Convergence of Elastic Mode Formulations of MPECs

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Outline

- MPECs: Notation, Definitions, Assumptions.
- Elastic Mode Formulations
- Local Relationships between elastic mode formulations and MPECs
- Elastic-Mode Formulation: Global Convergence:
  - Convergence of First-Order Points
  - Convergence of Second-Order Points
  - Numerical Examples

Study the formulation with complementary variables:

$$\begin{aligned} \min_x \quad & f(x) \text{ subject to} \\ & g(x) \geq 0, \quad h(x) = 0, \\ & 0 \leq G^T x \perp H^T x \geq 0, \end{aligned}$$

where

- G and H are $n \times m$ column submatrices of the $n \times n$ identity matrix (with no columns in common): *lower bounds*;
- $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}^p$, and $h : \mathbb{R}^n \to \mathbb{R}^q$ are twice continuously differentiable.

Theory extends to nonlinear functions $0 \leq G(x) \perp H(x) \geq 0$. We use bounds because they can be enforced explicitly by algorithms for the NLP subproblem; this leads to some nice properties.

Parametrized NLP Formulations

Regularized (Scholtes, 2001):

$$\text{Reg}(t): \quad \min_x \ f(x) \ \text{ subject to}$$
$$g(x) \geq 0, \quad h(x) = 0,$$
$$G^T x \geq 0, \quad H^T x \geq 0, \quad (G_i^T x)(H_i^T x) \leq t, \quad i = 1, 2, \ldots, m.$$

Elastic Mode for MPEC:

$$\text{Elastic}(c): \quad \min_{x,\zeta} \ f(x) + c\zeta + c(G^T x)^T (H^T x) \ \text{ subject to}$$
$$g(x) \geq -\zeta e_p, \quad \zeta e_q \geq h(x) \geq -\zeta e_q, \quad 0 \leq \zeta \leq \bar{\zeta},$$
$$G^T x \geq 0, \quad H^T x \geq 0,$$

Elastic Mode for NLP:

$$\text{Elastic}(c): \quad \min_{x,\zeta} \ f(x) + c\zeta \ \text{ subject to}$$
$$g(x) \geq -\zeta e_p, \quad \zeta e_q \geq h(x) \geq -\zeta e_q, \quad 0 \leq \zeta \leq \bar{\zeta},$$
$$G^T x \geq 0, \quad H^T x \geq 0, \quad (G^T x)^T (H^T x) \leq \zeta$$

Questions

1. How are solutions of Elastic(c) related to those of the MPEC? (Exactness)
2. For a fixed value of c, under what assumptions can we converge locally to a solution?
3. Can we devise a scheme for updating c_k that ensures desirable properties of the accumulation points?
4. If $\{(x^k, \zeta_k)\}$ is a sequence of approximate first-order (stationary) points for Elastic(c_k), what are the properties of accumulation points of this sequence?

But First, Some Definitions

Stationarity for MPEC at a feasible point x^*: Define active sets:

$$I_g \triangleq \{i \in \{1,2,\dots,p\} \mid g_i(x^*) = 0\},$$
$$I_G \triangleq \{i \in \{1,2,\dots,m\} \mid G_i^T x^* = 0\},$$
$$I_H \triangleq \{i \in \{1,2,\dots,m\} \mid H_i^T x^* = 0\},$$

Feasibility $\Rightarrow$
$I_G \cup I_H = \{1,2,\dots,m\}$

Multiplier tuple (λ,μ,τ,ν) defines **MPEC Lagrangian** :

$$L(x,\lambda,\mu,\tau,\nu) = f(x) - \lambda^T g(x) - \mu^T h(x) - \tau^T G^T x - \nu^T H^T x.$$

Constraint qualifications: **MPEC-LICQ:** $\mathcal{K}$ is linearly independent set (ensures that $(\lambda^*,\mu^*,\tau^*,\nu^*)$ satisfying stationarity is unique):

$$\mathcal{K} \triangleq \{\nabla g_i(x^*)\}_{i \in I_g} \cup \{\nabla h_i(x^*)\}_{i=1,2,\dots,q} \cup \{G_i\}_{i \in I_G} \cup \{H_i\}_{i \in I_H}.$$

Stationary points satisfy ...

$$\begin{array}{rcl} \nabla_x L(x^*, \lambda^*, \mu^*, \tau^*, \nu^*) & = & 0, \\ 0 \le \lambda^* \perp g(x^*) & \ge & 0, \\ h(x^*) & = & 0, \\ \tau^* \perp G^T x^* & \ge & 0, \\ \nu^* \perp H^T x^* & \ge & 0, \end{array}$$

...**AND**, from stronger to weaker concept, .

