Summarizing and Visualizing Graph Ensembles with Rank Statistics and Boxplots

Mukund Raj, Ian Ruginski, Robert M. Kirby and Ross T. Whitaker

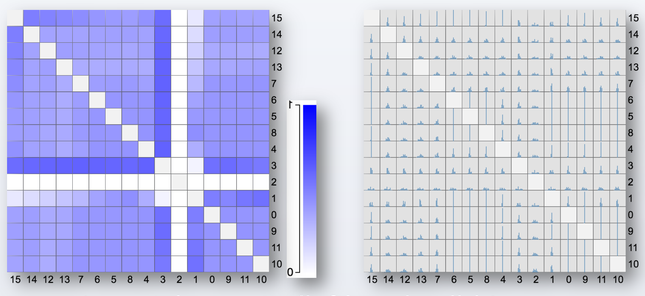
University of Utah

INTRODUCTION

- Ensembles of aligned, weighted graphs are important in many areas.
- In neuroscience graphs are used to represent functional and anatomical brain networks.
- Need to convey salient features in graph ensembles in terms of structure- and variability of edge weights in graph ensembles [1].
- Goals
  - Compare two ensembles.
  - Compare individual to ensemble.
- Questions when comparing
  - Is there a difference?
  - What is the direction of difference?
  - Is the difference significant?

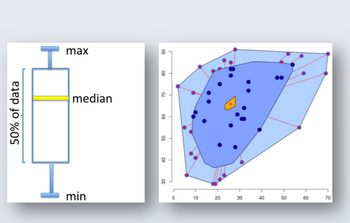
RELATED WORK AND BACKGROUND

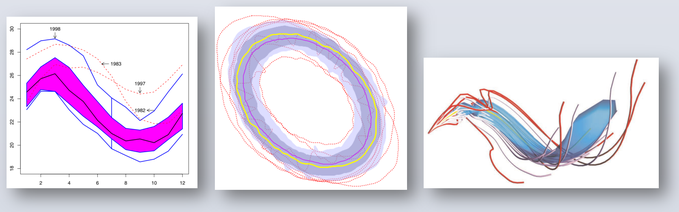
- Graph ensemble visualization
  - Heatmap
  - Cell histogram [2]



Figs. A heatmap (left) and cell histogram (right) visualization.

- Depth based visualizations
  - Tukey Boxplot (Points in 1-D)
  - Bagplot (Points in 2-D) [3]
  - ***Functional Boxplot (Functions)*** **[4]**
  - Contour Boxplot (Contours) [5]
  - Curve Boxplot (Multivariate curves) [6]



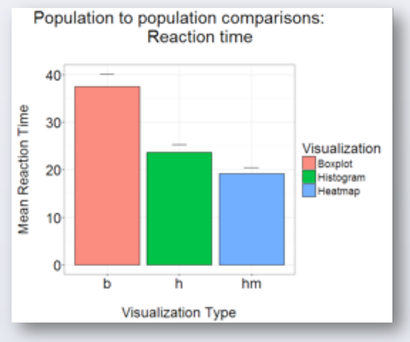


Figs. A Tukey boxplot (above-left), bagplot (above-right), functional boxplot (bottom-left), contour boxplot (bottom-center) and curve boxplot (bottom-right).

RESULTS

- Pilot user study
  - Goal: Evaluate network boxplot, heatmap and cell histogram.
  - Task: Match synthetic graph ensembles based on structure and variability.
  - Design:
    - Within subjects design with 6 subjects
    - Independent variable: Visualization type
    - Dependent variables: Accuracy and reaction time
  - Result summary: Using network boxplots, participants made more accurate judgments but also took more time to arrive at a decision.

Figs. Plots of accuracy (left) and reaction time (right).



- Interactive system for analyzing brain fMRI networks
  - Goal: Evaluate network boxplot using real brain imaging data.
  - Task: Comparing brain network ensembles from control and autistic groups.
  - Result summary: Noticeable differences between structure of population groups observed.

