Due to the index reduction , the (geometrical) joint and non interpenetration constraints, which define the feasible set

$$\mathcal{F} = \left\{ q \,\middle|\, \Theta^{(i)}(q) = 0,\ 1 \le i \le m, \Phi^{(j)}(q) \ge 0,\ 1 \le j \le n_{total} \right\}$$

are replaced by constraints at the velocity level.

This may create constraint drift, in which the constraint infeasibility keeps growing. In interactive simulation this is particulatly annoying, since geometrical inconsistency is easy to notice.

Preventing constraint drift

- Change the approach to a nonlinear and potentially nonconvex optimization problem.
- Perform a nonlinear projection after each LCP (and eventually preserving the good energy properties (Anitescu and Potra 2002)). If there is no friction the projection may be more costly than the LCP.
- Perform one step of an SQP applied to the nonlinear projection problem (Cline and Pai, 2003). Needs an additional Quadratic Program/step. Extension of (Ascher, Chin, Reich 1994) from DAE.
- Modify the right hand side of the LCP with an appropriate function of the infeasibility (parameter-free, (Jean, 1999,w/o analysis) and this work, (Anitescu and Hart 02)) and (Miller and Christiansen 02) and (Anitescu, Miller and Hart 03). This approach uses no additional subproblems or projections.