- Strong stationarity: $\tau_i^* \geq 0 \;\; \nu_i^* \geq 0, \;\; i \in I_G \cap I_H,$.
- M-stationarity: $\tau_i^* \nu_i^* \geq 0$ but not both τ_i^*, ν_i^* negative, for $i \in I_G \cap I_H$.
- C-stationarity: $\tau_i^* \nu_i^* \geq 0$ for $i \in I_G \cap I_H$.

Strong stationarity: there is no direction that decreases f but stays feasible to first order .

More Index Sets

At a strongly stationary x^* define **strongly** and **weakly** active subsets (Denote by superscripts $+$ and 0.)

$$
\begin{aligned}
I_g^+ &\triangleq \{i \in I_g \,|\, \lambda_i^* > 0 \text{ for some multiplier}\}, \\
J_G^+ &\triangleq \{i \in I_G \cap I_H \,|\, \tau_i^* > 0 \text{ for some multiplier}\}, \\
J_H^+ &\triangleq \{i \in I_G \cap I_H \,|\, \nu_i^* > 0 \text{ for some multiplier}\}, \\
I_g^0 &\triangleq I_g \setminus I_g^+, \quad J_H^0 \triangleq (I_G \cap I_H) \setminus J_H^+ \quad J_G^0 \triangleq (I_G \cap I_H) \setminus J_G^+,
\end{aligned}
$$

Different flavors of complementarity:

- USC: $J_G^+ = J_H^+ = I_G \cap I_H$
- PSC: $J_G^+ \cup J_H^+ = I_G \cap I_H$
- LSC: $I_G \cap I_H = \emptyset$

Critical directions for MPEC:

$$
\begin{aligned}
S^* \quad \triangleq \quad & \{s \,|\, \nabla h(x^*)^T s = 0\} \cap \\
& \{s \,|\, \nabla g_i(x^*)^T s = 0 \ \text{ for all } i \in I_g^+\} \cap \\
& \{s \,|\, \nabla g_i(x^*)^T s \geq 0 \ \text{ for all } i \in I_g^0\} \cap \\
& \{s \,|\, G_i^T s = 0 \ \text{ for all } i \in I_G \setminus I_H\} \cap \\
& \{s \,|\, H_i^T s = 0 \ \text{ for all } i \in I_H \setminus I_G\} \cap \\
& \{s \,|\, G_i^T s \geq 0 \ \text{ for all } i \in J_G^0\} \cap \\
& \{s \,|\, G_i^T s = 0 \ \text{ for all } i \in J_G^+\} \cap \\
& \{s \,|\, H_i^T s \geq 0 \ \text{ for all } i \in J_H^0\} \cap \\
& \{s \,|\, H_i^T s = 0 \ \text{ for all } i \in J_H^+\} \cap \\
& \{s \,|\, \min(H_i^T s, G_i^T s) = 0 \ \text{ for all } i \in J_G^0 \cap J_H^0\}.\}.
\end{aligned}
$$

Second-Order Conditions for MPEC

MPEC-SOSC: Let x^* be strongly stationary. There is $\sigma > 0$ such that for every $s \in S^*$, there are multipliers such that

$$s^T \nabla^2_{xx} L(x^*, \lambda^*, \mu^*, \tau^*, \nu^*) s \geq \sigma \|s\|^2.$$

Local Results - MA(2005a)-answers Q 1& 2

Theorem

If, the solution point x^* is strongly stationary, and MPEC-SOSC is satisfied at x^*, then for $c \geq c_0(x^*, \lambda^*, \mu^*, \tau^*, \nu^*)$.