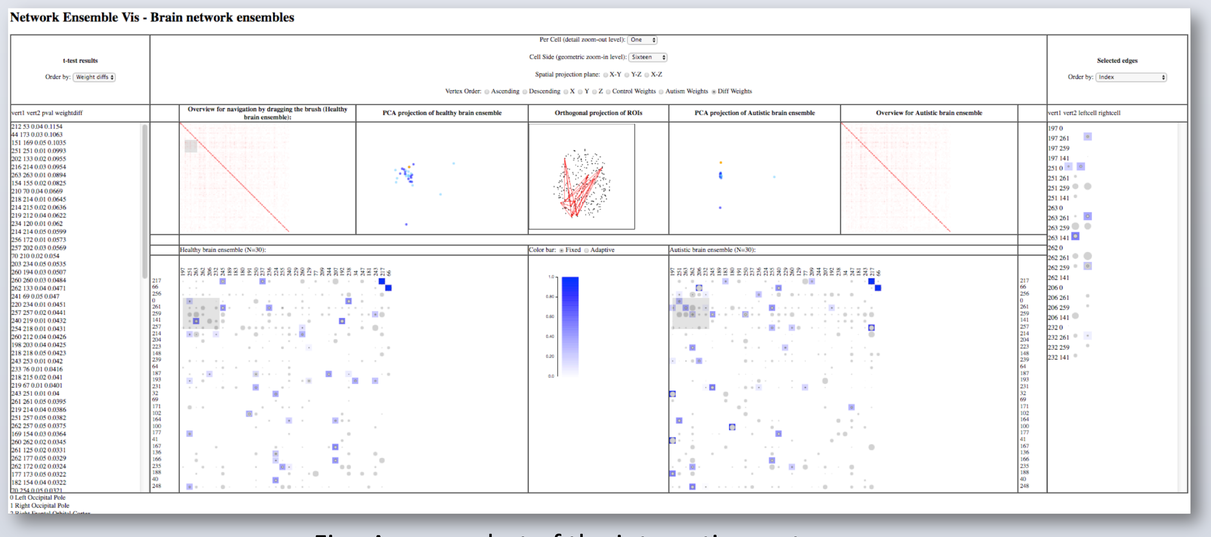


Fig. A screenshot of the interactive system

METHOD

- Step 1: Obtain weighted adjacency matrix representations of graphs in ensemble.

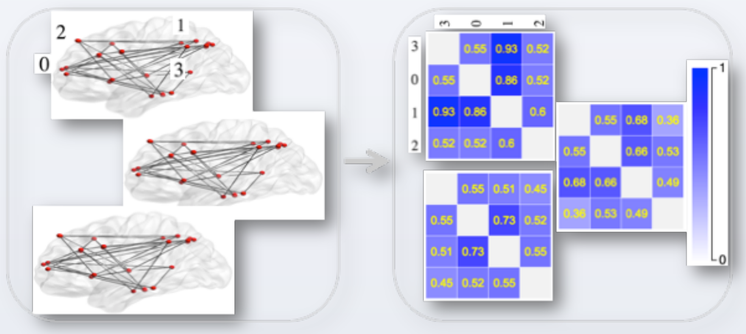
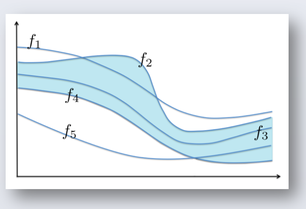


Fig. An impression of a brain network ensemble (left) and associated adjacency matrices (right).

- Step 2: Compute center outward order and rank statistics by performing functional band depth analysis of adjacency matrix ensembles.



$$\mathrm{fBD} = \mathrm{Prob}\big(f \in \mathrm{fB}(f_1, \ldots, f_j)\big)$$

Fig. A functional band 'fB' (left) and functional band depth 'fBD' formulation (above).

- Step 3: Render network boxplot visualization

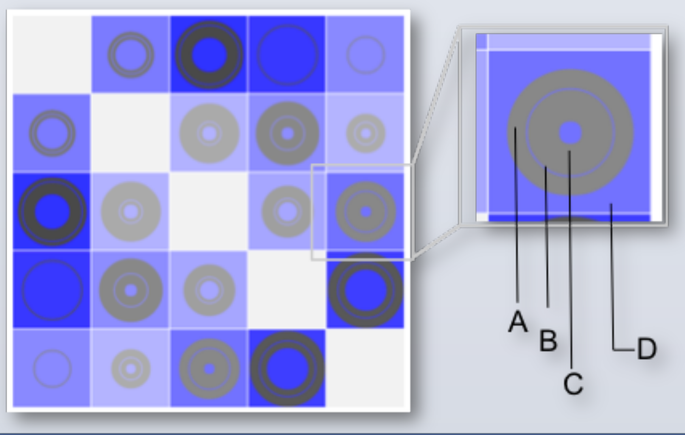


Fig. A network boxplot visualization. A' and 'C' indicate the 50 percentband while 'B' (radius) and 'D' (color) are two different encodings of the median.

FUTURE WORK

- Expert based evaluation of network boxplot based interactive system for analyzing fMRI network data.
- New user study.
  - Larger participant pool.
  - Include task to evaluate ensemble to individual comparison.

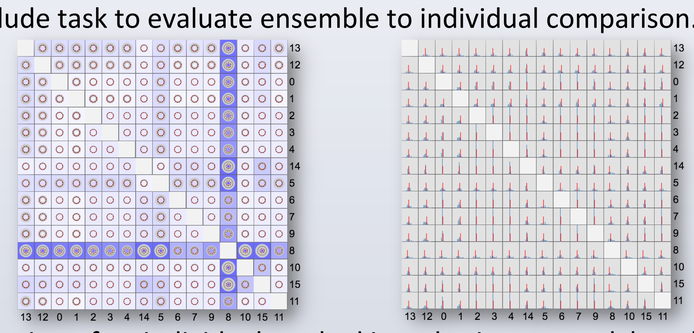


Fig. Representation of an individual marked in red using network boxplot (left) and cell histogram (center).

REFERENCES

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