- *$(x^*, 0)$ is a local minimum and an isolated stationary point of the elastic mode NLP problem.*
- *Elastic (c) satisfies MFCQ so its linearizations are feasible, the opposite of which were the main failure mode for NLP algorithms applied to MPEC.*
- *In addition, if MPEC-LICQ, MPEC-SOSC, and PSC hold then the elastic mode-Newton method is superlinearily convergent.*

Global results for mixed P parameterized VI- MA (2005b)

The partition $[A\ B\ C]$ is mixed P partition if

$$0 \neq (y, w, z) \in \mathbb{R}^{2n_c+l},\ Ay + Bw + Cz = 0 \Rightarrow$$
$$\exists i,\ 1 \leqslant i \leqslant n_c,\ \text{such that } y_i w_i > 0.$$

(OMPV)

$$\begin{array}{lll} \min\limits_{x,y,w,z} & f(x,y,w,z) & \\ \text{sbj.to} & g(x) & \leqslant 0 \ \ (\mu) \\ & h(x) & = 0 \ \ (\lambda) \\ & F(x,y,w,z) & = 0 \ \ (\theta) \\ & y, w & \leqslant 0 (\eta_{y,w}) \\ & y^T w & \leqslant 0 \ (\alpha_c) \end{array}$$

where the partition $[\nabla_y F, \nabla_w F, \nabla_z F]$ is a mixed P partition for any x. The **obstacle problem** has that property.

Partial answer to Q 3 & 4

Theorem

For a sequence of points (x^k, ζ^k) that ...

- *... are first-order ϵ_k approximate stationary points for Elastic NLP (c^k) such that $c^k \epsilon^k \to 0$, have C-stationary points at all accumulation points.*
- *... are second-order ϵ_k, δ_k approximate stationary points for Elastic NLP (c^k) such that $c^k \epsilon^k \to 0$, and accumulate at x^* that satisfies MPEC-LICQ, then x^* is an M-stationary points.*
- *If, in addition, x^* satisfies ULSC, then it is a strongly stationary point.*

Since the solves are inexact, the approach is implementable.

What is missing ? Better answer to Q 3 & 4

- It is still somewhat unsatisfactory that MPEC-LICQ, that is sufficient for a solution to be strongly stationary, is not sufficient for the accumulation point to be (strongly) stationary for Elastic mode (global convergence).
- The local results prove that: **if** c sufficiently large and fixed and **if** you start sufficiently close to a strongly stationary point then you converge.
- For robustness and implementability, we would need that **If** c^k satisfies an update rule and **if** the problem is solved inexactly and **if** the limit point satisfies MPEC-LICQ then the limit point is a strongly stationary point.

Elastic Formulation: Global Convergence

$$\text{Elastic}(c): \quad \min_{x,\zeta} \; f(x) + c\zeta + c(G^T x)^T (H^T x) \text{ subject to}$$
$$g(x) \geq -\zeta e_p, \;\; \zeta e_q \geq h(x) \geq -\zeta e_q, \;\; 0 \leq \zeta \leq \bar{\zeta},$$
$$G^T x \geq 0, \;\; H^T x \geq 0,$$

Approximate Optimality for Elastic(c)

ϵ-first-order point: Replace 0 in most of the KKT conditions for Elastic(c) by ϵ. However, still require all multipliers to be nonnegative, and enforce $G^T x \geq 0$, $H^T x \geq 0$ exactly.

Second-order point if there is some multiplier tuple satisfying KKT such that

$$\tilde{u}^T \nabla^2_{(x,\zeta)(x,\zeta)} L_c(x, \zeta, \lambda, \mu^-, \mu^+, \tau, \nu) \tilde{u} \geq 0,$$

for all $\tilde{u} \in \mathbb{R}^{n+1}$ in the gradient null space of active constraints at (x, ζ) ($\mu \to (\mu^+, \mu^-)$ from relaxation).

(ϵ, δ)-**second-order point** if for multipliers satisfying the ϵ-first-order definition we have

$$\tilde{u}^T \nabla^2_{(x,\zeta)(x,\zeta)} L_c(x, \zeta, \lambda, \mu^-, \mu^+, \tau, \nu)\tilde{u} \geq -C\|\tilde{u}\|^2,$$

for some fixed C and all $\tilde{u} \in \mathbb{R}^{n+1}$ that are in the gradient null space of all active bound constraints ($G^T x \geq 0$, $H^T x \geq 0$, $0 \leq \zeta \leq \bar{\zeta}$) at (x, ζ), and in the gradient null space of δ-active nonbound constraints $g(x) \geq -\zeta e_p$, $\zeta e_q \geq h(x) \geq -\zeta e_q$) at (x, ζ).

Sequence of Inexact First-Order Points

Given a sequence of inexact first-order points for Elastic(c_k), any accumulation point satisfying feasibility and CQ for the MPEC is C-stationary. Formally:

Theorem

$\{c_k\}$ positive, nondecreasing; $\{\epsilon_k\}$ is nonnegative with $\{c_k \epsilon_k\} \to 0$; (x^k, ζ_k) is an ϵ_k-first-order point of Elastic (c_k). If x^* is an accumulation point of $\{x^k\}$ that is feasible for MPEC and satisfies MPEC-LICQ, then x^* is C-stationary and $\zeta_k \to 0$ for the convergent subsequence.

Proof is long but fairly elementary.

To avoid the possibility of an infeasible limit, could increase c_k when the current approx solution is not sufficiently feasible.

Assume

- $\{f(x^k)\}$ is bounded below.
- $\{f(x^k) + c_k\zeta_k + c_k(G^T x^k)^T(H^T x^k)\}$ bounded above.
- **The update rule:** have sequences $\{\omega_k\} \to 0$, $\{\eta_k\} \to \infty$, such that $c_{k+1} \geq \eta_{k+1}$ when $\zeta_k + (G^T x^k)^T(H^T x^k) \geq \omega_k$.

Then any accumulation point of a sequence of ϵ_k-first-order points for Elastic(c_k) is feasible.

Sequence of Exact Second-Order Points – answer to Q4

$\{c_k\} \uparrow \infty$ and let each (x^k, ζ_k) be a second-order point for Elastic(c_k).

Theorem

Either there is finite termination at some c_k (with x_k feasible for MPEC), or else any accumulation point of $\{x^k\}$ is infeasible for MPEC or else fails to satisfy MPEC-LICQ.

Proof: First show $\zeta_k = 0$ for k sufficiently large. Then if $(G_j^T x^*)(H_j^T x^*) > 0$ for some j and accumulation point x^*, can identify a direction of arbitrarily negative curvature over the subsequence of k's. (Contradicts second-order assumption.) **Key: Finite exact complementarity $\Rightarrow$ c_k is fixed for $k \geq k_0$**

Other convergence properties are corollaries of the *inexact* case.

Sequence of Inexact Second-Order Points – answer Q4

(x^k, ζ_k) is an (ϵ_k, δ_k)-second-order point of Elastic(c_k).

Theorem

Let $\{c_k\}$ nondecreasing, $\{\epsilon_k\}$ has $\{c_k\epsilon_k\} \to 0$, and $\{\delta_k\} \to 0$. Assume that acc point x^* is feasible for MPEC, satisfies MPEC-LICQ. Then have c^* such that if $c_k \geq c^*$, k large, we have

(a) *x^* is M-stationary for MPEC.*

(b) *$\{c_k\}$ bounded $\Rightarrow x^*$ strongly stationary for MPEC.*

(c) *$\tau^k \perp G^T x^k$ and $\nu^k \perp H^T x^k \Rightarrow$* **finite exact complementarity** $(G^T x^k)^T (H^T x^k) = 0$ *(for k with x_k near x^* and $c_k \geq c^*$).*

Finite Exact Complementarity: Another Condition

Definition

The strengthened MPEC-LICQ (MPEC-SLICQ) holds at a feasible point x^* of MPEC if the vectors in each of the following sets are linearly independent:

$$\mathcal{K} \cup \{H_j\}, \text{ for } j \in I_G \backslash I_H, \quad \mathcal{K} \cup \{G_j\}, \text{ for } j \in I_H \backslash I_G,$$

where $\mathcal{K}$ is the usual set of active constraint gradients for MPEC.

Under similar conditions to the previous theorem, with MPEC-SLICQ replacing $\tau^k \perp G^T x^k$ and $\nu^k \perp H^T x^k$, get finite exact complementarity.

Algorithm Elastic-Inexact

Choose $c_0 > 0$, $\epsilon_0 > 0$, $M_\epsilon > M_c > 1$, and positive sequences $\{\delta_k\} \to 0$, $\{\omega_k\} \to 0$;

for $k = 0, 1, 2, \ldots$

find an (ϵ_k, δ_k)-second-order point (x^k, ζ_k) of $\mathrm{PF}(c_k)$ with Lagrange multipliers $(\lambda^k, \mu^{-k}, \mu^{+k}, \tau^k, \nu^k, \pi^{-k}, \pi^{+k})$;

if $\zeta_k + (G^T x^k)^T (H^T x^k) \geq \omega_k$,

set $c_{k+1} = M_c c_k$;

else

set $c_{k+1} = c_k$;

end (if)

choose $\epsilon_{k+1} \in (0, \epsilon_k / M_\epsilon]$.

end (for)

Numerical Results

Test on elastic-membrane-on-obstacle problems of Outrata, Kocvara, Zowe, as implemented in MacMPEC by Fletcher and Leyffer. Three problem sets , six variants of each (linear or parabolic obstacle, three levels of finite-element discretization for each):

- Incident set identification (`is`)
- Packaging problem with pliant obstacle (`pc`)
- Packaging problem with rigid obstacle (`pr`).

Implement Elastic-Inexact using filterSQP (Fletcher/Leyffer) as the NLP solver.

Parameters $c_0 = 10$, $\epsilon_0 = 10^{-3}$, $M_\epsilon = 15$, $M_c = 10$, $\omega_k = \min\{(k+1)^{-1}, c_k^{-1/2}\}$.

Aim: *Observe various features of the analysis: finite exact complementarity, second-order points for Elastic(c_k) at at the limiting MPEC, constraint qualifications.*

Means: Used AMPL scripts for implementation, dumped the derivative information on disk using "option auxfiles rc" and loaded it in Matlab using routines developped by Todd Munson.

Thanks to Sven Leyffer, Todd Munson.

Exact complementarity is satisfied at final point for all problems.
3/16 abnormal terminations:

Problem	Termination Message	c_k	Infeas
is-1-8	Optimal	10	
is-1-16	Optimal	10	
is-1-32	Small Trust Region	10	2.25e–07
is-2-8	Optimal	10	
is-2-16	Optimal	10	
is-2-32	Optimal	10^3	
pc-1-8	Optimal	10	
pc-1-16	Optimal	10^2	
pc-1-32	Optimal	10^3	
pc-2-8	Optimal	10^2	
pc-2-16	Optimal	10^5	
pc-2-32	Local Inf	10^4	6.06e–12
pr-1-8	Optimal	10^2	
pr-1-16	Optimal	10^3	
pr-1-32	Optimal	10^6	
pr-2-8	Optimal	10^2	
pr-2-16	Optimal	10^5	
pr-2-32	Local Inf	10^6	5.68e–13

Validates our **early satisfaction of exact complementarity** .

Constraint Qualifications and Second-Order Conditions

Define "numerically active" constraints using a tolerance of $\delta = 10^{-6}$.

Define $J_{\rm act}$ to be the matrix of numerically active constraint gradients.

For Q_2 spanning the null space of $J_{\rm act}$, we measure satisfaction of MPEC-SLICQ via

$$\chi_{\rm span} \triangleq \min \left(\min_{i \notin I_G} \| Q_2^T G_i \|_2, \min_{i \notin I_H} \| Q_2^T H_i \|_2 \right),$$

Satisfaction of second-order conditions measured by examining eigenvalues of $Q_2^T L Q_2$, where L is Hessian Lagrangian at the last NLP.

Problem	n_F	m_{act}	$\mathrm{cond}_2(J_{\mathrm{act}})$	$\chi\mathrm{span}$	$\lambda_{\mathrm{min}}\left(Q_2^T H Q_2\right)$
is-1-8	193	181	3.45e+03	1.95e–03	0
is-1-16	763	742	4.39e+04	6.84e–04	0
is-1-32	3042	3020	5.26e+05	3.90e–09	0
is-2-8	184	180	2.17e+03	5.66e–04	1.08e–04
is-2-16	750	745	6.46e+04	8.44e–05	4.10e–07
is-2-32	3032	3025	∞	0	-1.48
pc-1-8	228	228	1.96e+02	0	∞
pc-1-16	970	964	9.38e+03	1.91e–06	5.55e–02
pc-1-32	3997	3972	4.48e+04	1.22e–08	4.88e–01
pc-2-8	233	228	3.40e+03	1.27e–04	1.37e+00
pc-2-16	977	964	1.34e+04	4.34e–06	6.62e–01
pc-2-32	4001	3972	7.82e+04	7.61e–09	2.06e–01
pr-1-8	186	179	1.10e+03	2.96e–17	2.61e–07
pr-1-16	754	739	4.11e+03	1.35e–18	0
pr-1-32	3040	3011	8.99e+07	3.56e–19	4.34e–01
pr-2-8	185	179	3.22e+03	1.47e–18	4.88e–01
pr-2-16	743	739	3.07e+03	1.91e–23	2.12e–01
pr-2-32	3027	3011	7.62e+03	8.92e–24	1.79e–01

MPEC-LICQ is 15/16, approx second-order point 16/16, MPEC-SLICQ 10/16.

Examples

Example 1: Reg(t) does better(?)

$$\min_x \frac{1}{2}(x_2 - 1)^2 \text{ subject to } 0 \le x_1 \perp x_2 \ge 0,$$

Elastic(c_k) and PF(c_k) are both

$$\min_x \frac{1}{2}(x_2 - 1)^2 + c_k x_1 x_2 \text{ subject to } x_1 \ge 0,\ x_2 \ge 0,$$

for which $\nabla^2_{xx} L_{c_k}(x, \tau, \nu)$ is indefinite, implying that $x_1 = 0$ or $x_2 = 0$ at second-order points. Possible solutions of Elastic are $x^k = (0, 1)$—global solution of MPEC—or

$$x^k = \begin{bmatrix} \frac{1}{c_k} + \alpha \\ 0 \end{bmatrix}, \text{ for any } \alpha \ge 0$$

which are all local solutions satisfying MPEC-LICQ but are far

Reg(t_k) has solutions

$$x^k(\alpha) = \begin{bmatrix} \alpha \\ 1 \end{bmatrix}, \quad \alpha \in [0, t_k],$$

which approach MPEC solution.

Example 2: Elastic may do better.

Define "Robinson" function

$$F(y) \triangleq \int_0^y t^6 \sin(1/t)\, dt.$$

and define MPEC:

$$\min_x \; -x_1 - F(x_2) \;\; \text{subject to} \quad 0 \le x_1 \perp x_2 \ge 0,$$

MPEC has M-stationary point at $x^* = (0,0)$ with multipliers $\tau^* = -1$, $\nu^* = 0$, and strongly stationary points arbitrarily close to $(0,0)$ at $(0, (2n\pi + \pi)^{-1})^T$. But there is direction of unboundedness along x_1 axis.

Elastic(c) has second-order points that coincide with the x_2-axis MPEC strongly stationary points, with additional condition $x_2 \ge 1/c$. Other stationary points are not second-order points.

An NLP algorithm applied to Elastic(c) may find the direction of unboundedness or one of the x_2-axis stationary points, depending on how initialized etc.

Reg(t_k) for $\{t_k\} \uparrow \infty$ has a sequence of second-order sufficient solutions x^k approaching the M-stationary point 0.

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How likely is MPEC-SLICQ to hold?

Unfortunately, the set of h, g for which MPEC-SLICQ holds everywhere is not dense in the Whitney topology (as is the case for MPEC-LICQ, Scholtes 2001) ...

$x \in \mathbb{R}^3$, g vacuous, $h(x) = x_1^2 - x_2 + 1$, $G^T x = x_2$, $H^T x = x_3$ around $(0, 1, 0)$

.... since we need one more degree of freedom at the solution. We conjecture that for the problems for which that happens, the set of MPEC satisfying MPEC-SLICQ is dense in the sense above. for which that happens, the set of MPEC satisfying MPEC-SLICQ is dense in the sense above.